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WHOLESALE PRICING WITH ASYMMETRIC INFORMATION ABOUT A PRIVATE LABEL*

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A monopolistic manufacturer produces a branded good that is sold to final consumers by a monopolistic retailer who also sells a private label. The costs of the private label are unobserved by the manufacturer, which affects the terms of the contract offered by the manufacturer to the retailer. Given the revelation principle, the manufacturer distorts the quantity of the branded product downwards to learn those costs. The manufacturer can further reduce the retailer's information rent by distorting the quantity of the private label upwards—but this quantity is typically beyond its control. The optimum can nonetheless be achieved when combining a quantity discount with an end-of-year repayment.

I. INTRODUCTION

THIS APPLIED THEORY ARTICLE PRESENTS A mechanism design analysis of the wholesale contract proposed by the monopolistic manufacturer of a branded product to a monopolistic retailer if the retailer also sells a private label, whose costs are, however, unobserved by the manufacturer. Given the revelation principle, the manufacturer can learn the costs of the private label by distorting the quantity of the branded product downwards compared to the complete information benchmark, leaving the retailer an information rent. Based on this result, Yehezkel [2008] showed in a related model with asymmetric information about demand how the manufacturer can further reduce the retailer's information rent by conditioning the contract also on the quantity of the private label. These so-called market share contracts were studied, for example, by Majumdar and Shaffer [2009],

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Inderst and Shaffer [2010], Mills [2010], and Chen and Shaffer [2019]. While in the model of Yehezkel [2008] the quantity of the private label should be distorted downwards compared to the complete information benchmark, the present model with asymmetric information about costs suggests the opposite: The quantity of the private label should be distorted upwards.

This study was motivated by two observations: Brand manufacturers of fast-moving consumer goods have increasingly become subject to competition from private label products over the last decades. Private label "products encompass all merchandize sold under a retailer's brand. That brand can be the retailer's own name or a name created exclusively by that retailer."[1] The availability of private labels has allowed retailers to appropriate a greater share of industry profits, that is, the sum of profits made by retailers and manufacturers. This is the first observation motivating this study.

As the second observation, retail supermarkets have been reported to receive substantial payments from the manufacturers of branded products (Villas-Boas [2007]). Sometimes, those payments are easy to rationalize, such as slotting allowances that provide preferred retail shelf space, or merchandising support (Kim and Staelin [1999]; Klein and Wright [2007]). Those well-explicable payments are often made at the beginning of the period. Other payments are made at the end of the period, and they cannot be rationalized that easily. For example, marketing fees have been found to exceed the retailers' expenses for advertising so that part of those fees lacks an adequate service in return.[2]

Courts, policymakers, and authorities have sometimes expressed skepticism toward payments made by manufacturers to retailers as they might constitute *unfair trading practices*, which "are a collective name for very heterogeneous practices [… such as] receiving benefits without providing adequate services in return'; (European Commission [2017]). They are hypothesized to be particularly harmful because of the increasing bargaining power of retailers. Indeed, end-of-year payments are often paid especially to big retailers, whose bargaining power may have been enhanced by their use of private labels.[3] While these concerns

[1] Private Label Manufacturers' Association. "The Store Brands Story." *https://goo.gl/NrMmEJ* (accessed on 15 September 2022).

[2] Lebensmittelzeitung. "Wege aus der Sackgasse." 20 September 2002, *https://goo.gl/LrMFkP* (accessed on 15 September 2022).

[3] Lebensmittelzeitung. "So schnell geben wir nicht auf." 09 October 2015, *https://goo.gl/pmL186* (accessed on 15 September 2022). Alexander Italianer. "The Devil is in the Retail." Speech held at the conference on the study of *the economic impact of modern retail on choice and innovation in the EU food sector*, Brussels, 2 October 2014, *https://goo.gl/ax7M5J* (accessed on 15 September 2022).

about anticompetitive conduct may be valid in certain cases, the present article suggests that end-of-year payments can have procompetitive effects, too.

To show this, the article extends the seminal model of Mills [1995], who established that competition from a private label reduces double marginalization by strengthening the position of the retailer vis à vis the brand manufacturer, which lowers the wholesale and retail price of the branded good, raises the retailer's profit, lowers that of the manufacturer, and raises consumer surplus. These predictions are in line with the empirical findings of, for example, Draganska *et al.* [2010], Meza and Sudhir [2010], and Narasimhan and Wilcox [1998]. Mills [1999] later extended his model to non-linear pricing, as is in line with evidence provided by Villas-Boas [2007] for the yogurt market in the U.S., or by Bonnet and Dubois [2010] in the French market for bottled water.

Yehezkel [2008], to which the present article relates most closely, extended the Mills [1995] model by analyzing certain effects of asymmetric information. In his model, demand is observable by the retailer but not by the manufacturer (with costs being observable by both firms). The retailer thus has an incentive to understate demand to mislead the manufacturer into believing that the benefit to the retailer of accepting the contract and selling the branded product is low. At the same time, the retailer also has an incentive to overstate demand to mislead the manufacturer into believing that the retailer's profit from selling just the private label is high. Yehezkel [2008] shows that the first effect always dominates the second. Given the revelation principle, the manufacturer can learn demand by distorting the quantity of the branded product downwards. Moreover, if the manufacturer can also control the quantity of the private label, it can diminish the retailer's information rent by distorting the quantity of the private label downwards, too. This causes higher prices and harms final consumers. The market share contract thus has exclusionary effects.[4]

The present model, however, yields a very different result. It assumes asymmetric information about the costs of the private label that can be observed by the retailer but not by the manufacturer. Now, the retailer has an incentive to understate the costs of the private label to mislead the manufacturer into believing that the retailer's profit from selling just the private label is high. The manufacturer would also in this case optimally distort the quantity of the branded product downwards to learn the costs of the private label. But contrary to Yehezkel [2008], the manufacturer would want to diminish the retailer's information rent by distorting the quantity of the private label

[4] The exclusionary effects of market-share contracts were also studied by Chen and Shaffer [2019] in a "naked exclusion' model. By analyzing entry deterrence in a model with stochastic entry costs and a homogeneous good (that is, without a vertically differentiated private label) their study is, however, focused on a setting different from the one analyzed here.

upwards. The higher aggregate quantity benefits final consumers by causing prices weakly below those in the complete information benchmark, as will be demonstrated in this article.[5]

A practitioner might, however, argue that the menu derived from the mechanism design analysis is quite different from contracts observed in the industry. The menu derived from theory entails, for example, a lump-sum payment made by the retailer at the beginning of the period. But such payments have been observed in practice only infrequently (Villas-Boas [2007]). The menu also requires third-party enforcement, which is atypical for contractual relationships in markets for fast-moving consumer goods. The manufacturer must also control the quantity of the private label that, however, is typically beyond the manufacturer's command in practice.

Yet, a deeper investigation reveals that the contracts observed in practice are just variants of the optimal menu that, interestingly, possess some further desirable properties: The application of a quantity discount scheme may help avoiding the lump-sum payment from the retailer to the manufacturer, which is empirically uncommon. Moreover, the pricing function is optimally specified by the manufacturer to serve two further purposes. As its first purpose, the pricing function allows the manufacturer to collect an excess payment that is only refunded to the retailer at the end of the period upon observing the upwards-distorted sales of the private label. The manufacturer can, thus, incentivize the retailer to choose the "correct" quantity of the private label without being able to control it.

Because the quantities of the two products are strategic substitutes, the retailer responds by distorting the quantity of the branded product downwards. Therefore, as its second purpose, the manufacturer can choose the curvature of the pricing function such that it is a best response for the retailer to set the level of the branded product desired by the manufacturer, even if the retailer is formally free to choose also any other quantity. As the optimal menu benefits consumers, this article presents one reason why end-of-year payments may—contrary to the concerns of some courts, policymakers, and authorities—be considered procompetitive under the circumstances analyzed here.

The article is structured as follows. The model is presented in Section II. Section III demonstrates the complete information benchmark before turning to incomplete information about costs. Section IV demonstrates how the optimal mechanism can be implemented realistically. Section V concludes the article. Proofs are provided in the Appendix.

[5] Acconcia *et al.* [2008] study a related model where the upstream manufacturer can neither observe downstream demand nor the retailer's sales efforts. They compare contracts where only sales or sales and the retail price are contractible. Although conceptually related, the specific problem studied by Acconcia *et al.* [2008] differs from the one investigated here. The retailer in their model does not sell a private label so that the firms cannot condition the tariff on its sales, which is central to solving the asymmetric information problem analyzed in the present article.

II. THE MODEL

Consider a static, bilateral monopoly model with one upstream manufacturer and one downstream retailer, as was proposed by Mills [1995]. The downstream retailer sells two vertically differentiated products to final customers. One product is produced by the upstream manufacturer and the other by the downstream retailer; they are thus indexed by u and d. The sales in the downstream, retail market are made at prices p_u, p_d. The products have exogenously determined qualities s_u and s_d with $0 < s_d < s_u$. Hence, product u is thought of as a high-quality, branded product whereas product d is a lower quality private label. The quality differential is defined as $\Delta s = s_u - s_d > 0$. The qualities s_u and s_d are observed by the firms and the final customers.

To specify downstream demand, final customers' preference for quality is measured by the variable ϕ that is uniformly distributed in the interval $\phi \in [0, 1]$ with mass 1. Consumers' indirect utility function for the high-quality product is given by equation (1), and by equation (2) for the low-quality product.

$$v_u = \phi s_u - p_u, \tag{1}$$

$$v_d = \phi s_d - p_d. \tag{2}$$

The demand model was introduced by Mussa and Rosen [1978] and is in line with the discrete choice specifications used in the empirical studies of, for example, Draganska *et al.* [2010] or Meza and Sudhir [2010]. Other than in Yehezkel [2008], who assumed that the demand parameter ϕ is observed by the retailer but not by the manufacturer, ϕ is common knowledge in the present model.

The high-quality product is produced by the upstream manufacturer at constant marginal costs c_u. The downstream retailer obtains the private label at marginal costs c_d with $c_d \leq c_u$ (Inderst and Shaffer [2010]). As an extension to Mills [1995], the marginal costs $c_d(\theta) = \theta$ are a parsimonious function of the retailer's continuous type $\theta \in [0, 1]$. The cost differential is defined as $\Delta c(\theta) = c_u - c_d(\theta) \geq 0$ for all θ. Production does not require fixed costs. The type θ is distributed according to the density function $g(\theta)$ with $g(\theta) > 0$. The cumulative distribution function is denoted by $G(\theta)$ with $G(0) = 0$ and $G(1) = 1$. The variable $H(\theta)$ denotes the inverse hazard rate.

$$H(\theta) \equiv \frac{1 - G(\theta)}{g(\theta)}. \tag{3}$$

The inverse hazard rate $H(\theta)$ is assumed to be non-increasing in θ, as is standard. It is an important element of the optimal tariff if the manufacturer is incompletely informed about θ.

Therefore, let all parameters of the model be common knowledge except for the type θ, which is private information to the retailer. Consider a direct revelation mechanism and the timing of the game as follows:

1. The type θ is realized and observed by the retailer only.
2. The manufacturer (the principal) offers a menu $\langle q_u(\theta), q_d(\theta), T(\theta)\rangle$ to the retailer (the agent) that specifies combinations of the quantity $q_u(\theta)$ of the branded product, the quantity $q_d(\theta)$ of the private label, and a lump-sum payment $T(\theta)$ made at the beginning of the period. The menu includes $\langle 0, \cdot, 0\rangle$, that is, the retailer may choose not to deal with the manufacturer, in which case it decides freely about q_d. As a benchmark, the article also analyzes the menu $\langle q_u(\theta), T(\theta)\rangle$ that is conditional on the quantity of the branded product only.
3. The retailer reports $\hat{\theta} \in [0, 1]$ and receives $\langle q_u(\hat{\theta}), q_d(\hat{\theta}), T(\hat{\theta})\rangle$. Whenever necessary, I will denote the retailer's report $\hat{\theta}$ to distinguish it from the true θ.
4. The retailer chooses the downstream prices p_u, p_d such that the market clears at these quantities. The payments are made, and the profits $\pi_d\left(\hat{\theta}|\theta\right)$ of the downstream retailer and $\pi_u\left(\hat{\theta}|\theta\right) = T\left(\hat{\theta}\right) - c_u q_u\left(\hat{\theta}\right)$ of the upstream manufacturer are realized.

The manufacturer chooses the menu $\langle q_u(\theta), q_d(\theta), T(\theta)\rangle$ pursuing the objective of maximizing (4) subject to (IC) and (IR).

$$\max_{T(\cdot),q_u(\cdot),q_d(\cdot)} \int_0^1 \left[T(\theta) - c_u q_u(\theta)\right] g(\theta) d\theta, \tag{4}$$

$$\pi_d\left(\theta|\theta\right) \geq \pi_d\left(\hat{\theta}|\theta\right) \quad \forall \theta, \hat{\theta} \in [0, 1], \tag{IC}$$

$$\pi_d\left(\theta|\theta\right) \geq \pi_{d,n\ell}(\theta) \quad \forall \theta \in [0, 1]. \tag{IR}$$

Condition (IC) represents the retailer's incentive constraint. If it is satisfied, the retailer prefers $\langle q_u(\theta), q_d(\theta), T(\theta)\rangle$ after reporting the true θ to any other $\langle q_u(\hat{\theta}), q_d(\hat{\theta}), T(\hat{\theta})\rangle$ after reporting an incorrect $\hat{\theta}$. The profit $\pi_d(\hat{\theta}|\theta)$ is shown by (5).

$$\pi_d(\hat{\theta}|\theta) = R(\hat{\theta}|\theta) - q_d(\hat{\theta}|\theta) c_d(\theta) - T(\hat{\theta}). \tag{5}$$

The retailer's revenue $R(q_u, q_d)$ is presented in (6), where the second line makes use of the indirect utility functions (1) and (2). Further information on determining the inverse demand functions p_u and p_d is provided in the Appendix.

$$
(6) \qquad \begin{aligned} R(q_u, q_d) &= q_u \ \ p_u && + \ q_d \ p_d \\ &= q_u \ \left(s_u - s_u q_u - s_d q_d\right) && + \ q_d \ \left(s_d - s_d q_u - s_d q_d\right). \end{aligned}
$$

This function assumes $q_u, q_d > 0$. This assumption will be justified below in this section, where conditions will be presented that ensure positive demand.

In equation (5), the revenue $R(\hat{\theta}|\theta)$ is denoted as a function of $\hat{\theta}$ and θ only instead of being denoted as a function of q_u and q_d, which are fully specified by $\hat{\theta}$ and θ: The manufacturer chooses q_u based on the retailer's report $\hat{\theta}$. If the retailer is free to choose the quantity of the private label, it sets q_d based on the manufacturer's choice of $q_u\left(\hat{\theta}\right)$ and its type θ. The retailer's reaction function (7) is found by maximizing $\pi_d\left(\hat{\theta}|\theta\right)$ w.r.t. q_d.

$$
(7) \qquad q_d(q_u, \hat{\theta}|\theta) = \begin{cases} \frac{s_d - c_d(\theta)}{2s_d} - q_u\left(\hat{\theta}\right) & \text{if } \ q_u(\hat{\theta}) < \overline{q}_u \equiv \frac{s_d - c_d(\theta)}{2s_d}. \\ 0 & \text{otherwise} \end{cases}
$$

The retailer optimally sets $q_d(q_u, \hat{\theta}|\theta) = 0$ if the manufacturer offers a quantity of the branded product weakly above $\overline{q}_u$.

Condition (IR) represents the retailer's individual rationality constraint. The high-quality product is listed if the profit $\pi_d(\theta|\theta)$ of the downstream retailer when selling both products is weakly greater than its reservation profit $\pi_{d,n\ell}(\theta)$ when not listing ($n\ell$) the branded product, that is, selling the private label only. The profit $\pi_{d,n\ell}(\theta)$ is shown in (8).

$$
(8) \qquad \begin{aligned} \pi_{d,n\ell}(\theta) &= \max_{q_d}[R(0, q_d(\theta)) - q_d(\theta)c_d(\theta)] \\ &= \frac{\left[s_d - c_d(\theta)\right]^2}{4s_d}. \end{aligned}
$$

The profit $\pi_i(\theta)$ of a vertically integrated firm (see equation (9)) provides a benchmark when solving the model.

$$
(9) \qquad \pi_i(\theta) = R\left(q_u, q_d\right) - q_d c_d(\theta) - q_u c_u.
$$

Maximizing the industry profit $\pi_i(\theta)$ w.r.t. q_u and q_d gives the optimal quantities $q_u^*(\theta)$ and $q_d^*(\theta)$ as are shown in (10) and (11), as well as the optimal industry profit $\pi_i^*(\theta)$ shown in (12).

$$
(10) \qquad q_u^*(\theta) = 1 - \frac{\Delta s + \Delta c(\theta)}{2\Delta s},
$$

$$
(11) \qquad q_d^*(\theta) = \frac{\Delta s + \Delta c(\theta)}{2\Delta s} - \frac{s_d + c_d(\theta)}{2s_d},
$$

(12) $$\pi_i^*(\theta) = \pi_{d,n\ell}(\theta) + \frac{\left[\Delta s - \Delta c(\theta)\right]^2}{4\Delta s}.$$

Assumption (13) ensures that all relevant equilibrium and off-equilibrium quantities q_u and q_d, which will be explored in this article, are greater than zero, as is proven in the Appendix.

(13) $$\frac{\Delta s}{s_d} c_d(\theta) < \Delta c(\theta) < \Delta s - H(\theta) \quad \forall \theta \in [0, 1].$$

This precludes cases where the retailer maximizes profits by selling only one of the two products. Foreclosure concerns played a great role in the model of Yehezkel [2008] with asymmetric information about demand. In his model, the manufacturer requires the retailer to sell *too little* of the private label. Sometimes, the manufacturer would foreclose the private label altogether even if the product would be sold in the complete information benchmark. It will, however, be shown that in the present model with asymmetric information about the private label's marginal costs, the incompletely informed manufacturer requires the retailer to sell *too much* of the private label. Hence, foreclosure is not a concern in this case, and I will concentrate on a situation where both goods are sold.

III. THE MENUS

Section III(i) establishes the complete information benchmark. Section III(ii) presents the adverse selection problem occurring if the cost type θ is private information to the retailer. The manufacturer must leave the retailer an information rent when offering a menu $\langle q_u(\theta), T(\theta)\rangle$ that is conditional on q_u only. Section III(iii) demonstrates how the manufacturer can diminish the retailer's information rent by offering a menu $\langle q_u(\theta), q_d(\theta), T(\theta)\rangle$ that places an additional restriction on the quantity q_d of the private label.

III(i). *Complete Information*

Assume that the manufacturer observes the retailer's type θ (complete information; indexed by c) and offers $\langle q_{u,c}(\theta), T_c(\theta)\rangle$ so that the retailer earns $\pi_{d,c}(\theta)$ as defined in (14).

(14) $$\pi_{d,c}(\theta) = R\left(q_{u,c}(\theta), q_{d,c}(\theta)\right) - q_{d,c}(\theta) c_d(\theta) - T_c(\theta).$$

The manufacturer optimally sets the fixed fee $T_c(\theta)$ shown in (15) such that the retailer's individual rationality constraint binds in equality $(\pi_{d,c}(\theta) = \pi_{d,n\ell}(\theta))$.

(15) $$T_c(\theta) = q_{u,c}(\theta) c_u + \left[\pi_i\left(q_{u,c}(\theta), q_{d,c}(\theta)\right) - \pi_{d,n\ell}(\theta)\right].$$

This results in the manufacturer's profit shown in (16).

$$\pi_{u,c}(\theta) = \pi_i\left(q_{u,c}(\theta), q_{d,c}(\theta)\right) - \pi_{d,n\ell}(\theta). \tag{16}$$

Because $\pi_{d,n\ell}(\theta)$ is independent of $q_{u,c}$ the manufacturer chooses the quantity $q_{u,c}(\theta) = q_u^*(\theta)$ that maximizes industry profits $\pi_i^*(\theta)$. The retailer uses reaction function (7) to determine the quantity $q_{d,c}(\theta) = q_d^*(\theta)$ as in the vertically integrated situation (Yehezkel [2008]). Assumption (13) ensures $0 < q_{u,c}(\theta) < 1$ and $0 < q_{d,c}(\theta) < 1$.

III(ii). *Incomplete Information: Conditioning on q_u*

Now, assume that the manufacturer does not observe the retailer's type θ. The manufacturer offers a menu $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$ that conditions the tariff on the quantity of one product, that is, q_u only. This situation is indexed $i1$ (incomplete information, conditional on one product). It serves as a benchmark for the situation where the manufacturer controls the quantities of both goods as is analyzed in Section III(iii).

In a direct revelation mechanism, the retailer reports $\hat{\theta}$ and receives the pair $q_{u,i1}(\hat{\theta})$ and $T_{i1}(\hat{\theta})$. It earns the profit $\pi_{d,i1}(\hat{\theta}|\theta)$ shown in (17), where $q_{d,i1}(\hat{\theta}|\theta)$ denotes its best response to $q_{u,i1}(\hat{\theta})$ given its type θ.

$$\pi_{d,i1}(\hat{\theta}|\theta) = R(q_{u,i1}(\hat{\theta}), q_{d,i1}(\hat{\theta}|\theta)) - q_{d,i1}(\hat{\theta}|\theta)c_d(\theta) - T_{i1}(\hat{\theta}). \tag{17}$$

An adverse selection problem arises for all cost types but the lowest. The retailer has an incentive to understate the costs of the private label ($\hat{\theta} < \theta$) and, thus, exaggerate the reservation profit $\pi_{d,n\ell}(\hat{\theta})$, which would result in a lower payment $T_{i1}(\hat{\theta})$. Let $U_{i1}(\hat{\theta}|\theta)$ denote the retailer's additional profits earned over its complete information profits $\pi_{d,n\ell}(\theta)$ in this case.

$$U_{i1}(\hat{\theta}|\theta) = \pi_{d,i1}(\hat{\theta}|\theta) - \pi_{d,n\ell}(\theta). \tag{18}$$

Following the revelation principle, the manufacturer may choose $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$ to induce truth-telling ($\hat{\theta} = \theta$) and minimize the information rent $U_{i1}(\theta|\theta)$, which will be abbreviated as $U_{i1}(\theta)$. Using the functional forms of $\pi_{d,i1}(\hat{\theta}|\theta)$ and $\pi_{d,n\ell}(\theta)$, and applying the envelope theorem, one obtains the marginal information rents as are shown in (19).

$$\frac{\partial U_{i1}(\theta)}{\partial \theta} = q_u. \tag{19}$$

The sign of $\partial U_{i1}(\theta)/\partial\theta > 0$ shows that the retailer's information rent from understating its cost type θ rises in θ. Lemma 1 characterizes the fully revealing menu $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$ chosen by the manufacturer.

Lemma 1. The manufacturer chooses $q_{u,i1}(\theta)$ and $T_{i1}(\theta)$ as are shown by (20) and (21).

$$q_{u,i1}(\theta) = q_u^*(\theta) - \frac{H(\theta)}{2\Delta s}, \tag{20}$$

$$T_{i1}(\theta) = q_{u,i1}(\theta)c_u + \left[\pi_i(q_{u,i1}(\theta), \theta) - \pi_{d,n\ell}(\theta) - \int_0^\theta \frac{\partial U_{i1}(\hat{\theta}|\theta)}{\partial \hat{\theta}} d\hat{\theta}\right]. \tag{21}$$

For all $\theta < 1$, the manufacturer distorts $q_{u,i1}(\theta)$ downwards in comparison to the complete information quantity $q_u^*(\theta)$. The retailer reveals $\hat{\theta} = \theta$ and sets $q_{d,i1}$ according to best response function (7), which gives

$$q_{d,i1}(\theta) = q_d^*(\theta) + \frac{H(\theta)}{2\Delta s}. \tag{22}$$

Proof. See the Appendix. ■

In comparison to the complete information solution, the manufacturer distorts $q_{u,i1}(\theta)$ downwards for all but the retailer's highest cost type $\theta = 1$ (*no distortion at the top*), who demands most of the manufacturer's branded product. The downward distortion of q_u for $\theta < 1$ reduces the retailer's information rent that, however, still takes a positive value for all but the lowest cost type $\theta = 0$ (*no information rent at the bottom*). A downward distortion of q_u was also found by Yehezkel [2008] if the manufacturer cannot observe demand. The predictions of his model and the present one, where the manufacturer cannot observe c_d, differ however perceptibly if the manufacturer can also control q_d, which is shown next.

III(iii). *Incomplete Information: Conditioning on q_u and q_d*

The menu $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$ induces truth-telling ($\hat{\theta} = \theta$). The retailer is, however, free to choose q_d. The firm selects $q_{d,i1}(\theta)$ to maximize its profit $\pi_{d,i1}(\theta|\theta)$ and thereby also its information rent. If, however, the manufacturer is able to control both q_u and q_d through a menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ it can diminish the retailer's information rent $U_{i2}(\theta) \leq U_{i1}(\theta)$ and charge a weakly higher payment $T_{i2}(\theta) \geq T_{i1}(\theta)$, which lowers the retailer's profits $\pi_{d,i2}(\hat{\theta}|\theta)$ as defined by (23).

$$\pi_{d,i2}(\hat{\theta}|\theta) = R(q_{u,i2}(\hat{\theta}), q_{d,i2}(\hat{\theta})) - q_{d,i2}(\hat{\theta})c_d(\theta) - T_{i2}(\hat{\theta}). \tag{23}$$

This result, which was shown by Yehezkel [2008], follows naturally from the fact that the contract space of the menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ is a super-set of the menu $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$.

Yet, it is not obvious how the optimal contract should look like and whether consumers gain or loose from it. For example, Yehezkel [2008] assumed the retailer to possess private information about demand for the two products. In his model, the manufacturer imposes a maximum restriction on the quantity of the private label, or the manufacturer even forecloses the product altogether. This harms consumers. Should we expect a similar result also in the present model with asymmetric information about the production costs of the private label?

Proposition 1 demonstrates the properties of the tariff $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$. The analysis is done under the assumption that the manufacturer has the power to control q_d. Section IV shows under what conditions this may be the case.

Proposition 1. If the manufacturer offers the menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$, it optimally sets the same quantity of the branded product as in the case of $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$, which is shown by (24).

$$q_{u,i2}(\theta) = q_{u,i1}(\theta). \tag{24}$$

The manufacturer, however, chooses a higher quantity for the private label.

$$q_{d,i2}(\theta) = q_{d,i1}(\theta) + \frac{H(\theta)}{2s_d}. \tag{25}$$

It charges $T_{i2}(\theta)$ as defined in (26), so that $T_{i2}(\theta) \geq T_{i1}(\theta)$.

(26)

$$T_{i2}(\theta) = q_{u,i2}(\theta)c_u + \left[\pi_i(q_{u,i2}(\theta), q_{d,i2}(\theta)) - \pi_{d,n\ell}(\theta) - \int_0^\theta \frac{\partial U_{i2}(\hat{\theta}|\theta)}{\partial \hat{\theta}} d\hat{\theta}\right].$$

The fully revealing mechanism $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ diminishes the retailer's information rents ($U_{i2}(\theta) \leq U_{i1}(\theta)$) with

$$U_{i2}(\hat{\theta}|\theta) = \pi_{d,i2}(\hat{\theta}|\theta) - \pi_{d,n\ell}(\theta). \tag{27}$$

Proof. See the Appendix. ■

Part of Proposition 1 is in line with Yehezkel [2008]: The manufacturer chooses the same $q_{u,i2}(\theta) = q_{u,i1}(\theta)$ for both menus. Intuitively, when using $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$ the manufacturer has an incentive to lower q_u in order to reduce the retailer's information rent. This objective is no different when using $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ so that the manufacturer would not want to raise q_u above $q_{u,i1}(\theta)$. The firm also has no incentive to lower q_u below $q_{u,i1}(\theta)$ because this reduces $T_{i2}(\theta)$ by lowering the industry profit π_i, which can be distributed among the firms.

The manufacturer, however, modifies the quantity q_d of the private label. And this result differs from those of Yehezkel [2008]. In his model, the manufacturer distorts q_d downwards, whereas in my model the manufacturer distorts q_d upwards. This upward distortion depresses the retailer's profit $\pi_{d,i2}(\hat{\theta}|\theta)$ and information rent $U_{i2}(\hat{\theta}|\theta)$ because of two effects. Firstly, the high quantity $q_{d,i2}(\theta)$ can only be sold at a low price. Secondly, producing $q_{d,i2}(\theta)$ instead of the retailer's best response $q_{d,i1}(\theta)$ raises its costs. Both effects diminish the retailer's profit and, thus, information rents ($U_{i2}(\theta) \leq U_{i1}(\theta)$). This allows the manufacturer to extract a higher payment $T_{i2}(\theta) \geq T_{i1}(\theta)$ despite a lower joint profit. As is standard, one does not find a quantity distortion at the top (for $\theta = 1$) and no information rent at the bottom (for $\theta = 0$).

The differences between the predictions of Yehezkel [2008] and the present model can be explained as follows: Given that in his model the manufacturer cannot observe demand, the retailer has an incentive to understate demand. That way, the manufacturer would set the payment T suboptimally low, leaving the retailer with a profit above its reservation profit $\pi_{d,n\ell}(\theta)$. The manufacturer, however, reduces the retailer's information rents by setting a *low* quantity q_d that prevents the retailer from exploiting the, in fact, high demand. For certain parameter constellations, the manufacturer would even use an exclusive dealing contract to foreclose the private label. This is the case even if both products would be sold under complete information.

In my model, however, the manufacturer cannot observe the production costs of the private label, and the retailer has an incentive to understate these costs. That way, the manufacturer would overestimate the reservation profit that the retailer could earn when selling the private label only. The manufacturer would therefore set the payment T suboptimally low, leaving the retailer with a profit above its reservation profit $\pi_{d,n\ell}(\theta)$. The manufacturer reduces the retailer's profit and, thus, information rents by setting a *high* quantity q_d that can only be sold at low prices while being produced at high costs. Hence, other than in the model with asymmetric information about demand, asymmetric information about the private label's costs lowers foreclosure concerns instead of raising them.

In other words, in Yehezkel [2008], a retailer has a high willingness to pay for the branded product if the retailer is of the "high demand type", so that it wants a high q_d. In the present model, a retailer has a high willingness to pay for the branded product if it is of the "high cost type", so that it wants a low q_d. This affects the retailer's best response $q_d(q_u, \hat{\theta}|\theta)$ when it is not part of the contract; see equation (7). The sign of $\partial q_d(q_u, \hat{\theta}|\theta)/\partial\theta$, which is negative in the present model, flips depending on the nature of the private information; see equation (6) in Yehezkel [2008]. Therefore, the finding of Yehezkel [2008] (downward distortion of q_d) can easily be reversed depending on the nature of the private information.

TABLE I
EQUILIBRIUM PRICES

	$p_u = s_u - s_u q_u - s_d q_d$	$p_d = s_d - s_d q_u - s_d q_d$
$q_u^*(\theta), q_d^*(\theta)$	$\frac{s_u+c_u}{2}$	$\frac{s_d+c_d(\theta)}{2}$
$q_{u,i1}(\theta), q_{d,i1}(\theta)$	$\frac{s_u+c_u}{2} + \frac{H(\theta)}{2}$	$\frac{s_d+c_d(\theta)}{2}$
$q_{u,i2}(\theta), q_{d,i2}(\theta)$	$\frac{s_u+c_u}{2}$	$\frac{s_d+c_d(\theta)}{2} - \frac{H(\theta)}{2}$

III(iv). *Welfare Analysis*

The contracts $\langle q_{u,c}(\theta), T_c(\theta)\rangle$ under complete information, $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$ under incomplete information when conditioning on q_u only, and $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ when conditioning on both q_u and q_d affect consumer surplus. This can be seen from Table I showing the equilibrium prices p_u, p_d for all three cases.[6]

When the menu is conditional on q_u only, the retailer chooses $q_{d,i1}(\theta) \geq q_d^*(\theta)$ such that this increase just balances the manufacturer's downward distortion of $q_{u,i1}(\theta) \leq q_u^*(\theta)$, so that $q_{u,i1}(\theta) + q_{d,i1}(\theta) = q_u^*(\theta) + q_d^*(\theta)$ applies. The price p_u of the branded product weakly increases while p_d remains at the same level as in the complete information benchmark. Those weakly higher prices lower consumer surplus. A similar effect can be seen in the model of Yehezkel [2008] where, despite the different nature of the information asymmetry, the manufacturer also distorts the quantity q_u of the branded product downwards.

When the menu is conditional on both q_u and q_d, the manufacturer chooses $q_{u,i2}(\theta)$ and $q_{d,i2}(\theta)$ such that the total output rises above the complete information benchmark ($q_{u,i2}(\theta) + q_{d,i2}(\theta) \geq q_u^*(\theta) + q_d^*(\theta)$). The price p_u is the same as in the complete information benchmark, but p_d is weakly lower. Those weakly lower prices raise consumer surplus. This result is quite different from those obtained by Yehezkel [2008]. In his model, where the manufacturer is incompletely informed about demand, the manufacturer distorts both q_u and q_d downwards, which causes higher prices and lowers consumer surplus.

IV. APPLICATION

Industry observers might argue that the contracts observed in reality look quite different than the optimal menu. This section demonstrates how theory and practice can be reconciled: The observed contracts are actually variants of the menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ that are used to cope with certain

[6] Changes in the exogenous parameters c_u, s_u, and s_d have intuitive effects on the equilibrium outputs. One finds $\partial q_u/\partial c_u < 0$, $\partial q_d/\partial c_u > 0$, $\partial q_u/\partial s_u > 0$, $\partial q_d/\partial s_u < 0$, $\partial q_u/\partial s_d < 0$, and $\partial q_d/\partial s_d > 0$ in all three cases.

difficulties in practice. For example, the manufacturer may not be able to control q_d, which can be solved by end-of-year repayments, as is shown in Section IV(i). Evidence also suggests that a lump-sum payment T is uncommon in practice (potentially due to the retailer's financing constraints), and that the retailer may, additionally, be free to choose quantities other than $q_{u,i2}(\theta), q_{d,i2}(\theta)$. Section IV(ii), therefore, illustrates how T can be replaced by empirically observed quantity discount schemes that – at the same time – incentivize the retailer to choose the "correct" quantities.

IV(i). *End-Of-Year Repayment*

While the manufacturer can restrict deliveries of the branded product to distort the quantity downwards to $q_{u,i2}(\theta)$, the manufacturer typically lacks the power to control the retailer's sales of the private label to distort its quantity upwards to $q_{d,i2}(\theta)$. Lemma 2, therefore, establishes how the manufacturer can overcome its inability to control q_d if the contract is enforced by a third party. Instead of requiring the retailer to set $q_d = q_{d,i2}(\theta)$, the manufacturer uses a tariff $T_f(q_d, \theta)$ conditional on θ where a third party imposes a fine $F(\theta)$ on the retailer in case it sets $q_d \neq q_{d,i2}(\theta)$. The index f stands for *fine*. Choosing $F(\theta)$ sufficiently high incentivizes the retailer to set $q_d = q_{d,i2}(\theta)$. This obvious result serves as a stepping stone for the further analysis.

Lemma 2. The retailer adheres to $q_{d,i2}(\theta)$ if the manufacturer proposes a menu $\langle q_{u,i2}(\theta), T_f(q_d, \theta), \overline{F}(\theta)\rangle$ with

$$T_f(q_d, \theta) = T_{i2}(\theta) + F(\theta)\left(1 - I(q_d, \theta)\right), \tag{28}$$

$$F(\theta) > \overline{F}(\theta) \equiv \frac{H(\theta)^2}{4s_d}, \text{ and} \tag{29}$$

$$I(q_d, \theta) = \begin{cases} 1 & \text{if} \quad q_d = q_{d,i2}(\theta), \\ 0 & \text{if} \quad q_d \neq q_{d,i2}(\theta), \end{cases} \tag{30}$$

although the retailer is free to choose quantities other than $q_{d,i2}(\theta)$.

Proof. Lemma 1 showed that the retailer chooses its best response to $q_{u,i2}(\theta)$, which is $q_{d,i1}(\theta)$, if the manufacturer does not impose a restriction on q_d. Now, setting $q_d \neq q_{d,i2}(\theta)$, so that $I(q_d, \theta) = 0$, is punished by a fine $F(\theta)$. The retailer then earns the profit shown in the second row of (31).

$$\begin{aligned} &R(q_{u,i2}(\theta), q_{d,i2}(\theta)) - q_{d,i2}(\theta)c_d(\theta) - T_{i2}(\theta) > \\ &R(q_{u,i2}(\theta), q_{d,i1}(\theta)) - q_{d,i1}(\theta)c_d(\theta) - T_{i2}(\theta) - F(\theta). \end{aligned} \tag{31}$$

The first row of (31) shows the retailer's profit from setting the quantity $q_{d,i2}(\theta)$ as desired by the manufacturer. Using the functional forms of $q_{u,i2}(\theta)$, $q_{d,i2}(\theta)$, and $q_{d,i1}(\theta)$, it is straightforward to show that inequality (31) applies if $F(\theta)$ satisfies (29). ■

There is not much evidence that manufacturers actually relied on such fines. One observes, however, that manufacturers of fast-moving consumer goods frequently make end-of-year (re)payments to retailers (Kim and Staelin [1999]; Bloom *et al.* [2000]; Klein and Wright [2007]; Villas-Boas [2007]). Based on this evidence, Lemma 3 suggests that instead of using a fine, the manufacturer may also use a scheme where it collects $F(\theta)$ as a deposit at the beginning of the period, which is part of the total payment $T_e(q_d, \theta)$ and refunded at the end of the period upon observing $q_d = q_{d,i2}(\theta)$. The index e stands for *end-of-year repayment*.

Lemma 3. The retailer adheres to $q_{d,i2}(\theta)$ if the manufacturer proposes a menu $\langle q_{u,i2}(\theta), T_e(q_d, \theta), \overline{F}(\theta)\rangle$ with

$$(32) \qquad T_e(q_d, \theta) = [T_{i2}(\theta) + F(\theta)] - F(\theta)I(q_d, \theta) \text{ and } F(\theta) > \overline{F}(\theta),$$

although the retailer is free to choose quantities other than $q_{d,i2}(\theta)$.

The proof of Lemma 3 is straightforward and can be omitted. One sees that $T_e(q_d, \theta) = T_f(q_d, \theta)$ and, therefore, $\pi_{d,e}(\theta) = \pi_{d,f}(\theta)$ apply for all values of q_d so that the retailer adheres to $q_{d,i2}(\theta)$ as follows from Lemma 2.[7]

Note that both contracts must be enforced by a third party. In case of $\langle q_{u,i2}(\theta), T_f(q_d, \theta), \overline{F}(\theta)\rangle$, this party ensures that the retailer pays the fine upon setting $q_d \neq q_{d,i2}(\theta)$. In case of $\langle q_{u,i2}(\theta), T_e(q_d, \theta), \overline{F}(\theta)\rangle$, it ensures that the manufacturer makes the end-of-year repayment upon observing $q_d = q_{d,i2}(\theta)$.

It is well-known that agreements can be self-enforcing if the parties interact repeatedly (see Telser [1980], for example). Repeated interaction is indeed a common feature of markets for fast-moving consumer goods. To provide one example where the menu $\langle q_{u,i2}(\theta), T_e(q_d, \theta), \overline{F}(\theta)\rangle$ with the end-of-year repayment is self-enforcing, assume that after the current period the manufacturer-retailer pair interacts in one additional period. The manufacturer has learned the retailer's type by then so that the firms earn their complete information profits $\pi_{u,c}(\theta)$ and $\pi_{d,c}(\theta)$ in this future period. Recall that, in this period, the retailer earns $\pi_{d,c}(\theta) = \pi_{d,n\ell}(\theta)$ whether it sells the branded product or not. Therefore, assume that the retailer lists the branded product if the manufacturer has made the repayment $F(\theta)$ in the first period,

[7] An application of deposit-refund schemes to public good games was provided by Gerber and Wichardt [2009].

and it de-lists the branded product otherwise. After normalizing the manufacturer's discount factor to 1, one finds that the manufacturer makes the repayment if inequality (33) applies.

$$F(\theta) < \pi_{u,c}(\theta). \tag{33}$$

Inequality (33) shows that the manufacturer refunds $F(\theta)$ if it earns a higher profit $\pi_{u,c}(\theta)$ in the second period from continuing the business relationship than from withholding $F(\theta)$ in the first period. The menu $\langle q_{u,i2}(\theta), T_e(q_d, \theta), \overline{F}(\theta)\rangle$ is self-enforcing in this case.[8] Otherwise, the firms must rely on third-party enforcement. Or they have to resort to menu $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$, which is conditional on q_u only, if third-party enforcement is not an option.

IV(ii). *Quantity Discount*

Lemmas 2 and 3 demonstrated how the manufacturer can incentivize the retailer to sell $q_{d,i2}(\theta)$, instead of playing its best response $q_{d,i1}(\theta)$ to $q_{u,i2}(\theta)$, even if the manufacturer cannot formally control the retailer's choice of q_d. The menus $\langle q_{u,i2}(\theta), T_f(q_d, \theta), \overline{F}(\theta)\rangle$ with the fine, and $\langle q_{u,i2}(\theta), T_e(q_d, \theta), \overline{F}(\theta)\rangle$ with the end-of-year repayment, which are variants of the optimal menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$, *required* the retailer to sell the quantity $q_{u,i2}(\theta)$ of the branded product. In reality, however, retailers are often free to select also quantities other than $q_{u,i2}(\theta)$.

Therefore, Lemma 4 shows how the manufacturer can make it a best response (hence the index b) for the retailer to choose $q_{u,i2}(\theta)$, which requires two small modifications of the menu $\langle q_{u,i2}(\theta), T_e(q_d, \theta), \overline{F}(\theta)\rangle$: The threshold for the deposit $F(\theta)$ is raised to $\tilde{F}(\theta) > \overline{F}(\theta)$. The combination of the quantity restriction $q_u = q_{u,i2}(\theta)$ and the lump-sum payment $T_e(q_d, \theta)$ is replaced by a two-part tariff $T_b(q_u, q_d, \theta)$ where the retailer may purchase any quantity q_u of the branded product at a price equaling the marginal production costs c_u.

Lemma 4. The retailer adheres to $q_{d,i2}(\theta)$ and $q_{u,i2}(\theta)$ if the manufacturer proposes a menu $\langle T_b(q_u, q_d, \theta), \tilde{F}(\theta)\rangle$ with

$$\begin{aligned} T_b(q_u, q_d, \theta) &= \left[T_{i2}(\theta) + F(\theta) - q_{u,i2}(\theta)c_u\right] + q_u c_u - F(\theta)I(q_d, \theta) \\ &= T_e(q_d, \theta) + (q_u - q_{u,i2}(\theta))c_u, \text{ and} \end{aligned} \tag{34}$$

$$F(\theta) > \tilde{F}(\theta) \equiv \frac{H(\theta)^2}{4s_d} \cdot \frac{s_u}{\Delta s}, \tag{35}$$

[8] Conditions other than (33) will, of course, emerge under different assumptions. If, for example, the cost-type θ follows a random walk so that the manufacturer faces the same asymmetric information problem in every consecutive period, an infinitely repeated game may be needed to prevent the manufacturer from withholding the repayment.

although the retailer is free to choose quantities other than $q_{d,i2}(\theta)$ and $q_{u,i2}(\theta)$.

Proof. The manufacturer uses a two-part tariff $T_b(q_u, q_d, \theta)$, according to which it charges a fixed fee $T_{i2}(\theta) + F(\theta) - q_{u,i2}(\theta)c_u$ and a variable payment $q_u c_u$. The fixed fee collects the payment $T_{i2}(\theta)$ and the deposit $F(\theta)$ but is net of the production costs of the branded product. The retailer purchases the branded product at a price equaling the manufacturer's marginal costs c_u. The deposit $F(\theta)$ is returned upon observing $q_d = q_{d,i2}(\theta)$, in which case $I(q_d, \theta) = 1$ applies. The tariff is specified such that $T_b(q_u, q_d, \theta) = T_{i2}(\theta)$ and $\pi_{d,b}(\theta) = \pi_{d,i2}(\theta)$ apply if the retailer chooses $q_{u,i2}(\theta), q_{d,i2}(\theta)$.

Recall that, in Proposition 1, $q_{u,i2}(\theta)$ was determined as the best response to $q_{d,i2}(\theta)$ if the branded product is obtained at marginal costs c_u. Therefore, in a first step, I prove that this is also the case now because $T_b(q_u, q_d, \theta)$ is specified such that $\partial T_b(q_u, q_d, \theta)/\partial q_u = c_u$ applies. Setting $q_{u,i2}(\theta)$ is a best response if the mechanism incentivizes the retailer to set $q_{d,i2}(\theta)$, and if the mechanism is fully revealing. Maximizing $\pi_{d,b}(\theta) = R(q_u, q_d) - c_d(\theta)q_d - T_b(q_u, q_d, \theta)$ with respect to q_u gives the retailer's best response function (36).

(36)
$$q_{u,b}(q_d(\theta)) = \begin{cases} \frac{1}{2s_u}\left[s_u - \frac{\partial T_b(q_u,q_d,\theta)}{\partial q_u} - 2s_d q_d(\theta)\right] & \text{if } q_d(\theta) < \frac{1}{2s_d}\left[s_u - \frac{\partial T_b(q_u,q_d,\theta)}{\partial q_u}\right] \\ 0 & \text{otherwise} \end{cases}.$$

Plugging $q_{d,i2}(\theta)$ and $\partial T_b(q_u, q_d, \theta)/\partial q_u = c_u$ in (36) proves $q_{u,b}(q_{d,i2}(\theta)) = q_{u,i2}(\theta)$. The second-order condition of the maximization problem is given by (37).

$$\text{(37)} \qquad \frac{\partial^2 \pi_{d,b}(\theta)}{\partial q_u^2} = -2s_u - \frac{\partial^2 T_b(q_u, q_d, \theta)}{\partial q_u^2}.$$

Because of $\partial^2 T_b(q_u, q_d, \theta)/\partial q_u^2 = 0$, (37) is negative for all q_u.

In a second step, I prove that the retailer finds it optimal to set $q_{d,i2}(\theta)$ if the mechanism is fully revealing. This is the case if the deposit $F(\theta)$ is set high enough to prevent double deviations from $q_{u,i2}(\theta), q_{d,i2}(\theta)$: In Lemmas 2 and 3 the manufacturer controlled q_u, so that there could only be single deviations, meaning that the retailer chose q_d according to best response function (7). Now, there may be double deviations, where the retailer chooses q_d and q_u according to best response functions (7) and (36). Given $\partial T_b(q_u, q_d, \theta)/\partial q_u = c_u$, those functions intersect at the quantities $q_u^*(\theta)$ and $q_d^*(\theta)$ that maximize the firms' joint profits. If the retailer chooses $q_d^*(\theta) \neq q_{d,i2}(\theta)$ the manufacturer will, however, withhold the deposit $F(\theta) > \tilde{F}(\theta)$. The threshold $\tilde{F}(\theta)$ in (35) was chosen such that $\pi_{d,b}(q_u^*(\theta), q_d^*(\theta), \theta) < \pi_{d,b}(q_{u,i2}(\theta), q_{d,i2}(\theta), \theta)$ applies, which makes double deviations unprofitable for the retailer. Note that $\tilde{F}(\theta) > \overline{F}(\theta)$ applies, with

$\overline{F}(\theta)$ being defined in (29). The deposit for preventing double deviations is, therefore, higher than the deposit required for preventing single deviations.

As a third step, recall that $\langle T_b(q_u, q_d, \theta), \tilde{F}(\theta)\rangle$ was specified such that $T_b(q_u, q_d, \theta) = T_{i2}(\theta)$ and $\pi_{d,b}(\theta) = \pi_{d,i2}(\theta)$ apply if the retailer chooses $q_{u,i2}(\theta)$ and $q_{d,i2}(\theta)$. The retailer, thus, reveals its type θ truthfully as follows from Proposition 1. This proves Lemma 4. ■

The menus proposed in Lemmas 2 to 4 assume the payment of a lump-sum fee at the beginning of the period. This is, however, at odds with the observation that in "mainstream retail sectors such as grocery retailing or departmental stores, retailers do not seem to pay lump-sum fees to manufacturers" (Iyer and Villas-Boas [2003]). Similarly, Villas-Boas [2007] establishes "that retail supermarkets do not often pay fixed fees to their manufacturers", whereas the "existence of quantity discounts is common practice in [the food retail] industry." Therefore, Draganska *et al.* [2010] proposed that a "fruitful avenue for future research would be to explore how to incorporate quantity discounts into the negotiation process" between a manufacturer and a retailer.

This is what Lemma 5 does. Based on the contributions of Jeuland and Shugan [1983] and Kolay *et al.* [2004], who showed that a lump-sum payment may be replaced by a quantity discount, Lemma 5 demonstrates how the manufacturer can collect $T_{i2}(\theta) + F(\theta)$ by charging a high price on the initial units of the branded product while granting a quantity discount on the additional units. The index r thus stands for *rebate*.

Lemma 5. The retailer adheres to $q_{d,i2}(\theta)$ and $q_{u,i2}(\theta)$ if the manufacturer proposes a menu $\langle T_r(q_u, q_d, \theta), \tilde{F}(\theta)\rangle$ with

(38)
$$T_r(q_u, q_d, \theta) = \begin{cases} x(\theta)\left(q_u \cdot q_{u,i2}(\theta) - \frac{q_u^2}{2}\right) + q_u c_u - F(\theta) I_r(q_d, \theta) & \text{if } q_u \leq q_{u,i2}(\theta), \\ T_b(q_u, q_d, \theta) & \text{if } q_u > q_{u,i2}(\theta), \end{cases}$$

(39)
$$I_r(q_d, \theta) = \begin{cases} 1 & \text{if } q_d = q_{d,i2}(\theta) \text{ and } q_u > 0, \\ 0 & \text{otherwise,} \end{cases}$$

(40)
$$x(\theta) = \frac{2\left[T_{i2}(\theta) + F(\theta) - c_u q_{u,i2}(\theta)\right]}{q_{u,i2}(\theta)^2},$$

and $F(\theta) > \tilde{F}(\theta)$, although the retailer is free to choose quantities other than $q_{d,i2}(\theta)$ and $q_{u,i2}(\theta)$, and although the menu does not entail a lump-sum payment.

Proof. The payment function satisfies $T_r(0, q_d, \theta) = 0$ for $q_u = 0$, so that it does not entail a lump-sum payment. For $q_u > 0$, the shape of $T_r(q_u, q_d, \theta)$ is determined by (41) and (42).

$$\frac{\partial T_r(q_u, q_d, \theta)}{\partial q_u} = \begin{cases} x(\theta)(q_{u,i2}(\theta) - q_u) + c_u & \text{if} \quad q_u \leq q_{u,i2}(\theta), \\ c_u & \text{if} \quad q_u > q_{u,i2}(\theta). \end{cases} \tag{41}$$

$$\frac{\partial^2 T_r(q_u, q_d, \theta)}{\partial q_u^2} = \begin{cases} -x(\theta) & \text{if} \quad q_u \leq q_{u,i2}(\theta), \\ 0 & \text{if} \quad q_u > q_{u,i2}(\theta). \end{cases} \tag{42}$$

For $q_u \leq q_{u,i2}(\theta)$, the manufacturer charges a positive price $\partial T_r(q_u, q_d, \theta)/\partial q_u$ that falls linearly with a slope of $-x(\theta)$ toward c_u as q_u approaches $q_{u,i2}(\theta)$. This decrease constitutes a quantity discount. Given $\partial^2 T_r(q_u, q_d, \theta)/\partial q_u^2 < 0$, the payment function rises concavely in q_u, and it reaches $T_r(q_{u,i2}(\theta), q_d, \theta) = T_b(q_{u,i2}(\theta), q_d, \theta)$ at $q_{u,i2}(\theta)$, which is ensured by choosing $x(\theta)$ according to (40). For $q_u > q_{u,i2}(\theta)$, the function satisfies $T_r(q_u, q_d, \theta) = T_b(q_u, q_d, \theta)$, so that it rises linearly because q_u is sold at a price c_u. As before, the payment is combined with a refund $F(\theta) > \tilde{F}(\theta)$ for setting $q_d = q_{d,i2}(\theta)$, which is only paid in cases with $q_u > 0$.

I will show that the retailer's profit is maximized when choosing $q_{u,i2}(\theta), q_{d,i2}(\theta)$. To see this, consider the retailer's profit function (43) that results if the retailer is assumed to choose q_d according to best response function (7).

$$\pi_{d,r}(q_u, \theta) = q_u \left(\Delta s - \Delta s q_u + c_d \right) + \pi_{d,n\ell}(\theta) - T_r(q_u, q_d, \theta). \tag{43}$$

For the moment, assume $q_d \neq q_{d,i2}(\theta)$ so that $I_r(q_d, \theta) = 0$. The first and second derivatives of the profit function are given by (44) and (45).

$$\frac{\partial \pi_{d,r}(q_u, \theta)}{\partial q_u} = \Delta s - 2\Delta s q_u + c_d - \frac{\partial T_r(q_u, q_d, \theta)}{\partial q_u}, \tag{44}$$

$$\frac{\partial^2 \pi_{d,r}(q_u, \theta)}{\partial q_u^2} = -2\Delta s - \frac{\partial^2 T_r(q_u, q_d, \theta)}{\partial q_u^2}, \tag{45}$$

Because of the discontinuity in $\partial T_r(q_u, q_d, \theta)/\partial q_u$ at $q_u = q_{u,i2}(\theta)$, one must distinguish the cases $q_u > q_{u,i2}(\theta)$ and $q_u \leq q_{u,i2}(\theta)$.

For $q_u > q_{u,i2}(\theta)$, the payment function was specified such that $T_r(q_u, q_d, \theta) = T_b(q_u, q_d, \theta)$ applies. Given $\partial T_r(q_u, q_d, \theta)/\partial q_u = c_u$, the relevant deviation point is the same as in Lemma 4, which is $q_u^*(\theta), q_d^*(\theta)$. Lemma 4 already proved that the retailer receives a higher profit when setting $q_{u,i2}(\theta), q_{d,i2}(\theta)$ as compared to setting $q_u^*(\theta), q_d^*(\theta)$, which is because of $F(\theta) > \tilde{F}(\theta)$.

For $q_u \leq q_{u,i2}(\theta)$, solving $\partial \pi_{d,r}(q_u, \theta)/\partial q_u = 0$ for q_u gives

(46) $$q_{u,r}^{dev}(\theta) = q_{u,i2}(\theta) + \frac{H(\theta)}{2\Delta s - x(\theta)}.$$

Re-arranging $q_{u,r}^{dev}(\theta) \leq q_{u,i2}(\theta)$ yields $2\Delta s - x(\theta) \leq 0$. This weak inequality, however, implies $\partial^2 \pi_{d,r}(q_u, \theta)/\partial q_u^2 = -(2\Delta s - x(\theta)) \geq 0$, so that the deviation point with $q_{u,r}^{dev}(\theta)$ is a *minimum* of the retailer's profit function. Because both the retailer's revenue and the payment function $T_r(q_u, q_d, \theta)$ are concave in q_u, a deviation point with $q_{u,r}^{dev}(\theta)$ exists if $T_r(q_u, q_d, \theta)$ is "more concave" than the revenue function. Starting from $q_u = 0$, the profit initially falls when increasing q_u, before it rises again once q_u is raised beyond $q_{u,r}^{dev}(\theta)$.

The retailer would optimally choose one of the two corner solutions. One is found at $q_{u,i2}(\theta), q_{d,i2}(\theta)$, which is the combination of quantities desired by the manufacturer. The other corner solution is found at $q_u = 0$, where q_d would be set according to best response function (7), which gives (47).

(47) $$q_{d,n\ell}(\theta) = \frac{s_d - c_d(\theta)}{2s_d}.$$

The retailer would make the profit $\pi_{d,r}(0, q_{d,n\ell}, \theta) = \pi_{d,n\ell}(\theta)$. However, once the retailer chooses $q_{d,i2}(\theta)$ and $q_{u,i2}(\theta)$, it receives the refund $F(\theta)$ and earns the profit $\pi_{d,r}(q_{u,i2}(\theta), q_{d,i2}(\theta), \theta) = \pi_{d,i2}(\theta) \geq \pi_{d,n\ell}(\theta)$.

This proves that the retailer selects $q_{d,i2}(\theta)$ and $q_{u,i2}(\theta)$ although the firm is free to choose also quantities other than that, and although the menu does not entail a lump-sum payment. Because of $\pi_{d,r}(q_{u,i2}(\theta), q_{d,i2}(\theta), \theta) = \pi_{d,i2}(\theta)$ the retailer reveals its type truthfully, which follows from Proposition 1. This proves Lemma 5. ■

After characterizing the optimal menu in Proposition 1, Lemmas 2 and 3 showed how a fine or refund $F(\theta)$ can be used so that the retailer sets the optimal $q_{d,i2}(\theta)$. Lemma 4 demonstrated how the condition on q_u can be dropped when using a two-part tariff with a fixed and a variable payment, where the branded product is sold at a price of c_u. Following Jeuland and Shugan [1983] and Kolay *et al.* [2004], Lemma 5 demonstrated how the fixed fee can be replaced by a quantity discount with decreasing prices. The equivalence of both types of tariffs may help to explain the observations made, for example, by Iyer and Villas-Boas [2003], Villas-Boas [2007], or Draganska *et al.* [2010] according to which quantity discount schemes are more common in retail markets than tariffs with a fixed fee.

V. CONCLUSION

This article presents a mechanism design analysis of the optimal wholesale contract proposed by the monopolistic manufacturer of a branded product to a monopolistic retailer if the retailer also sells a private label whose costs are unobserved by the manufacturer. The retailer has an incentive to understate the costs of the private label and benefit from a lower payment to the manufacturer. Given the revelation principle, the manufacturer can induce truth-telling by requiring the retailer to sell a suboptimally low quantity of the branded product. Yehezkel [2008] showed that the manufacturer can further reduce the retailer's information rent by imposing a restriction on the quantity of the private label, too.

Such market share contracts, which are conditional on the quantities of both goods, can have exclusionary effects. If the manufacturer cannot observe aggregate demand (as in Yehezkel [2008]) the manufacturer would want to distort the quantity of the branded product downwards along with that of the private label. While this raises concerns that the private label might be foreclosed, a quite different effect is suggested by the model analyzed in the present article where the manufacturer cannot observe the costs of the private label. In this case, the manufacturer would want to distort the quantity of the private label upwards. This is an effect contrary to exclusion, and it even creates a benefit for consumers by contributing to lower prices.

One might, however, be concerned that this suggests anticompetitive effects of a different type. Because the manufacturer is typically unable to control the quantity of the private label, it incentivizes the retailer to distort the quantity of the private label upwards. This is done by collecting an excess payment through high prices of the branded product, which is only repaid to the retailer at the end of the period after observing that the retailer had indeed sold the high quantity of the private label. Courts, policymakers, and authorities have sometimes expressed skepticism toward payments that manufacturers had to make to retailers. This is especially the case if the manufacturer did not receive a specific service in return, and if these payments were made to a powerful retailer with a strong private label, as evidenced by high sales of this product and low sales of the branded product. Such payments might be considered *unfair trading practices*.

Yet, it "is often difficult to distinguish [unfair trading practices] from what might be considered normal competitive behavior" (European Commission [2017]). And indeed, the model proposed in this article suggests an efficiency rationale for such repayments. Here, the repayment is part of a fully-revealing, self-enforcing mechanism that allows the manufacturer to discriminate between different retailer types and to raise its profit by diminishing the retailer's information rents. The model shows that the aggregate quantity is even higher and prices lower than in the complete information benchmark, so that the market share contract with end-of-year repayments benefits final consumers.

Future research might analyze how these results must be modified when assuming an upstream and/or downstream oligopoly—a conceptually straightforward but analytically demanding extension. Such a model would also allow analyzing the effects of an exchange of information about the costs of the private label among the producers of branded products. This is relevant for competition policy because, for example, the conduct of several producers of drugstore products, who had exchanged information about retailers in the downstream market, was considered a violation of competition laws by the German Federal Cartel Office.

APPENDIX

Determining revenue function (6). Equating $v_u = v_d$ as defined in (1) and (2) yields the location $\hat{\phi}$ of the indifferent consumer. All consumers with preferences $\phi \in [\hat{\phi}, 1]$ demand the branded product as is shown by demand function (A1).

$$q_u = 1 - \hat{\phi} \quad \text{with} \quad \hat{\phi} = \frac{p_u - p_d}{\Delta s}. \tag{A1}$$

Let $\phi_{0,d}$ define the critical value of ϕ where $v_d(\phi_{0,d}) = 0$ applies. Hence, all consumers with preferences $\phi \in [\phi_{0,d}, \hat{\phi})$ demand the private label as is shown by demand function (A2)

$$q_d = \hat{\phi} - \phi_{0,d} \quad \text{with} \quad \phi_{0,d} = \frac{p_d}{s_d}. \tag{A2}$$

Inverting the system of demands (A1) and (A2) yields the inverse demand functions shown in (6). ■

Derivation of Assumption (13). A vertically integrated firm sets q_u^*, q_d^* as defined in (10) and (11), which are also chosen in competition if the manufacturer observes the retailer's type θ (see Section III(i)).

$$q_u^*(\theta) = 1 - \frac{\Delta s + \Delta c(\theta)}{2\Delta s}. \tag{10}$$

$$q_d^*(\theta) = \frac{\Delta s + \Delta c(\theta)}{2\Delta s} - \frac{s_d + c_d(\theta)}{2 s_d}. \tag{11}$$

If the manufacturer does not observe the retailer's type and conditions the menu on q_u only, it optimally sets $q_{u,i1}(\theta)$, and the retailer responds by setting $q_{d,i1}(\theta)$ as are shown in (20) and (22) (see Lemma 1).

$$q_{u,i1}(\theta) = q_u^*(\theta) - \frac{H(\theta)}{2\Delta s}, \tag{20}$$

$$q_{d,i1}(\theta) = q_d^*(\theta) + \frac{H(\theta)}{2\Delta s}, \tag{22}$$

If the manufacturer can also control q_d, it sets $q_{u,i2}(\theta)$ and $q_{d,i2}(\theta)$ as defined by (24) and (25) (see Proposition 1).

$$q_{u,i2}(\theta) = q_{u,i1}(\theta), \tag{24}$$

(25)
$$q_{d,i2}(\theta) = q_{d,i1}(\theta) + \frac{H(\theta)}{2s_d},$$

These are also the equilibrium quantities relevant in Lemmas 2 to 5. The off-equilibrium quantities are $q_u^*(\theta), q_d^*(\theta)$ in Lemmas 4 and 5. In Lemma 5, the retailer also considers setting $q_{u,r}^{dev}(\theta) \in [0, q_{u,i2}(\theta)]$, which results in $q_{d,r}^{dev}(\theta) \leq q_{d,n\ell}(\theta)$ as is defined in (47).

(47)
$$q_{d,n\ell}(\theta) = \frac{s_d - c_d(\theta)}{2s_d}.$$

One sees that $q_{u,i2}(\theta) = q_{u,i1}(\theta) \leq q_u^*(\theta)$ and $q_d^*(\theta) \leq q_{d,i1}(\theta) \leq q_{d,i2}(\theta) < q_{d,n\ell}(\theta)$ apply, so that $q_u, q_d > 0$ require (A3) and (A4).

(A3)
$$q_{u,i2}(\theta) > 0 \Leftrightarrow \Delta c(\theta) < \Delta s - H(\theta),$$

(A4)
$$q_d^*(\theta) > 0 \Leftrightarrow \frac{\Delta s}{s_d} c_d(\theta) < \Delta c(\theta).$$

Combining (A3) and (A4) gives (13). ■

Proof of Lemma 1. At the optimum, the IR-constraint must be binding for a retailer of the lowest type so that $U_{i1}(0) = 0$. When using $U_{i1}(0) = 0$ and plugging $\pi_{d,i1}(\hat{\theta}|\theta)$ from (17) in $U_{i1}(\hat{\theta}|\theta)$ from (18), one can solve for $T_{i1}(\theta)$ as is shown in (21). Plugging (21) in the manufacturer's maximization problem (4) and integrating by parts yields the manufacturer's expected profit (A5).

(A5)
$$\max_{q_u} \int_0^1 \left[\pi_i(q_u, q_d, \theta) - \pi_{d,n\ell}(\theta) - H(\theta) \frac{\partial U_{i1}(\theta)}{\partial \theta} \right] g(\theta) d\theta.$$

Optimizing (A5) w.r.t. q_u gives the optimal output as is shown in (20). Assumption (13) ensures $0 < q_{u,i1}(\theta) < 1$ and $0 < q_{d,i1}(\theta) < 1$. ■

Proof of Proposition 1. Given the revelation principle, the menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta) \rangle$ is fully revealing. Therefore, write the information rents $U_{i2}(\hat{\theta}|\theta)$ as $U_{i2}(\theta)$, and determine the marginal information rents (A6) using $\pi_{d,i2}(\hat{\theta}|\theta)$ from (23), $\pi_{d,n\ell}(\theta)$ from (8), and the envelope theorem.

(A6)
$$\frac{\partial U_{i2}(\theta)}{\partial \theta} = \left[\frac{s_d - c_d(\theta)}{2s_d} - q_d \right].$$

Using the definition of $U_{i2}(\theta)$ from (27), the functional forms of $\pi_{d,i2}(\hat{\theta}|\theta)$ and $\pi_{d,n\ell}(\theta)$, along with condition $U_{i2}(0) = 0$, one can solve for $T_{i2}(\theta)$ as was stated in (26). The manufacturer's profits and maximization problem are then given by

(A7)
$$\max_{q_u, q_d} \int_0^1 \left[\pi_i(q_u, q_d, \theta) - \pi_{d,n\ell}(\theta) - H(\theta) \frac{\partial U_{i2}(\theta)}{\partial \theta} \right] g(\theta) d\theta.$$

Solving the first-order conditions of (A7) yields $q_{u,i2}(\theta)$ and $q_{d,i2}(\theta)$ as were shown in (24) and (25).

Because the menu $\langle q_{u,i2}(\theta), q_{d,i2}(\theta), T_{i2}(\theta)\rangle$ is a super-set of the menu $\langle q_{u,i1}(\theta), T_{i1}(\theta)\rangle$, the manufacturer's choice $q_{d,i2}(\theta) \geq q_{d,i1}(\theta)$ implies $\pi_{u,i2}(\theta) \geq \pi_{u,i1}(\theta)$. The higher aggregate output causes $\pi_i(q_{u,i2}(\theta), q_{d,i2}(\theta)) \leq \pi_i(q_{u,i1}(\theta), q_{d,i1}(\theta))$, so that $\pi_{u,i2}(\theta) \geq \pi_{u,i1}(\theta)$ can only be true if $T_{i2}(\theta) \geq T_{i1}(\theta)$, which requires $U_{i2}(\theta) \leq U_{i1}(\theta)$. ■

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