Specific Preschool Executive Functions Predict Unique Aspects of Mathematics Development: A 3-Year Longitudinal Study

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Abstract

This study assessed the extent to which executive functions (EF), according to their factor structure in 5-year-olds ($N = 244$), influenced early quantity number competencies, arithmetic fluency, and mathematics school achievement throughout first and second grade. A confirmatory factor analysis resulted in updating as a first, and inhibition and shifting as a combined second factor. In the structural equation model, updating significantly affected knowledge of the number word sequence, suggesting a facilitatory effect on basic encoding processes in numerical materials that can be learnt purely by rote. Shifting and inhibition significantly influenced quantity-to-number-word-linkages, indicating that these processes promote developing a profound understanding of numbers. These results show the supportive role of specific EF for specific aspects of a numerical foundation.

Key words: executive functions, number sense, mathematics, kindergarten, primary school

Mathematical development in kindergarten and primary school encompasses a wide range of abilities with increasing complexity: Starting with drawing digits and reciting the number word sequence like a memorized poem, continuing with understanding the linkage between number words, digits and quantities, and concluding with performing mathematical operations like adding and subtracting. Multiple factors determine the ease and the success of this development. A plethora of studies revealed the impact of the visuospatial sketchpad (e.g., Friso-van den Bos, van der Ven, Kroesbergen & van Luit, 2013), IQ and processing speed (e.g., van der Sluis, de Jong & van der Leij, 2007), or phonological awareness (e.g., Krajewski & Schneider, 2009a) on mathematics. One additional main source of influence may stem from the executive functions (EF), a cognitive construct that has recently gained much attention (e.g. Bull & Lee, 2014).

Executive functions in childhood

EF are mechanisms that regulate, coordinate, and control information processing and behavior whenever new or complex tasks are to be accomplished, or when automatized responses have to be inhibited (Miyake et al., 2000). Somewhat earlier in 1996, Baddeley already proposed four specific functions that compose the central executive, that is, the coordination of concurrent task performance, switching of retrieval strategies, selective attention and inhibition of irrelevant stimuli as well as holding and manipulating information in long-term memory. Miyake and colleagues, however, described three basic EF, updating, inhibition, and shifting, which have been shown to be moderately correlated with each other. *Inhibition* is the ability to suppress a previously learnt, dominant response and replace it with a more adequate one. *Shifting* is defined as the ability to flexibly shift between mental sets, rules, and tasks. *Updating* can be understood as the ability to monitor information processing in form of adding relevant and deleting irrelevant information in working memory. Many EF

researchers use the term updating and the term working memory synonymously when referring to the process of holding information in mind and working with it cognitively (e.g., Diamond, 2013, p. 137). In our article we follow this definition that encompasses both the updating and manipulation of information aspects as overlapping processes of working memory and updating, but that does not include inhibition or attention shifting sensu Baddeleys definition of the central executive (Baddeley, 1996). Therefore, in the following literature overview the term updating is consistently used whenever referring to either updating or working memory tasks in the above sense**.**

The performance and structure of EF undergo significant changes from childhood to adolescence. Inhibition develops quite early in childhood and is almost fully developed by the end of the first decade of life, whereas shifting, as the most complex EF, develops later (Garon, Bryson & Smith, 2008). This development is reflected in the structure of EF. Many studies aimed at determining the structure from early childhood to adolescence. The results have been mixed especially concerning the period when EF structure changes from an undifferentiated one-factor to a multidimensional model. Confirmatory factor analyses with data from children between three and six years of age predominantly resulted in a single-factor model representing a general and unspecific cognitive process (e.g., Hughes, Ensor, Wilson, & Graham, 2010; Wiebe et al., 2011). During the transition to school, a more differentiated structure of EF emerges. Confirming their differentiation hypothesis, Lee, Bull, and Ho (2013) showed a two-factor model, with updating and a combined inhibition-switch-factor, in a confirmatory factor analysis for 5- to 13-year-olds, and a full three-factor model only for 15-year-olds. Additional evidence for a two-factor structure in 7- to 9-year-olds was found in the structural equation model of van der Sluis, de Jong, and van der Leij (2007), with the factors shifting and updating, and, in the study of van der Ven, Kroesbergen, Boom, and Leseman (2012), with the updating and combined shifting-inhibition two-factor model.

In sum the findings concerning the differentiation of EF in children shortly before entering school are still rather heterogeneous, with some studies showing a one-factor model (e.g., Wiebe, Espy & Charak, 2008) and other studies showing a two-factor model (e.g. Miller et al., 2013) or even the full three-factor model (e.g., Lehto, Juujärvi, Kooistra, & Pulkkinen, 2003). Methodological differences are discussed to explain this heterogeneity, i.a., the times of measurement, the comprehensiveness of the applied measures as well as the validity of the EF tasks, e.g., the independence of shifting measures from inhibitory aspects (Li et al., 2013).

Executive functions and mathematical abilities

Many other studies aimed at determining the relevance of EF for educational development. The existing research suggests that EF generally play a significant albeit moderate role in the prediction of mathematical abilities in preschool and primary school. For example, the review of Friso-van den Bos and colleagues (2013) showed that mathematics correlated significantly with inhibition $r = .27$, $p < .001$, shifting, $r = .28$, $p < .001$, and updating $r = .34 - .38$, $p < .001$. Bull and Lee (2014) attributed the updating factor, in particular, to play a fundamental role in mathematical development and performance. LeFevre and colleagues (2013) provided empirical evidence for the influence of EF in mathematical development: In their study a common EF-factor consisting of inhibition, shifting, and working memory and updating, respectively, (tested with the backward digit span in addition to measurements of the phonological loop and the visual-spatial sketchpad) predicted concurrent mathematical knowledge and fluency in simple arithmetic tasks in second graders as well as fluency in the same tasks in third grade. Similar results were obtained by Clark, Pritchard, and Woodward (2010) who tested updating, shifting, and inhibition in four-year-olds. Their exploratory factor analysis revealed one common EF-

factor that predicted a significant part of variance in mathematical fluency at the end of first grade.

When using separate EF instead of one general EF-factor, relationships between certain EF and mathematical abilities have been found. Numerical abilities during preschool (e.g., aggregate of counting, subtraction, addition, and geometry) tend to be strongly influenced either by inhibition alone (e.g., motor inhibition task in Blair & Razza, 2007) or inhibition and updating (e.g., the Stroop task and the color span backwards task in Roebers et al., 2011). Miller, Müller, Giesbrecht, Carpendale, and Kerns (2013), however, showed no influence of inhibition but a strong relation of updating (assessed by backward span tasks) on general preschool mathematics.

Mathematical abilities in primary school may be influenced by working memory, respectively updating. In a mediation analysis, Monette, Bigras, and Guay (2011) showed that only preschool updating (tested with verbal and visual-spatial backward span tasks) but not inhibition nor shifting abilities predicted mathematics in first grade. Bull, Espy, and Wiebe (2008), however, concluded from latent growth modeling that good preschool inhibition, superior abilities in using the phonological loop and the visual-spatial sketchpad, and planning abilities constitute the scaffolding for mathematical development in seven- and eight-year-olds. The least agreement has been obtained in the role of shifting in mathematics. Van der Sluis and colleagues (2007) emphasized the role of shifting for flexible switching strategies in complex multistep arithmetic problems. Other studies concluded that shifting is of no significance for mathematical achievement or loses its impact whenever updating and inhibition are taken into account as well (e.g., Blair & Razza, 2007; Monette et al., 2011). Further studies revealed common factors of shifting and inhibition (e.g., van der Ven et al., 2012) so that the specific predictive value of shifting for mathematics is difficult to estimate. Furthermore, other researchers simply did not administer shifting tasks because they argued

that shifting could not be differentiated from inhibition abilities in preschool children (Kroesbergen, van Luit, van Lieshout, van Loosbroek, & van de Rijt, 2009).

It is not yet clear which role specific EF play for different aspects of mathematical development. Either researchers assumed the EF construct to be unidimensional (e.g., LeFevre et al., 2013), or they combined different mathematical abilities to a single factor with the objective to predict general mathematical ability rather than predicting specific mathematical abilities (e.g., Blair & Razza, 2007). One study addressed this issue recently and examined the links between distinct preschool EF and several mathematic abilities, namely, written calculation, arithmetical facts, and mathematical problem-solving simultaneously (Viterbori, Usai, Traverso, & De Franchis, 2015). A confirmatory factor analysis resulted in a two-factor structure with an inhibition and an updating-shifting factor (updating was assessed with the backward digit span task and the dual request span task, shifting with the semantic fluency task and the DCCS). The structural equation model showed that inhibition had no influence on any mathematics tasks. The authors explained this unexpected absence of influence with the nature of the inhibition tasks used in their study, which consisted predominantly of response inhibition rather than interference control, another inhibitory aspect that should be of greater importance for math performance. The structural equation model displayed, however, a strong influence of the updating and shifting abilities on written calculation in first grade and arithmetic fact retrieval and problem solving in third grade. These findings may be interpreted in two ways. Firstly, Viterbori and colleagues assessed updating with numerical (i.e., a backward digit span) and visual-spatial material (i.e., a dual request selective task which involves remembering a starting point on a chessboard while performing a concurrent task on that board) rather than using non-numerical and non-spatial material. Thus, part of the strong influence of the updating-shifting factor on mathematic fact retrieval in third grade may be due to this shared task characteristic.

Secondly, as stated by Viterbori and colleagues, some critical variables were missing in the longitudinal path model, such as early numerical competencies, which have been shown to have a strong impact on mathematical performance in school (Krajewski & Schneider, 2009a). Their inclusion in a path model may alter the contribution of EF (i.a., updating and shifting) on mathematical performance in school notably.

Numerical development and mathematical school achievement

In this context, Krajewski's developmental model of *quantity number competencies* (*QNC*; Krajewski & Schneider, 2009a,b) serves as a suitable approach to describe early numerical competencies. This theoretical model postulates three levels of development through which children acquire a deep understanding of number. On the first level, children learn to discriminate quantities and, independently, learn to recite the number-word sequence, while both components do not necessarily have to be linked together (i.e., basic numerical skills, QNC Level I). Later on, children recognize that number words are associated with quantities and, subsequently, they can arrange number quantities along the number-word sequence, initially, in rough categories like two corresponding to "a bit" or ten being "much" and, later on, naming the exact number along the number-word sequence more precisely (i.e., quantity-number concept, QNC Level II). On the third level, children understand that parts of a quantity as well as differences between two quantities can be described with precise number-words representing a third quantity (concept of number relationships, QNC Level III). Early numerical abilities, as defined in the QNC-model, have been empirically shown to be strong predictors of later mathematical school achievement. Especially the insight of the link between quantity and number-word representations (QNC level II) is important for further mathematical development (Krajewski & Schneider, 2009b).

Processing speed and mathematical abilities

As listed above, a number of variables are of concern for mathematic development. One more variable of particular interest is the speed of information processing because of its influential role in the prediction of mathematical school achievement (Krajewski & Schneider, 2009b) on the one hand *and* its overlap with EF performance on the other hand (Salthouse, 2005; van der Sluis et al., 2007). Already Miyake and colleagues (2000) quote executive tasks involve other cognitive processes. In the study of van der Sluis and colleagues (2007) a substantial amount of variance in EF tasks is explained by non-executive processing demands of applied measures such as naming speed. In addition, Passolunghi and Siegel (2001) and Geary (2005) suggest that a slower access to number representations stored in long-term memory may diminish the immediate recall of numerical information in children with arithmetic disabilities. Besides, for mathematical problems that can be solved through fast retrieval of arithmetic facts stored in long-term memory, the amount of variance explained by the speed of information processing as compared to the amount of variance explained by EF has to be clearly determined (van der Sluis et al., 2007).

Hypothesized influences of executive functions on mathematical development

In our literature review we referred to the empirical evidence that executive functioning correlates with and predicts mathematical performance in preschool and school. We argue that further research is needed to more closely investigate how specific components of EF predict specific mathematical competencies. It is the central aim of the present study to answer this research question.

The influence of EF on mathematics as described above can best be understood when looking at the processes that underlie numerical information processing. Thus, our hypotheses are based on the following considerations.

Learning the number-word sequence *(Basic numerical abilities QNC level I)* does not only require the repetition of single number-words but also needs constant *updating* of the relevant segment of the number-word sequence in working memory for the correct reproduction. For instance, when learning to name the preceding numbers of the number-word "six", it is initially necessary to activate the sequence "four-five-six", as well as to keep reciting the words in that sequence. Simultaneously, this sequence has to be turned in a backward order, namely "six-five-four", and then the single number words, "five" or "four", can be separated from that backward sequence. As updating is defined as a process of adding and deleting information in order to be left with merely relevant information (Miyake et al., 2000), it can be hypothesized that updating plays an important role in developing a correct long-term representation of the number-word sequence.

Moreover, after activating the number word "six" for backward reciting it is further irrelevant for recalling the preceding number-word "five" and needs no more attention. Being a novice in counting, it may need deliberate *inhibition* to keep irrelevant information out of working memory, a procedure which reminds of Baddeleys definition of one executive function, that is, the selective attention to one stimulus while inhibiting disrupting effects of other stimuli (Baddeley, 1996).

Understanding the concept of number and the link to their respective quantities (*Quantity to number-word linkage, QNC level II*) is more than the mere connection of number-words and their quantitative meaning. First of all, when exposed to numerical situations, the child has to focus on numerosity and, at the same time, block out irrelevant and potentially interfering characteristics like color, type, and expanse. Chan and Mazzocco (2017) found that the salience of competing features affects responses to number. Thus, focusing on numerosity should be easier when a child has superior abilities of *inhibiting* competing (non-numerical) information and *shifting* to relevant (numerical) information even

if alternative, salient stimuli are present. Hence, a deeper understanding of numbers requires identifying and holding in mind all relevant information. Moreover, it requires shifting flexibly between all three types of number representations: Spatial quantities, phonological number-words, and visual digits (Dehaene, 1992). McLean and Hitch (1999) demonstrated that children with arithmetic difficulties show deficits in shifting between retrieval plans, a finding that may apply for shifting between different numerical representations as well.

This line of arguments can be extended to the influence of EF on *mathematical school achievement*. The more numerical information is needed to be processed and updated, and the more long-term memorized fact knowledge and consecutive steps are required in solving a mathematical problem, the more deliberate inhibition of competing but irrelevant details and shifting between concurrent information and procedures is necessary. These prerequisites concern many of the tasks that are commonly set to measure school achievement in the regular curriculum. Arithmetic tasks in school demand *shifting* between different calculation strategies and sub-solutions in multi-step procedures (e.g., van der Sluis et al., 2007), *inhibiting* prepotent, but task-irrelevant information, e.g., when only paying attention to parts of information at one time (McLean & Hitch, 1999), and inhibiting automatized strategies in favor of more adequate new ones (Bull & Sherif, 2001). We, therefore, hypothesize that inhibition and shifting may not only influence the understanding of quantity-to-number-word linkage (QNC level II), but also affect mathematic school achievement. Furthermore, *updating* may help keeping information in mind and activating information from long-term memory in order to combine current and retrieved information during the calculation procedure. This consideration is supported through the finding that children with mathematical disabilities showed impairments in holding and manipulating information retrieved from long term memory (McLean & Hitch, 1999). Further on, in the study of Toll, van der Ven, Kroesbergen and van Luit (2010), working memory (measured by a keep track

task, odd one out task and digit span backward task) resembled a powerful predictor for mathematical difficulties in first and second grade.

Sufficient *updating* skills, however, may also play an indirect role for mathematical school achievement via enhanced encoding processes and resulting *arithmetic fluency*. To build a strong and easily retrievable association between the addition problem "3+4" and its result "7", for example, the information has to be sufficiently repeated and linked in working memory. In line with this argument, Viterbori and colleagues (2015) found that lower working memory leads to weaker associations between an arithmetic problem and its specific result. Since updating facilitates holding only relevant information in working memory, it can be implied that it may promote the association between a specific task and its specific result and, subsequently, reduce the number of repetitions in working memory needed to store mathematical fact knowledge in long-term memory. For that reason, it can be hypothesized that performance on tasks that can be solved through knowledge learned by rote, like simple addition problems from zero to ten (i.e., arithmetic fluency), are influenced by updating ability. Arithmetic fluency, that is, an easy and fast retrieval of information in long term memory, in turn, should promote school achievement to the regular math curriculum (Krajewski & Schneider, 2009b). Accordingly, an indirect influence of updating on mathematic school achievement is to be expected. As it is not necessary to deal with complex (relevant and irrelevant) information to retrieve simple addition facts, we hypothesize that inhibition and shifting may be of lesser significance for arithmetic fluency.

Last but not least, because of the above described relevance of processing speed for both EF and mathematics, we also include a task to assess the speed of naming well-known objects and dice into our statistics. Although we expect an influence of naming speed on fluently retrievable mathematical facts (number word sequence and simple addition

problems), in particular with regards to the numerical material (i.e., naming of dice), we hypothesize that EF nevertheless remain equally influential in the aforementioned ways.

Summary

The construct of EF promises to account for a significant amount of variance in mathematical abilities because focusing and shifting attention as well as updating relevant information are essential for higher cognitive activities. A number of studies showed an influence of EF on mathematical abilities. These studies focused on the influence of either *separate* EF on *general* mathematic abilities or *general* EF on *specific* mathematic abilities, with the influence varying as a consequence of whether the structure of EF was found to be uni- or multidimensional. The current state of research is, therefore, limited in two ways: Firstly, the factor structure of EF in five-year-olds still needs confirmation. Secondly, to our knowledge, there are no studies which investigated the possible differential impact of separate EF on separate mathematical abilities throughout mathematical development such as early quantity number competencies on different levels (i.e., number-word sequences, level I, and quantity to number-word linkage, level II), mathematical fluency (i.e., speeded calculation of simple addition problems in the range up to ten), and mathematical school achievement in the regular math curriculum. This shortage leads to the following two questions: Is the factor structure of EF in five-year-olds still unidimensional or do two or even three separate but related functions exist? Do separate EF in preschoolers (according to the given factor structure) predict later basic and higher quantity-number competencies, arithmetic fluency, or mathematic school achievement differently?

Method

Participants and Design

Two-hundred-and-sixty-two kindergarten children were recruited for a longitudinal study with parental consent and without any particular selection criteria from 34 kindergartens in both urban and rural areas in middle Germany. Concomitant with school enrollment the children placed across 46 schools. They were tested twice a year during kindergarten and first grade and once at the end of second grade (October 2011 - July 2014). Specifically, testing occurred at the beginning of their last year in kindergarten (T1: M_{age} = 5 years; 8 months, SD = 3.5 months, 126 boys, 118 girls), at the end of kindergarten (T2: M_{age} = 6;3), the beginning of first grade (T3: M_{age} = 6;8), the end of first grade (T4: M_{age} = 7;3), and at the end of second grade (T5: M_{age} = 8 years;3 months; SD = 3.6 months, 105 boys, 87 girls). Participation drop-outs were reported to be due to delayed (N=14) and preterm school enrollment (N=7), relocation (N=16), and withdrawal from participation for personal reasons (N=33). The majority of participants were German native speakers (88%) and the parents' level of educational as assessed during the last year of kindergarten ranged from low (father; mother: 17.3%; 10.3%) to middle (26.8%; 37.1%) and high (52.6%; 50.7%).

Materials and Procedure

During kindergarten, testing was divided into four sessions (30 min each), distributed across one or two weeks and administered individually in a standardized order. During primary school, testing occurred in three sessions distributed over one or two weeks depending on the organizational conditions of the school (3x45 min or 1x45 min + 1x90 min) and was administered in small groups of three to eight children. Due to time constraints, both the inhibition and shifting tasks were only administered at T1 while the updating tasks were administered only at T2 seven months later.

Inhibition. Two tasks served as indicators of inhibitory ability. In the Stroop task (Jansen, Mannhaupt, Marx, & Skowronek, 2002), children were shown a sheet with six rows

and a total of 24 pictures of wrong colored fruits and vegetables (e.g., a blue tomato). Children were asked to name the true color of each fruit or vegetable as quickly as possible. The dependent variable was the sum of correct answers divided by the time taken on the task.

In the Knock-and-Tap task (Korkman, Kirk, & Kemps, 1998), the experimenter demonstrated three different moves with his hand (i.e., a knock, side fist, or tap on the table). Children were instructed to inhibit the impulse to imitate the experimenters' move, but to respond instead with a different move of their hand: the knock should be countered with a side fist, the side fist with a knock, and the tap with no hand movement at all. The task consisted of two test phases with 12 items each. The dependent measure was the sum of correct trials.

Shifting. The first task used for indicating shifting ability was the Dimensional Change Card Sort (DCCS; Zelazo, 2006). The experimenter displayed one card at a time with either a blue boat or a red rabbit on it. In the first trial, cards had to be sorted according to their shape (boat vs. rabbit), and in the second trial according to the color of the objects (red vs. blue). In the third trial, children had to shift flexibly between the two sorting rules depending on a third feature: If the card had a black border, it had to be sorted by the objects' color, and if it had no border, it had to be sorted by the objects' shape. To ensure that the children did not forget the sorting rules, the rules were repeated by the experimenter before each trial. The sum of the correctly sorted cards in the third trial, consisting of 12 items, served as the dependent variable.

In the Auditory Attention and Response Set task (Korkman et al., 1998), children listened to a series of 180 monosyllabic words like "house," "milk," "wind," and the color words "red," "blue," "yellow," "black," and "white" in a mixed order. Whenever they heard the color word "red" they had to point to the red spot on a sheet with four colored spots in the first trial. In the second trial, the instruction changed to include a shifting element. Here,

children had to point to the yellow spot when hearing the color word "red," to the red spot when hearing "yellow," and the blue one when hearing "blue." The dependent measure was the sum of correct answers in the shifting condition.

Updating. In the computerized Picture Memory task (analogous to the Letter Memory task used in Miyake et al., 2000), children were shown one of eight different pictures at a time (i.e., monosyllabic black-and-white objects like a ball or a house). The picture was displayed for two seconds before it disappeared. After a five-second pause a new picture emerged and was displayed for another two seconds. During the following pause, preceding the next picture, children had to name the last two objects that had just appeared. As there were eight pictures in each sequence, children had to update and name two pictures for seven times (e.g., displayed sequence of pictures: house - ball - ice - tree - …; correct answers: "house – ball;" "ball – ice;" "ice – tree;" …). The dependent variable was the sum of correctly named trials for a total of three blocks.

The computerized Complex Object Span task (Hasselhorn et al., 2012) involved remembering a sequence of pictures (e.g., an apple, a candle, a piece of cheese) in the correct order. The pictures were presented one at a time for two seconds each, after which children were distracted in that they had to decide whether the shown object was eatable or not. The test length varied as a function of the correct answers by the child and ended when two trials of sequences with the same number of pictures had been recalled incorrectly. The maximum sequence length consisted of five items. The outcome measure was the sum of correctly recalled sequences.

Quantity Number Competencies Level I (QNC I). To assess basic numerical skills, that is, number words isolated from quantities, children were asked four times to name the following number word for a given number, and to recite the number-word sequence from one onwards. The procedure was stopped by the experimenter, when a child reached the

number word thirty-one. In addition, there were four items that asked children to name the preceding number word of a given one, and to count backwards from five to zero (MBK 0; Krajewski, in press). This knowledge about the number word sequence was assessed at the end of kindergarten (T2).

Quantity Number Competencies Level II (QNC II). To assess the level of insight in quantity to number-word linkage, the following four tasks that are part of the test of basic mathematical competencies (MBK 1; Ennemoser, Krajewski & Sinner, in press) were administered at the beginning of first grade (T3). Every child was given a workbook with either Arabic numerals or pictorial illustrations. All instructions were given orally and the dependent variable was the correct number of each trial.

In the Number Comparison task, children were presented with two digits (e.g., 5 and 3) and were instructed to circle the digit that represents the smaller quantity. This task consisted of four items.

The Numerical Seriation task consisted of three items presenting an incomplete sequence of objects with increasing quantities, such as a hand with one, two, three, or more outstretched fingers. In every sequence, one picture of the sequential pictures was left out (e.g., a hand showing three fingers) and the children had to mark the picture that presented the missing element from a set of suggested pictures (e.g., hands showing three, five, or eight fingers).

The Number-Line task required children to mark the correct position of a given number on a number line. Four trials involved a number line ranging from zero to ten and one number line ranged from zero to 100. Answers within a certain error range (max. deviation of 15%) around the actual position were rated as a correct answer.

In the Number Concept task, children were asked to draw a certain number of objects as well as compare the quantity of various elements on pictures (e.g., boxes with five, six and

seven apples and a box with six apples that were spread further apart) and find the one with the highest quantity. There was a total of four trials.

The validity of the administered quantity number competencies tasks is documented in a number of studies. For example, Krajewski and Schneider (2009a) showed in one of their longitudinal studies that basic numerical skills (QNC level I) and higher quantity number competencies (QNC level II), assessed at preschool, correlated with mathematic school achievement in Grade 3 with $r = .64$ and $r = .62$, respectively. Furthermore Krajewski and Schneider (2009b) demonstrated that children with mathematical difficulties in grade 4 could have been distinguished from typically performing children by their weak quantity number competencies, measured with the MBK 0, in kindergarten. Similarly, performance in the MBK 1 correlates with mathematics attainment in second grade ($r = .71$) and teachers' judgement of mathematics abilities in fourth grade ($r = .60$; Ennemoser et al., in press).

Arithmetic fluency. In this task, children had to solve as many problems as they could within 40 seconds on a worksheet with twenty single-digit addition problems (e.g., 3+2; 2+7). It was administered at the end of first grade (T4) and the dependent variable was the sum of correct trials.

Mathematics school achievement. Mathematical school achievement was assessed at the end of grade 2 (T5) with the "Deutscher Mathematiktest für zweite Klassen," a measure for school children in second grade that is representative for the mathematics curriculum in Germany (DEMAT 2+; Krajewski, Liehm, & Schneider, 2004). It includes the measurement within three mathematical domains: Arithmetics (e.g., subtraction, addition, and division), applied arithmetics (e.g., comparison of lengths and calculation problems with money and text problems), and geometry. The workbook provided children with tasks of Arabic numbers, pictorial representations, and text problems for the calculation, geometry, and word problems, respectively. Instructions were given verbally by the experimenter for each subtest

and the time specifications were being obeyed. The sum of correct trials served as the dependent variable.

Naming speed of objects and dice. At T1, children were asked to name 18 familiar objects (e.g., house, tree, ball) and 18 dice (numbers from one to six), as quickly as possible to assess the speed of retrieval from long-term memory for verbal and numerical material. The objects and dice were presented on separate worksheets and in two rows consisting of nine items each. Time on task was measured with stopwatches and documented in seconds; milliseconds. The dependent variable was the sum of correctly named items divided by the time children took to name the 18 objects or dice.

Results

Descriptive Statistics

On average, 15.6% of the data points were missing for each time of measurement, with lower percentages at T1 (11%) and a higher ones at T5 (28%). A complete dataset was obtained for only 170 children. At first, we conducted our analyses with the raw data of all children, irrespective of missing data points, and then imputed those by using the regression imputation function with AMOS 22 (Arbuckle, 2013). Finally, we used the covariance matrix to check for equivalences in our findings. Analyzing the data with the complete data set only would have yielded in reduced power due to a minimized sample size and in a potential bias due to specific drop outs.

Descriptive statistics and the results of the correlational analysis can be found in Table 1. The reliabilities of the tasks used in this study ranged from .34 and .92, indicating, at least, sufficient internal consistency for the majority of tasks. Although the Number Concept task showed poor reliability, it was still considered in the analyses because of its content-related relevance and because it has generally been found to be a valid indicator for numerical

development. The results of the Kolmogorov-Smirnov-Z-Tests yielded significant results for all tasks except for the Stroop and the Picture Memory task. Yet, for all measures except for the Knock-and-Tap task, which proved to be too easy, the curtosis values did not exceed the limit of seven as requested by West, Finch, and Curran (1995).

Due to this ceiling effect, the correlations between the Knock-and-Tap task and other EF tasks were either low (correlation with the Auditory attention and response set task $r = .13$, $p < .05$), or not significant. Correlations within the other EF tasks ranged from $r = .15$ ($p < .05$) to $r = .44$ ($p < .01$). Moreover, the correlation between the two QNC level I tasks was high ($r = .60$, $p < .01$) whereas the correlation within QNC level II tasks again ranged from $r = .21$ to $r = .44$ ($p < .01$). There were also significant small to moderate crossover correlations between the Stroop task and the shifting tasks and mathematics, as well as moderate correlations between the updating tasks and mathematics.

---Table 1---

Confirmatory Factor Analysis

Three models were tested to examine the structure of preschool EF. First, a three-factor model consisting of the correlated but independent latent factors inhibition, shifting, and updating was examined. Second, a two-factor model with a combined inhibition and shifting factor and a separate updating factor was considered (with correlations between inhibition and shifting being constrained to one and their correlations to updating being set equal). Third, an undifferentiated model with all three constructs loading on one factor (with correlations between the three factors being constrained to one) was tested. A nested-model approach was applied to compare the three-factor model with the other models while controlling for age at T1.

Of the three selected models, the undifferentiated EF-model with correlations between all factors constrained to one displayed the poorest fit (see Table 2). Both the two- and the three-factor model showed very good model fit indices that did not differ significantly from each other in their chi square values. Yet, the two-factor model resulted in a slightly better fit and the use of a covariance matrix or imputed data did not yield different results.

--- Table 2 ---

Figure 1 depicts the factor loadings and correlations of the three-factor and the two-factor model. The correlation between the updating and the combined inhibition and shifting factor was strong for both models, yet, there was still about 67% of unshared variance left. The correlation between the two latent factors inhibition and shifting was very high ($r = .96$) in the three-factor model, which led us to infer that the two-factor EF-model with a separate updating and a combined inhibition and shifting factor would be the most plausible fit for this data set. This result answers our question concerning the EF structure in that EF are already differentiable EF in preschool.

--- *Figure 1* ---

Prediction of Mathematics through Executive Functions

We specified a structural equation model with interrelations among updating, inhibition and shifting, QNC skills on level I (number word sequence) and level II (quantity to number-word linkage), arithmetic fluency, and mathematic school achievement. Naming speed of objects and dice were included as additional predictors and in order to control for processing speed.

In the *initial model*, we estimated all paths from EF and naming speed on basic numerical skills to mathematical school achievement. Table 1 shows the factor loadings,

which ranged from $\lambda = .33$ to .79 within the EF constructs, from $\lambda = .78$ to $\lambda = .83$ within QNC level I, from $\lambda=.52$ to $\lambda=.70$ within QNC level II, from $\lambda=.88$ to $\lambda=.92$ in the school mathematics test and with $\lambda = .47$ in the geometry task. The initial model fit the data well ($\chi^2(110) = 133.994$; $\chi^2/df = 1.218$, $p = .060$; $CFI = .981$; $RMSEA = .029$). In a second *restricted model*, all twelve paths with non-significant beta weights smaller than .10 were set to zero. This restricted model is shown in Fig. 2 and proved to have an even better fit ($\chi^2(122) = 139.032$; $\chi^2/df = 1.140$, $p = .139$; $CFI = .987$; $RMSEA = .023$). These two models differed slightly in two beta-weights: While the path from quantity-number concept to arithmetic fluency did not reach significance in the initial model ($\beta = .42$, $p = .126$), it did so in the restricted model ($\beta = .50$, $p < .001$). However, the magnitude of the beta-weight for the path from inhibition and shifting to quantity-number concept (QNC level II) was slightly higher in the initial ($\beta = .51$, $p = .004$) than in the final model ($\beta = .39$, $p = .001$).

--- *Figure 2* ---

The *restricted model* depicts the overall mathematical development. The ability to count forward and backward (QNC level I) strongly predicted the quantity-number concept (QNC level II; $\beta = .64$; $p < .001$), which, in turn, contributed strongly to mathematic school achievement ($\beta = .73$; $p < .001$), and arithmetic fluency ($\beta = .50$, $p < .001$), while arithmetic fluency itself had no significant influence on mathematic school achievement (estimate in the initial model: $\beta = .04$, $p = .628$).

Furthermore, the restricted model shows specific influences of EF on different mathematical skills. Updating influenced the ability to count forward and backward (QNC level I; $\beta = .44$, $p < .001$) but did not influence higher numerical abilities (estimate in the initial model: QNC level II; $\beta = -.10$, $p = .511$; Arithmetic fluency; $\beta = .11$, $p = .388$; Mathematic school achievement $\beta = .09$, $p = .504$). Inhibition and shifting, however, showed

a moderate association to the quantity-number concept (QNC level II; $\beta = .39$, $p = .001$), but no significant effect on basic numerical skills (estimate in the initial model: QNC level I; $\beta = .02$, $p = .885$), arithmetic fluency (estimate in the initial model: $\beta = -.02$, $p = .909$), or mathematic school achievement (estimate in the initial model: $\beta = -.03$, $p = .901$).

Naming speed of dice influenced arithmetic fluency ($\beta = .23$, $p < .001$) and basic numerical abilities (QNC level I; $\beta = .38$, $p < .001$), whereas naming speed of objects had no significant influence on the mathematic domain.

Due to missing values in the raw data, the restricted model was computed again by using imputed data as well as the covariance matrix. Using imputed data changed the model fit only marginally ($\chi^2(122) = 152.676$; $\chi^2/df = 1.251$, $p = .031$; $CFI = .985$; $RMSEA = .031$) without any change in the magnitude of the effect or the significance level of the beta-weights. Using the covariance matrix yielded a comparable model fit ($\chi^2(122) = 143.332$; $\chi^2/df = 1.175$, $p = .091$; $CFI = .980$; $RMSEA = .030$), but lowered the magnitude of the beta-weight of the QNC level II on mathematic school achievement ($\beta = .50$; $p < .01$).

In one last step we extended the restricted model with a manifest variable in order to control for the impact of age. Again we found only a marginal change in model fit ($\chi^2(132) = 148.188$; $\chi^2/df = 1.123$, $p = .159$; $CFI = .987$; $RMSEA = .021$), and a slightly higher influence of inhibition and shifting on QNC level II ($\beta = .42$; $p < .01$). Age at T1 correlated only with naming speed of dice ($r = .23$; $p < .01$), and inhibition and shifting ($r = .32$; $p < .01$). The correlation between age and updating turned out to be non-significant ($r = .15$, $p = .062$).

In sum, these results met our hypothesis, that different EF might play different roles for various mathematical abilities. In detail, updating predicted knowledge of the number word sequence (QNC level I), whereas the combined inhibition and shifting factor was relevant for understanding the number concept (QNC level II). In contrast, when controlling for early numerical competencies, neither the inhibition-shifting factor influenced QNC level

I, nor updating was relevant for arithmetic fluency, nor any EF factor was directly predictive for school mathematics. As expected, numerical naming speed (naming dice), but not object naming speed was associated with both the knowledge of the number word sequence and arithmetic fluency.

Discussion

The major aim of this study was to investigate the predictive value of specific EF on various aspects of mathematical abilities. Therefore, we applied an EF test battery in preschool and initially analyzed the EF factor structure in our sample. Further, we used several measures of mathematical abilities that were applied longitudinally during preschool to second grade, and assessed the EF factors' contribution to later mathematical achievement.

Two-factor structure of preschoolers' executive functions: Updating vs. inhibition and shifting

Our first question concerned the structure of EF in five-year-olds. We aimed to provide further support either for already emerged separate but related EF (e.g., Lee et al., 2013), or of an undifferentiated executive control process (e.g., Wiebe et al., 2011). According to the developmental course of inhibition and updating in kindergarten and preschool (Garon et al., 2008), a multi-dimensional model was to be expected. A confirmatory factor analysis with our data revealed a two-factor structure. Moreover, the correlations and factor loadings corresponded to those of previous research (e.g., van der Sluis et al., 2007) and displayed a separate yet correlated updating factor. Inhibition and shifting were so strongly correlated that the presumption of independent inhibition and shifting factors did not hold true for the preschoolers in our study. Notably, unlike Wiebe and colleagues (2011) and Hughes and colleagues (2010) who found only a one-factor EF

structure, the present study included shifting tasks in addition to measures of inhibition and updating.

Moreover, in contrast to other studies, we applied only a few rules in our two shifting tasks and reduced the influence of verbal working memory by regularly repeating the sorting rules during the shifting phase of the DCCS task. This procedure may have raised the likelihood that separate EF factors were discovered. However, we assume that inhibitory processes may have facilitated the performance in shifting tasks (see Garon et al., 2008; van der Ven et al., 2012). On the one hand, this influence might be caused by shared task requirements, as for example, shifting to a new rule is easier when the old rule is quickly and reliably inhibited. On the other hand, the relation between inhibition and shifting might be caused by the children's level of development. While inhibition emerges very early in life the more complex shifting ability develops later on (Garon et al., 2008).

Replication of earlier findings in the math domain: Importance of specific precursors

Our second question concerned the prediction of mathematic development. Here, the focus was to clarify if the individual preschool EF (i.e., updating, inhibition and shifting) predicted basic and higher quantity-number competencies, arithmetic fluency, and mathematic school achievement. The broad assessment of mathematical abilities throughout the ongoing numerical development and the identified two distinct EF factors allowed a detailed view on their interrelations.

The structural equation model replicated the finding that content-specific precursors, namely, quantity number competencies, are fundamental predictors of mathematical achievement in school (e.g. Jordan, Glutting & Ramineni, 2010; Krajewski & Schneider, 2009a,b). 53% of variance in later mathematic school achievement and 25% of variance in later arithmetic fluency were explained by the understanding of the quantity number-word

linkage in preschool (QNC level II; *ß* = .73 and *ß* = .50, respectively). When controlling for early number competencies, facts learned by rote alone (i.e., arithmetic fluency) were not sufficient for mathematical school achievement (Krajewski and Schneider, 2009b).

Impact of updating on basic numerical skills (Knowledge of number word sequence)

While several studies demonstrated an influence of updating on mathematics in general (e.g., Monette et al., 2011) we investigated this relationship in more detail. The structural equation model displays a relationship between updating and early mathematical development in that updating explained 19% of variance in basic numerical skills (i.e., number-word sequence, QNC level I). This result indicates that updating has a particularly potent effect on a very early level in numerical development, such as learning the number-word sequence, and, therefore, has a vital influence on further mathematical development. The impact of updating on higher numerical abilities (e.g., the quantity-number concept, QNC Level II) and later mathematic achievement was shown to be indirect only, mediated by basic numerical abilities. This finding leads to the conclusion that updating may facilitate encoding processes and, as a result, may promote prompt storing of the correct number-word sequence forwards and backwards in long term memory. Finally, when the flexible use of the number-word sequence has been automatized, or in other words, if one is able to recite it blindfolded, no more updating processes should be needed to maintain number words while linking them to quantities and quantity relations on the next developmental levels. This may explain the missing direct paths from updating to higher levels in the mathematical development.

Kolkman, Hoijtink, Kroesbergen & Leseman (2013), focused on basic EF and its connection to basic mathematical abilities as well. In contrast to our results they found that updating played a more important role than inhibition or shifting for QNC level II (Numerical

magnitude tasks and number line tasks). So they concluded that better updating skills help to acquire a better number understanding. As Kolkman and colleagues did only include higher numerical abilities (QNC level II) in their analyses but no basic numerical skills (QNC level I), their results are not contradictory to our findings. In contrast, our results suggest that the direct influence from updating on number understanding is mediated by a flexible use of the number-word sequence.

It is not yet clear as to why our hypothesis was not met that updating would show a direct influence on arithmetic fluency as well (i.e., rapid solving of simple addition tasks that can learned by rote). Perhaps this might also be due to a mediating effect of the early numerical competencies (QNC levels I and II), which would make updating less directly influential for rapid automatized solving of simple addition tasks.

Impact of inhibition and shifting on higher numerical competencies (Understanding of the quantity to number-word linkage)

In contrast to updating, preschool inhibition and shifting abilities showed a significant influence on the learning of contents that require a conceptual understanding of number. This combined EF factor explained 15% of variance in the performance on tasks measuring the quantity-number concept (QNC Level II) one year later, at the end of grade 1. Grasping the sense of numbers involves shifting between different kinds of information as well as blocking out irrelevant information like shape, color, or size of objects, that means inhibition in order to successfully focus on a setting's numerosity (see Chan & Mazzocco, 2017; Viterbori et al., 2015). Furthermore, we suggest that inhibition and shifting processes are relevant when number words, quantities, and digits have to be flexibly combined at QNC level II.

Inhibition and shifting were not relevant for the learning and utilization of the number-word sequence in our study. This supports the idea that the number-word sequence

26

(QNC level I) can be acquired without a strong necessity to coordinate flexibly between the three different kinds of numerical presentations and, therefore, without a deep insight in the numerical system. Similarly, inhibition and shifting did not directly support arithmetic fluency, indicating that solving simple addition problems in the number space up to ten, can, at least in part, be managed without a deeper understanding. However, what could be implied as a result of our findings is that there might be a mediated effect of inhibition and shifting on the more complex mathematic school achievement through the direct influence of the mediating abilities on QNC level II. Alternatively, the long interval between measurement times may have resulted in the lack of a significant effect of preschool EF on 2rd grade scholastic achievement.

Impact of numerical naming speed on access to numerical knowledge in long-term memory

Naming speed that we included as a variable for basic cognitive ability, showed no direct influence on mathematical school performance but varying influence on the storing of numerical facts in long term memory depending on the type of stimuli. Only fast access to numerical stimuli in long-term memory (i.e., speed of naming the numbers on dice) had a positive effect on the performance of numerical fact knowledge, namely, reciting the number-word sequence forward and backward (QNC level I) and arithmetic fluency (explained variance: 14 % and 15%, respectively). Speed of naming objects, however, correlated moderately to highly with EF but displayed no significant influence on the mathematical domain. This result affirms the finding of Landerl, Bevan, and Butterworth (2004), who found that children with difficulties in mathematics were only impaired in numerical processing but not in access to non-numerical contents in long term memory. Yet, including speed of naming numerical material did not delete the influence of EF on numerical abilities in our study. This result strengthens our conclusion that EF are essential for building a solid

foundation of numerical development, which, in turn, has a significant impact on mathematic school achievement.

Practical implications for supporting children's mathematical learning

There are some directions for practical implication of our findings. Firstly, assuming developmental improvement of EF and their influence on numerical abilities leads to consider EF training. Secondly, taking into account children's limited EF (instead of training it) leads to focus on reducing EF demands in mathematical learning settings.

Results of studies on *EF training* are mixed. As an example, while in a study by Thorell and colleagues (2009) a basic EF training of inhibition with preschoolers improved performance only in trained, but not in transfer tasks, Blair and Raver (2014) focused on complex EF in daily learning activities in kindergarten (reflect, plan, persist and use strategies in learning activities). This approach resulted in improvement of widespread abilities, e.g. of mathematics, especially in children with low SES. Jacob & Parkinson (2015) reviewed that the potential of an EF training to improve concurrent and future achievement is rather small. Further on, they quote that there is still lack of a study that randomly assigns participants to EF trainings and collects the outcome in EF and achievement. Diamond and Ling (2016) state that EF are malleable through training, but transfer to untrained cognitive abilities may only be narrow. They see the inclusion of EF training in the everyday curriculum as a premise for successful transfer, whereas implementation as an add-on only may limit positive effects. Further, they argue that a program's variety of novelty, complexity, and diversity of trained tasks may serve as another key for generalization on other cognitive tasks and long lasting improvement. However, as long as the question of effectiveness of EF training is not homogeneously answered, there may be other practical conclusions we can draw from our results. This leads us to concentrate on the reduction of EF demands in learning settings.

According to our data, firstly, updating is involved in the acquisition of QNC level I. Thus, impaired updating may hinder learning the number-word sequence forwards and backwards and accessing preceding and following numbers in the sequences adequately. It may *reduce updating demands*, if exercises comprise only very circumscribed content and the same content is offered repeatedly until it is absolutely automatized. It should not be sufficient to spend time with teaching the child to count from 1 onwards (e.g., up to 20). Moreover, children have explicitly to be taught to recite the number-word sequence backwards as well. But, most notably, they have to be taught to determine single items in the ascending and descending number-word sequences again and again to convert this in procedural knowledge. Otherwise, children without sufficient updating skills may fail in learning that because their working memory is overloaded with permanently activating and keeping the forward sequence in mind, while, simultaneously, turning it in a backward order to finally separate single items from that backward sequence.

Secondly, we found an impact of inhibition and shifting on the conceptual understanding of number (QNC level II). Thus, impaired inhibition and shifting abilities may retard a child in becoming aware that number words are linked to quantities and quantity relations. To *reduce inhibition and shifting demands* and support children in grasping these principles, creating adequate arrangements of learning settings should be helpful. E.g., when numbers are introduced through storytelling, attention may often be guided to narrative facts. As a consequence, focusing on numeral information is hindered. Therefore, a setting should clearly focus on the numerical aspects to help the child identifying relevant numerical information and suppressing present alternative salient stimuli (see Chan & Mazzocco, 2017). This consideration includes learning material. Material which is reduced to its pure abstract numerical meaning and which avoids seductive and irrelevant details lessens inhibition and shifting demands. In addition, material which serves as a visible representation of the abstract

idea of number will help to generate an adequate inner representation of number. Particularly, to foster a numerical understanding of numbers, all numbers should be represented by the same visual material, e.g., two bricks represent number two, identical three bricks represent number three and identical four bricks represent number four. In this way numbers differ only quantitatively, but not in numerical irrelevant aspects like kind of object, size or color. Hence, children do not have to inhibit irrelevant stimuli and they do not need to shift their attention to relevant numerical information. Moreover, a clear instruction that directs attention only to relevant numerical information reduces inhibitory and shifting efforts.

Limitations of the study

Our study is limited in some ways. One limitation is that we did not include other important predictors like visual-spatial working memory or intelligence. Our sample size turned out to be too small to integrate more control variables into the structural equation model. This is particularly regrettable when recalling the findings of Jacob and Parkinson (2015). In their review the association between EF and achievement was reduced significantly after including child background characteristics and IQ. Regarding our results it therefore stands to reason if the EF measures actually just captured parts of variance of general cognitive ability or socioeconomic status. Nevertheless, we included speed of retrieval from long-term memory to control for overlapping variance with EF performance as recommended by van der Sluis et al. (2007). We found that EF still produced separable variance and kept predictive power.

Furthermore, there are some limitations in our assessment of EF. Firstly, a more continuous assessment of EF would have been preferable for depicting the longitudinal development of EF and their simultaneous association with mathematic school achievement. Indeed, it can be assumed that EF possibly remain a considerable source of influence on

scholastic achievement. Nevertheless, in our study we could show that its influence is not limited to school abilities (e.g., van der Ven et al., 2012) but even starts in preschool. In this way, EF take indirect effects on math at school through early numerical competencies. Secondly, although only seven months apart the administration of different EF tasks at two measurement points is not ideal and may have altered the results. As EF develop rapidly in this age the confirmatory factor analysis may have revealed a different factor solution with data collected at the very same point of measurement. Similarly, the overlapping points of measurement of updating and basic numerical abilities may have raised the observed *ß*-weight between these two latent variables. We approached this methodological shortcoming, which was due to limited time capacities at each point of measurement, by controlling for age in our models.

Contributions to the literature

In summary, this study was aimed at closely investigating the relationship between EF and mathematical abilities. The first main result provides support for a two-factor structure of EF in preschoolers shortly before they entered school. Secondly and more importantly, this study finds an impact of non-domain-specific updating on very basic numerical abilities (i.e., correct utilization of the number-word sequence), as well as an influence of inhibition and shifting on higher numerical abilities (i.e., understanding of the quantity-number concept). While it's well known that children with mathematical learning disabilities usually have very low updating capacities, our results lead to the conclusion that difficulties in updating as well as in inhibition and shifting hinder building a solid fundament of mathematics even very early in development. In terms of practical applications, our findings suggest that fostering numerical abilities early on in children is particularly effective when mathematical education programs use materials and instructions that are explicit, clean of irrelevant details, and focus

31

on the numerical structure of number-words, quantities and digits to make numerical information more salient. Moreover, it is important to automatize numerical facts at all developmental levels. As a consequence, the demands of EF are minimized and the acquisition of numerical understanding can be optimized and facilitated.

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Tables and Figures

Table 1 *Descriptive Statistics, Correlations, and Factor Loadings of all Measures*

		Inhibition		Shifting		Updating		Naming Speed		QNC I		QNC II				Fluency	Mathematics		
	Indicator	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	Stroop (sec)*T1*	-																	
2	Knock and tap *T1*	.09	-																
3	DCCS *T1*	.23**	-.00	-															
4	Auditory Attention and Response Set *T1*	.42**	.13*	.30**	-														
5	Picture Memory Task *T2*	.32**	.12	.17*	.37**	-													
6	Complex memory span *T2*	.20**	.04	.21**	.24**	.44**	-												
7	Naming speed objects *T1*	.44**	.05	.08	.19**	.15*	.17*	-											
8	Naming speed dices *T1*	.50**	.13	.16*	.41**	.44**	.33**	.53**	-										
9	Number word sequence forward *T2*	.22**	.08	.09	.29**	.39**	.33**	.27**	.44**	-									
10	Number word sequence backward *T2*	.29**	.04	.11	.24**	.40**	.33**	.27**	.15**	.60**	-								
11	Number comparison *T3*	.24**	.06	.14*	.28**	.37**	.28**	.22**	.34**	.48**	.52**	-							
12	Numerical seriation *T3*	.24**	-.03	.19**	.28**	.17*	.14*	.14*	.22**	.59**	.23**	.21**	-						

39

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
13	Number line *T3*	.29**	.03	.28**	.29**	.33**	.27**	.21**	.37**	.34**	.36**	.44**	.32**	-					
14	Number concept*T3*	.10	.02	.16*	.21**	.20**	.21**	.14*	.26**	.28**	.26**	.21**	.33**	.32**	-				
15	Arith. fluency *T4*	.35**	.03	.15*	.30**	.40**	.31**	.26**	.53**	.43**	.45**	.37**	.34**	.44**	.29**	-			
16	Arithmetics *T5*	.15*	.08	.16*	.27**	.32**	.28**	.17*	.31**	.41**	.40**	.38**	.29**	.37**	.17*	.41**	-		
17	Applied Arithmetics *T5*	.23**	.13	.15*	.30**	.39**	.27**	.14	.35**	.43**	.41**	.42**	.31**	.41**	.22**	.39**	.72**	-	
18	Geometry *T5*	.15*	.01	.05	.14	.31**	.10	.13	.20**	.20**	.18*	.19**	.20**	.12	.19**	.29**	.34**	.32**	-
	Indicator	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	N	232	242	233	237	238	232	235	237	238	238	226	226	226	226	220	189	191	192
	Minimum	.06	0	2	0	0	0	0.28	0.11	2	0	0	0	0	0	0	0	0	0
	Maximum	.73	24	12	36	21	8	1.65	1.78	7	5	4	3	5	4	20	20	12	4
	M	.36	22.3	7.3	23.2	11.1	2.4	0.83	0.93	5.9	4.0	3.2	1.9	3.3	3.6	12.0	12.0	8.0	1.6
	SD	.13	3.2	2.2	8.2	5.2	1.2	0.22	0.33	1.3	1.4	1.1	1.1	1.5	0.7	4.7	5.2	3.2	1.4
	Factor loadings[1]	.55	.37	.40	.72	.92	.49	-	-	.78	.83	.59	.52	.70	.52	-	.88	.92	.47
	Cronbach's a[2]	-	.81	.58	-	.86	-	-	-	.76	.76	.64	.62	.71	.34	.92	.81	.67	.68
	Skewedness	0.24	-3.61	0.56	-0.73	-0.07	-0.33	0.11	-0.44	-1.05	-1.39	-1.27	-0.55	-0.79	-1.89	-0.30	-0.45	-0.82	.035
	Kurtosis	-0.10	17.00	0.03	-0.18	-0.58	1.66	0.55	0.15	0.248	1.00	0.75	-1.04	-0.36	4.22	-0.27	-0.81	-0.30	-1.13

Note. Spearman-Rho for correlations*; Z* from Kolmogorov-Smirnov test are for normal distribution; [1]Factor loadings in the initial model; [2]Cronbach's alpha for Stroop, Auditory Attention and response set, Complex memory span and Naming speed could not be generated due to variable characteristics (time variable, increasing task difficulty);

* *p* <.05; ** *p* <.01; Kindergarten: *T1*= Time of measurement 1, 5;8 years; *T2*= 6;3 years; School entry: *T3*= 6;8 years; *T4*= 7;3 years, *T5*= 7;8 years.

Table 2

Model Fit Indices of the Confirmatory Factor Analysis

Model	X^2	df	X^2/df	p	CFI	RMSEA	X^2_{Diff}
Raw data incl. missings							
1. Undifferentiated	33.855	9	3.762	.000	.875	.102	1-2: 25.731**
							1-3: 25.794**
2. Two factor	8.125	8	1.016	.421	.999	.008	2-3: .063; *p* =
3. Three factor	8.062	6	1.344	.234	.990	.036	.969
Imputed data							
1. Undifferentiated	37.775	9	4.197	.000	.889	.110	1-2: 29.436**
							1-3: 29.473**
2. Two factor	8.339	8	1.042	.401	.999	.013	1-2: .037; *p* =
3. Three factor	8.302	6	1.384	.217	.991	.038	982
Covariance matrix							
1. Undifferentiated	29.528	9	3.281	.001	.875	.110	1-2: 22.372**
							1-3: 22.458**
2. Two factor	7.157	8	.895	.520	1.000	.000	1-2: .086; *p* =
3. Three factor	7.071	6	1.178	.314	.993	.031	.958

Note. X^2=Chi-squared value; *df*=degrees of freedom; *CFI*=Comparative fit index; *RMSEA*=Root mean error of approximation; **p <.01.
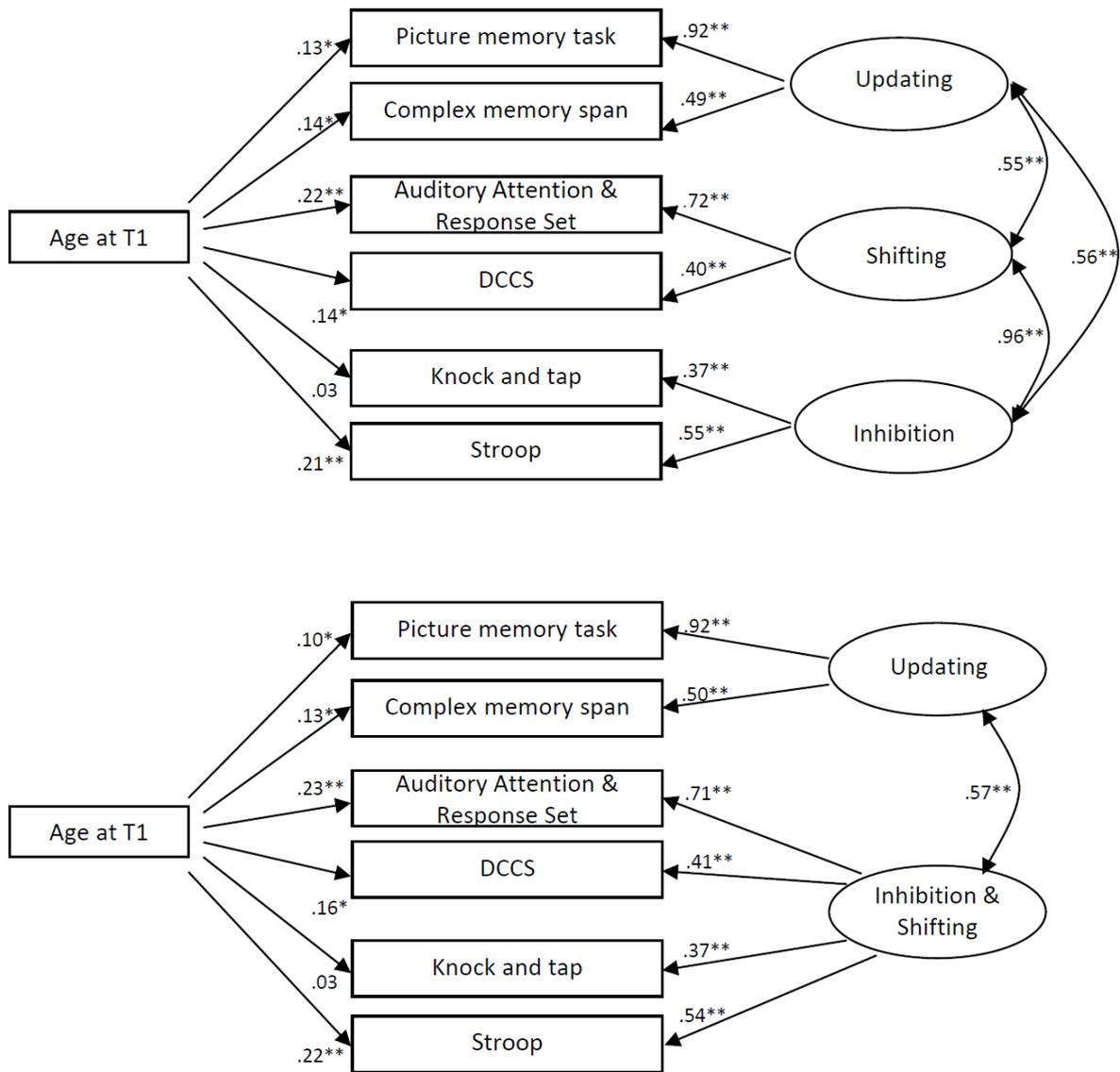
Figure 1. Confirmatory factor analysis: Structure of EF. * = *p* <.05; ** = *p* <.01.
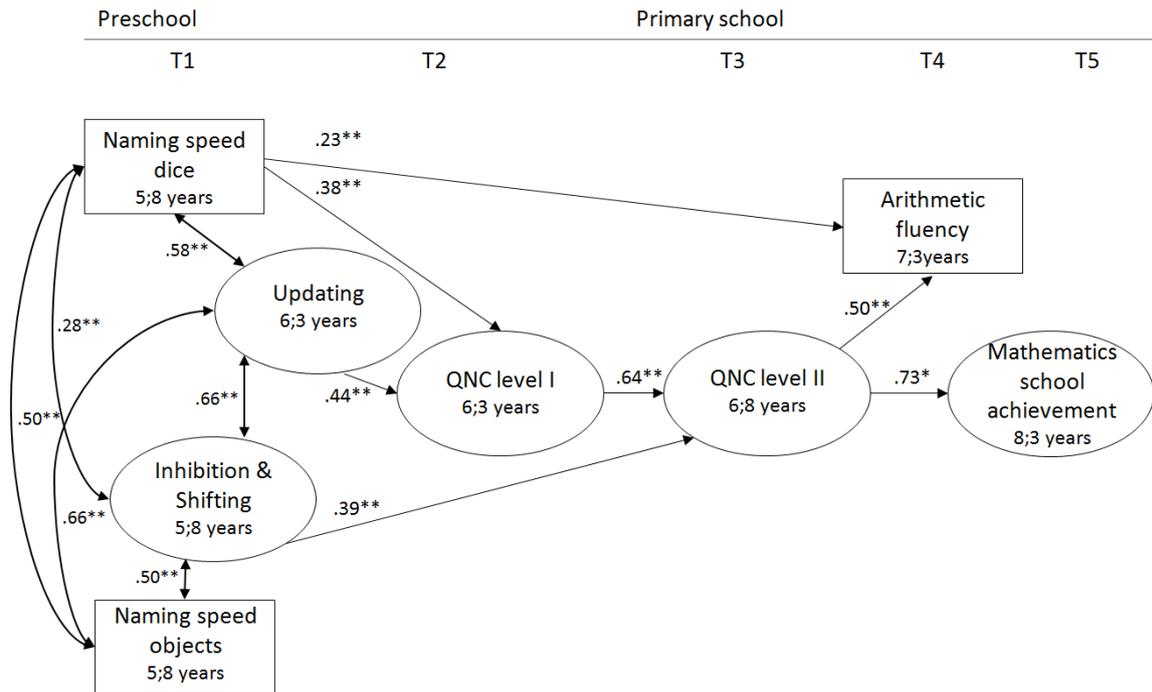
Figure 2. Structural equation model: Influence of EF on mathematical development. Model fit: $\chi^2(122) = 139.032$; $\chi^2/df = 1.140$, $p = .139$; *CFI* = .987; *RMSEA* = .023; * = $p < .05$; ** = $p < .01$.