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Play-Hysteresis in Supply as Part of a Market Model *

von

MATTHIAS GÖCKE*

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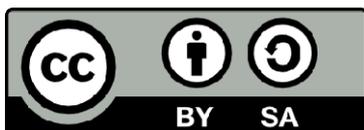
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Corresponding author:

Prof. Dr. Matthias Göcke, Justus-Liebig-University of Giessen, Department of Business
Administration and Economics, VWL IV, Licher Straße 62, 35394 Giessen, Germany, phone: +49
(0)641/99-22241, fax: +49 (0)641/99-22249, e-mail: Matthias.Goecke@wirtschaft.uni-giessen.de.

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Abstract

Consequences of path-dependent supply side on the market equilibrium are illustrated. Supply is only a subsystem of the entire market with its forcing variable (price) being endogenous from the perspective of the entire market. This results in feedbacks on the equilibrium of price and quantity if transient exogenous disturbances occur. Aggregate hysteresis is modelled by continuous dynamics showing similarities to ‘mechanical play’. This contrast the standard firm level modelling of hysteresis resulting from discontinuous (activity/inactivity) switches. Play dynamics are captured in a simple linearized way, just by adding two parameters to a supply equation.

JEL Codes: C39; C51; C61.

Keywords: path-dependence; play-hysteresis; modelling techniques

1. Introduction

The term “hysteresis” – originally stemming from physics and magnetism – generally describes permanent effects of a temporary stimulus, i.e. a past and only temporary change of the relevant economic determinants (technically: the input or the forcing variables) results in a permanent change of the economic behaviour (as the output or dependent variable).¹ Hysteresis characterizes systems with *path-dependent multiple equilibria*: As a consequence, the observed behaviour of the system does not only rely on the current levels of the forcing variables, but also depends on the initial conditions and the past realisations of the input variables (*Cross/Allan*, 1988, p. 26). Typically, hysteresis in economics is based on sunk adjustment costs: standard examples are hiring-/firing costs in labour markets and entry-/exit-costs in international export markets.² The starting point is usually the path-dependent behavioural pattern on the micro level of a single unit (firm), being – under consideration of the past spending of sunk-costs – active on a market or not. Thus, the path-dependent switching of the activity status at specific triggers is to be modelled on a micro level. However, aggregation over a multiplicity of heterogeneous agents is not straightforward and results in a more complex aggregate path-dependent pattern of the entire aggregate economic system. The aggregate path-dependence (as may be known from the magnetic hysteresis-loop of an entire piece of iron) is not characterised by discontinuous switches (between activity and inactivity), but by a smooth/continuous transition between different “branches” of the input-output-relation, which occurs when the direction of the movement of the forcing variable changes. In this paper a simple method is applied to model this dynamics on an aggregate level by a procedure which shows similarities to the phenomenon of play in mechanics. By adding only two additional parameters to a linear relation, the complex path-dependent pattern on an aggregate level is captured by an approximation based on linear segments.

¹ The terms 'input' and 'output' are used in a technical manner and *not* in a narrow economic sense (as e.g. production output and factor input).

² For labour markets see *Blanchard/Summers* (1986), *Lindbeck/Snower* (1986), and *Bentolila/Bertola* (1990); for international trade see *Baldwin* (1989), *Baldwin/Krugman* (1989) and *Dixit* (1989, 1990). See *Cross* (1993), *Göcke* (2002) and *Cross/Grinfeld/Lamba* (2009) for an overview of hysteresis in economics.

Furthermore, due to the dynamic complexity, hysteresis in economics is often modelled with an exogenous forcing variable. However, the forcing variable of the hysteretic subsystem (as e.g. the exchange rate for the entry/exit decision in international markets, or the wage for hiring/firing decisions) is usually an endogenous variable from the perspective of the whole economic system. Thus, permanent effects of transient changes of the forcing variable eventually result in a feed-back effect of the system on the equilibrium level of the forcing variable itself (*Baldwin/Lyons*, 1994; *Cross/McNamara/Pokrovskii/Kalačev*, 2010, pp. 25 ff.). Due to the simplicity of the (linearized) play dynamics, this feedback-effect can be captured easily. In this paper a standard market supply&demand model is extended by play dynamics on the supply side. The forcing variable of the hysteretic sub-system (i.e. on the supply side) is the price level and the dependent variable is the supply quantity. Both are simultaneously determined by the whole market system of supply and demand. For different demand elasticity situations the resulting permanent equilibrium effects on price and quantity caused by transient exogenous demand shocks are calculated.

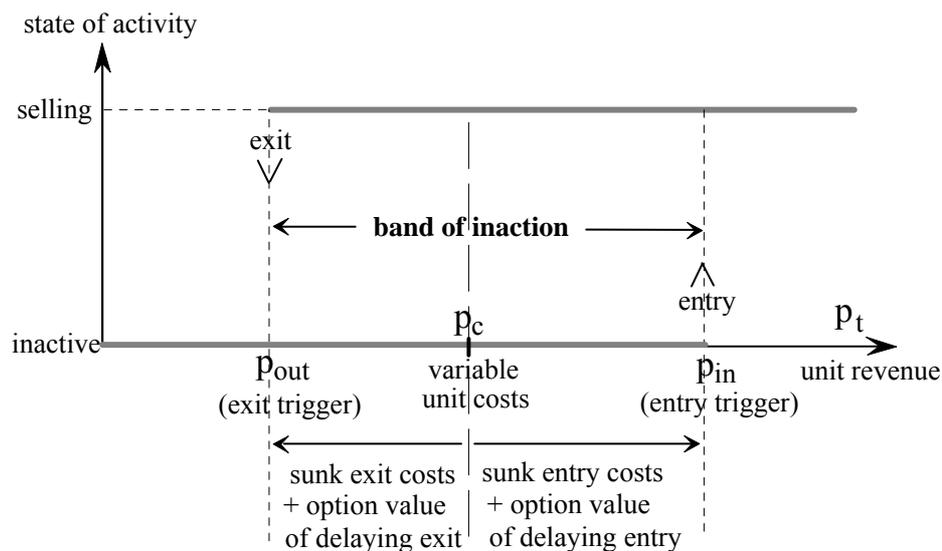
The rest of the paper is structured as follows: After presenting the microeconomic implications of (sunk-cost) hysteresis (Ch. 2), an intuition of the consequences of aggregating over heterogeneous agents is given and the linearized approximation of the aggregate dynamics by play-hysteresis is described (Ch. 3). In Ch. 4 a supply side with play dynamics is integrated into a market model, and permanent (“remanence”) effects of transient demand shocks on the equilibrium are derived for different demand elasticity situations. Chapter 5 concludes.

2. Hysteresis in a microeconomic perspective

Consider a simple microeconomic example with sunk market-entry costs (*Baldwin*, 1989; *Dixit*, 1989): In order to sell in the market, a previously inactive firm must expend market-entry investments, e.g. in setting up a distribution and service network or for introductory sales promotion. These entry cost are sunk, since the expenses are firm-specific and cannot be regained if the firm later wants to leave the market. An inactive firm will only enter the market if the sunk entry costs are covered by revenues. Thus, the price that triggers an entry (p_{in} in Fig. 1) exceeds the variable unit costs (p_c). Moreover, if sunk exit costs are relevant, an

active firm will only exit if the losses under continuation of activity are larger than the sunk exit costs. Hence the exit trigger p_{out} is lower than variable unit costs. Entry and exit triggers differ in a situation with sunk entry and/or exit costs. The micro path-dependence is based on discontinuous switches of the activity state if entry or exit triggers are passed. Between both triggers a ‘band of inaction’ occurs (Baldwin, 1989, pp. 7 f.). Inside this band, the current level of the input/forcing variable (price) does not unambiguously determine the current state of the output/dependent variable (firm’s activity), since the relation shows two path-dependent equilibria (‘branches’).³ If a *temporary* change of the input variable leads to a switch between these equilibria/branches,⁴ a *permanent* effect on the output variable (called “*remanence*”) remains. This after-effect is the constituting feature of hysteresis.

Fig. 1 – *Discontinuous micro hysteresis loop (‘non-ideal relay’): market activity of a single firm*



Uncertainty about the future development concerning the determinants of the firm’s profits reinforces the hysteresis characteristics via option value effects.⁵ Since an exit will destroy sunk investments in the market, an active firm may stay even if it is currently losing money

³ Krasnosel’skii/Pokrovskii (1989, p. 263 and p. 271) call this dynamic pattern “*non-ideal relay*”. See Brokate/Sprekels, 1996, pp. 23 f., for a general description of relay-hysteresis. The original magnetic hysteresis of a single iron-crystal (i.e. at micro level) shows exactly this pattern.

⁴ Passing of microeconomic triggers usually results from “large shocks”. Thus, studies implicitly relying on non-ideal relay-hysteresis, point out the difference between large shocks triggering permanent effects and small ones that do not. See e.g. the titles of Baldwin/Krugman, 1989, and Baldwin/Lyons, 1994, and see the abstract of Evans/Honkapohja, 1993.

⁵ For a comprehensive treatment of uncertainty effects see Dixit/Pindyck (1994).

due to low prices. If the low price would later prove to be only transitory, an immediate exit could turn out to be a mistake. Hence, under uncertainty the opportunity of a “wait-and-see”-strategy shifts the exit-trigger to the left, and analogously the entry-trigger to the right (since a currently favourable price could turn out to be only transitory). I.e. the “band of inaction” is widened by uncertainty.

This first example refers to the supply of final products. However, sunk adjustment costs of changing market activity in general can result in hysteresis effects on markets.⁶ A prominent example on factor markets is hysteresis on labour markets based on sunk hiring and firing costs (*Blanchard/Summers*, 1986, and *Bentolila/Bertola*, 1990).

Beside sunk-costs, several other economic factors may result in hysteretic path-dependence. E.g. Learning-by-doing based on production activity results in permanently reduced unit costs (and with this in an increased supply) based on a temporarily increased production quantity. On the demand side, the penetration of a (new) market may require a temporary decrease in prices. After risk-averse consumers have made favourable experiences with the temporarily cheap product, the willingness to pay more for a now well-known product is increased. All these mechanisms are based on transient factors resulting in permanent effects. The temporary increase of training costs resulting in cost reducing experience or the initial revenue reduction in order to open a market can in a general view be seen as (sunk) “investments” in future profits (since these expenditures can not be regained). The ex-ante decision (before the sunk costs were paid) differs from the ex-post situation (when the “investment” was carried out). As the relevant marginal costs respectively the revenues are changed, the same exogenous situation results in a different path-dependent reaction. Thus, a temporary exogenous disturbance can have permanent effects – which characterises hysteresis.

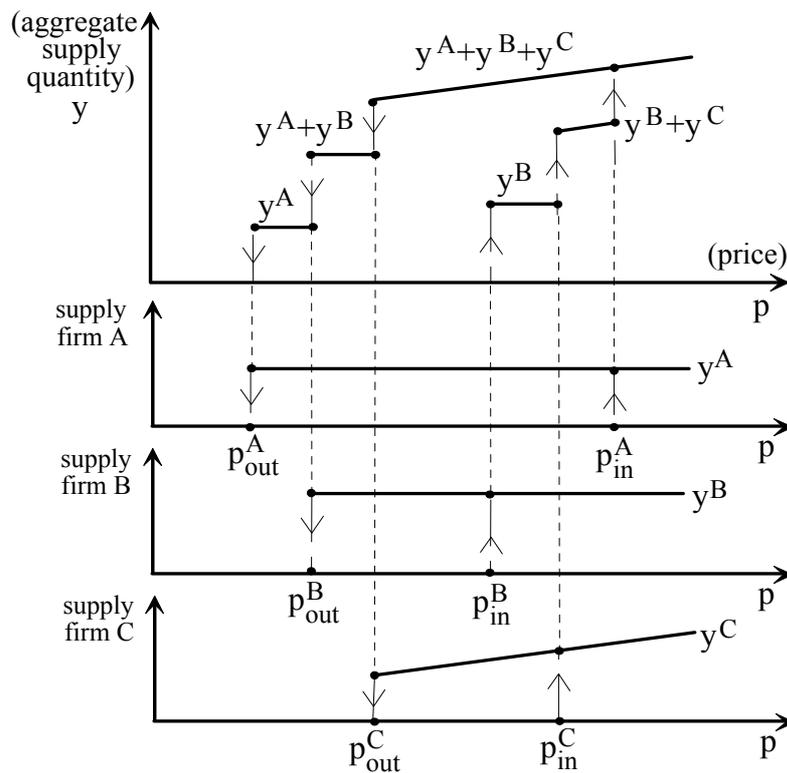
3. Aggregate market supply with play-hysteresis

On a microeconomic level of a single economic unit (i.e. firm) hysteresis occurs via a band of inaction, i.e. a gap between two triggers. *Belke/Göcke* (2001, 2005) focus on the shape of a

⁶ See *Froot/Klemperer*, 1989, p. 638, for a systematisation of factors generating hysteresis on the supply and on the demand side.

macro hysteresis loop and on the *consequences of aggregation*.⁷ Aggregation is not trivial if heterogeneity, e.g. regarding the level of variable costs, the value of sunk exit/entry costs and/or the level of uncertainty about future market situation is taken into account, i.e. if the entry and exit triggers are different between firms. In the realistic case of heterogeneity, a transition from the micro level of a firm to the aggregate level of entire supply in a market leads to a *change of the hysteresis pattern*.

Fig. 2 – Aggregation for 3 heterogeneous firms



In order to give an intuition of the implications of aggregation, Fig. 2 shows a very simple example of only 3 firms, with heterogeneous non-ideal relay reactions. The individual firm's supply is depicted for firms A, B and C in the lower part of the diagram, while the dynamics of the aggregate supply is illustrated in the upper part of Fig. 2. If no firm was initially active (i.e. for a very low initial price level), a monotonously increasing price will result in an entry of firm B at price p_{in}^B , firm C will start activity at price $p_{in}^C > p_{in}^B$, and a price-level above $p_{in}^A > p_{in}^B$ will cause an entry of firm A. If later on the price decreases monotonously, for a price lower than p_{out}^C the firm C, and for $p < p_{out}^B < p_{out}^C$ the firm B will exit. If the price falls to $p < p_{out}^A$, no firm will be active anymore. If the micro behaviour is characterized by non-ideal relays,

⁷ For a suitable aggregation procedure from micro to macro hysteresis – the Preisach (1935) model – see Mayergoyz (1986). For applications to economics see e.g. Amable/Henry/Lordon/Topol (1991), Cross (1994), Piscitelli/Cross/Grinfeld/Lamba (2000), and Göcke (2002).

the aggregate supply loop for all firms together shows a kind of “stairway” (i.e. a step function) for increasing and for decreasing prices – with a band-of-inaction between the “stairway-up” and the “stairway-down” region. The higher the number hysteretic firms which are underlying the aggregation procedure, the smaller is the relative size of the individual firm’s “steps”, converging towards a more and more continuously looking aggregate reaction on both “stairways”.

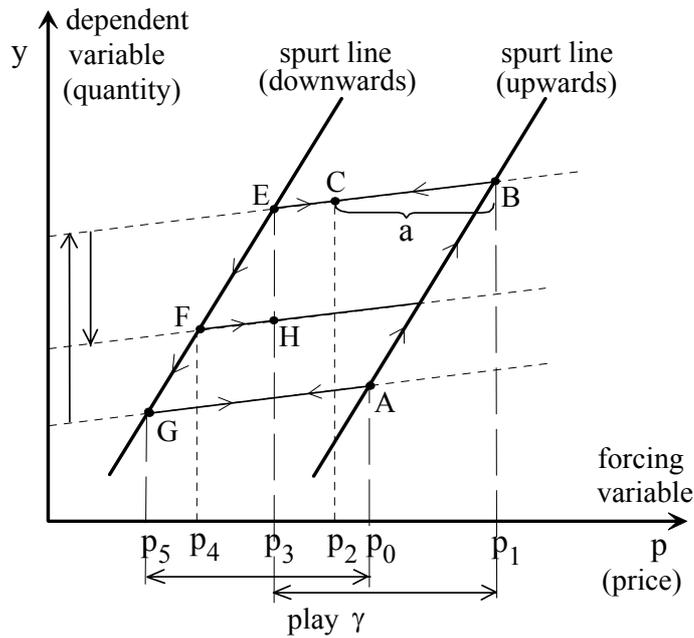
Belke/Göcke (2001, 2005) show – based on an explicit aggregation procedure – that even the aggregate behaviour is characterized by areas of weak reactions which can – corresponding to play in mechanics – be called “play”.⁸ As far as changes occur inside some play area, there are no persistent aggregate effects from small changes in the forcing variables. However, if changes go beyond the play area, sudden strong reactions (and persistence effects) of the output variable occur.⁹ However, play-hysteresis is in two aspects different to the micro non-ideal relay-loop. First, the play-loop shows *no discontinuities*. Second, analogous to mechanical play (e.g. when steering a car) the *play/inaction area is shifted with the history* of the forcing variable: Every change in the direction of the movement of the forcing variable starts with traversing a play area. Only after this play is passed, a stronger reaction (called “spurt”) will result, if the forcing variable continues to move in the same direction.

Fig. 3 gives an impression of play dynamics for the simple case of linear segments – as described by *Belke/Göcke* (2001, 2005). In our example, the dependent variable is the aggregate supply quantity y on a market and the forcing variable is the price level p . Preceding price increases had led to an initial situation in starting point A (price p_0) located on the upward leading (right) spurt line. Changing direction (i.e. now the price decreases) results in entering the play area. A weak play reaction results until the entire play area of absolute width $\gamma (>0)$ is passed. The downward leading spurt line starts in point G at p_5 (with: $\gamma = p_0 - p_5$). In the play area (between points A and G) only a weak reaction of the dependent variable y results from changes in the forcing variable p . A further decrease of p would induce a strong response of y along the (left) downward leading spurt line.

⁸ For play hysteresis, see *Krasnosel'skii/Pokrovskii* (1989), pp. 6 ff., and *Brokate/Sprekels* (1996, pp. 24 f. and pp. 42 ff.). For an example of implicit play-hysteresis in economics see *Delgado* (1991, Fig. 2, p. 472) where the price-stickiness as a result of menu-costs is analysed.

⁹ See *Pindyck* (1988), pp. 980 f., *Dixit/Pindyck* (1994), pp. 15 f., for a non-technical description of “spurts” based on a microeconomic sunk cost mechanism.

Fig. 3 – Linear play-hysteresis and spurt areas



Alternatively, think of a price increase starting from p_0 (A) up to p_1 (point B) and a subsequent decrease to p_2 (C). The corresponding reaction of y first evolves along the right spurt line from $A \rightarrow B$. With this movement the relevant play area is vertically upward-shifted, from line GA to line EB ($\gamma = p_0 - p_5 = p_1 - p_3$). Now a decrease from p_2 (C) to p_3 (E) takes place in a play area.¹⁰ This play area is partially penetrated in point C by an extent 'a'. A further price decrease $p_2 \rightarrow p_3 \rightarrow p_4$ (with trajectory points $C \rightarrow E \rightarrow F$) leads to passing the entire play width γ in point E (p_3), followed by a strong reaction on the downward leading (left) spurt line until point F. On this spurt-down line, a further price decrease suddenly leads to a strong decrease of the supply quantity. However, this (continuous) change in behaviour is not a constant trigger level as in the micro loop, but path-dependent, since the play lines are vertically shifted by movements along the spurt lines. The play area is shifted in the opposite direction as before, so that for a subsequent increase back to $p_4 \rightarrow p_3$ the reaction is described by a weak play reaction ($F \rightarrow H$).

Actually, interpreted in terms of Fig. 2 the spurt-lines are a kind of continuous “stairway-up/-down” reaction due to aggregation over a large number of heterogeneous firms, and the width γ of the play area is related to the distance between both “stairways”. Of course, using play

¹⁰ In the case of mechanical play there would be even no reaction of y inside the play area (Krasnosel'skii/Pokrovskii, 1989, p. 8).

dynamics with linear segments and a constant play width¹¹ is a simplified way to capture macro/aggregate dynamics. The slope of the branches of the aggregate loop depends on the distribution of the trigger values of the firms and is in general non-linear.¹² However, even non-linear aggregate loops can be seen as *approximated* by the *kinked* play-loop.

In the following, we present the basic principles of a play algorithm which was developed by *Belke/Göcke* (2001) for the analysis of play-hysteresis in employment.¹³ The change (Δp) in the forcing/input variable p may occur either inside the play area inducing a weak reaction or on a spurt line resulting in a strong reaction of the dependent/output variable y (Δy). The movement of p inside the play area is Δa (cumulated as ‘ a ’), and the movement in the spurt area is Δs . We consider a special case, when Δp starts from a spurt-line and enters a play area, denoted as Δp_j . This corresponds to trajectory $B \rightarrow C \rightarrow E$ in Fig. 3. In the past, the movement of p has led to $(j-1)$ changes between the left and the right spurt line. The new change Δp_j may enter the play area to an extent of a_j (in Fig. 3 for point C the distance to point B illustrates distance ‘ a_j ’) or even pass the entire play γ (at point E) and enter the opposite spurt line by the last part Δs_j (i.e. $E \rightarrow F$). These considerations are summarized by:

$$(1) \quad \Delta p_j = a_j + \Delta s_j \quad \text{with: } \Delta s_j = \begin{cases} \text{sgn}(\Delta p_j) \cdot (|\Delta p_j| - \gamma) & \text{if } (|\Delta p_j| - \gamma) > 0 \\ 0 & \text{else} \end{cases}$$

The change (Δy) in the output variable y caused by Δp_j is composed of the weak play reaction ($B \rightarrow E$) and – occasionally – by a strong spurt reaction ($E \rightarrow F$). Let the parameter α denote the slope of the weak play area and $(\alpha + \beta)$ the strong spurt slope:

$$(2) \quad \Delta y_j = \alpha \cdot a_j + (\alpha + \beta) \cdot \Delta s_j \quad \text{with: } |\alpha| < |\alpha + \beta|$$

The play line is shifted vertically by spurt movements. The cumulated vertical displacement V_{j-1} of the relevant play line as a result of all previous movements on both spurt lines is:

¹¹ Since uncertainty results – due to option value effects – in a widening of the band of inaction on a micro-level, increased uncertainty which is prevalent on the whole market (for all firms) will result in a widening of the play area on the aggregate level. For an integration of these effects into a play loop (and an econometric estimation of play dynamics in a situation with variable exchange rate uncertainty for exporting firms) see *Belke/Göcke* (2005).

¹² This would result from using the explicit *Preisach* (1935)/*Mayergoyz* (1986) aggregation procedure.

¹³ Based on Portuguese firm-level data, *Mota* (2008), pp. 99 ff., and *Mota/Varejão/Vasconcelos* (2012) use this linear play-algorithm to estimate and compare *aggregate* employment hysteresis with micro level adjustment patterns. *Belke/Göcke/Günther* (2012) apply the algorithm empirically in order to estimate play dynamics for German exports.

$$(3) \quad V_{j-1} = \beta \cdot \left[\sum_{i=0}^{j-1} \Delta s_i \right] = \beta \cdot s_{j-1} \quad \text{with: } s_{j-1} \equiv \sum_{i=0}^{j-1} \Delta s_i$$

The dependent variable is determined by the shift V_{j-1} resulting from past spurts and by the current reaction Δy_j :

$$(4) \quad y_j = C^* + V_{j-1} + \Delta y_j = C^* + \beta \cdot \sum_{i=0}^{j-1} \Delta s_i + \alpha \cdot a_j + (\alpha + \beta) \cdot \Delta s_j$$

$$\Rightarrow y_j = C^* + \beta \cdot \sum_{i=0}^j \Delta s_i + \alpha \cdot \Delta p_j = C^* - \alpha \cdot \sum_{i=0}^{j-1} \Delta p_i + \beta \cdot \sum_{i=0}^j \Delta s_i + \alpha \cdot (\sum_{i=0}^{j-1} \Delta p_i + \Delta p_j)$$

$$\Rightarrow y_j = C + \alpha \cdot p_j + \beta \cdot s_j \quad \text{with: } C \equiv C^* - \alpha \cdot \sum_{i=0}^{j-1} \Delta p_i \text{ and } p_j = \sum_{i=0}^j \Delta p_i$$

Eq. (4) shows that the complex dynamics of the play loop are captured by a simple linear equation, where only an artificial variable s_j is added. This “*spurt variable*” s_j summarizes all preceding and present spurt movements leading to shifts of the play area. According to eq. (1) the spurt variable s_j is just the series p_j of the original forcing variable where all small movements (a_j) inside the play areas (with width γ) are filtered out. The coefficient β of this “filtered” input series s_j is the *difference in slope* between the play and the spurt reaction regarding price changes. Summarizing, the complex dynamics are captured in a simple linear(ized) way, just by adding only two new parameters to the model: (1) play width γ (for filtering price ‘p’ to get spurt ‘s’), and (2) the slope difference β of spurt sections compared to play sections.

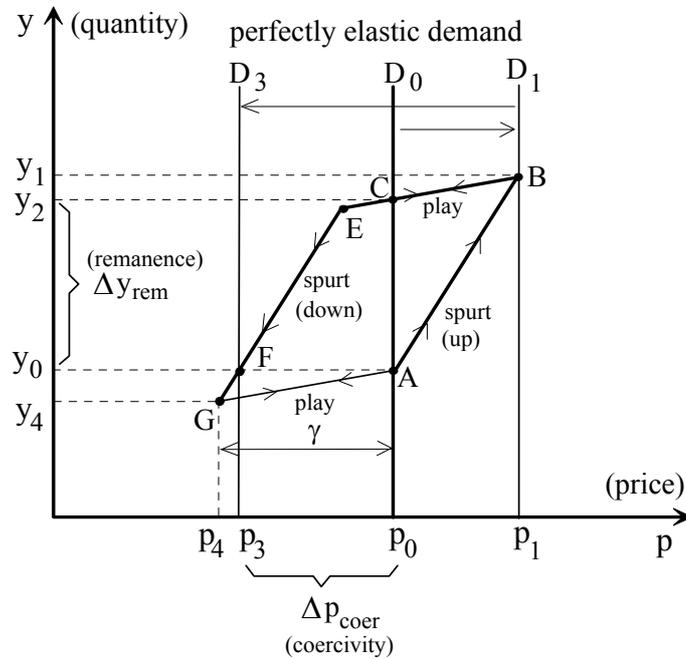
4. Play on the supply-side in a market model

4.1 Perfectly elastic demand and exogenous price

In a situation with perfectly elastic demand, the price level is completely determined by demand. In Fig. 4 (where – as it is *not* common in economics – the price/input is on the horizontal abscissa and the resulting quantity/output is on the vertical ordinate), perfect demand elasticity is represented by a vertical demand curve. Actually, from the supply side’s perspective, the price level is exogenous. Thus, in this special case the forcing variable p of the supply-subsystem of the entire market model is an exogenous variable. Implicitly, the case

of exogeneity of the forcing hysteresis variable is often assumed if hysteresis is modelled in economics.¹⁴

Fig. 4 – *Supply with play and perfectly elastic demand (i.e. exogenous price)*



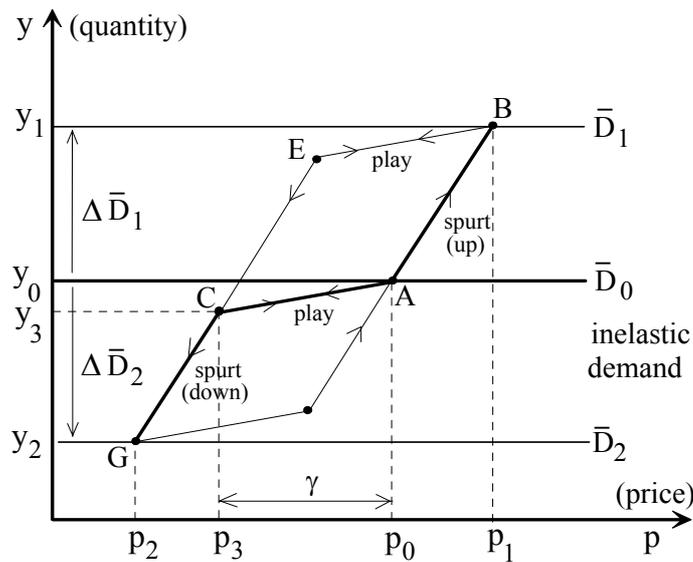
The interpretation is analogous to the explanations in the previous section. However, we can use this simple special case in order to present some further definitions. Starting from an initial situation in point A (Fig. 4, with demand D_0 , price p_0 , and quantity y_0), a demand/price increase to D_1 (p_1) results in a strong spurt reaction on the spurt-up line with slope $(\alpha + \beta)$ to point B with quantity y_1 . A later price decrease back to D_0 (p_0) takes place on a play line with slope α (point C and y_2). Although the price is on its initial level (p_0) again, an after effect – called “*remanence*” – on the quantity remains: i.e. the distance between A and C, resp. $\Delta y_{rem} = (y_2 - y_0)$. With a further price decrease, passing the play area in point E and going on along the spurt-down line, for demand D_3 (price p_3) in point F the initial quantity y_0 is regained. This kind of “overshooting” of the forcing variable $\Delta p_{coer} = (p_3 - p_0)$, which is necessary to reach the initial state of the dependent variable, is called “*coercivity*” or “*coercive force*”. The initial point A is reached again (and a full hysteresis-loop is completed), if the price continues to decrease until p_4 (point G) and if then a price increase passes the play area up to point A.

¹⁴ Counterexamples are e.g. *Baldwin/Lyons* (1994), *Ljungqvist* (1994) and *Göcke* (2001) for a foreign trade subsystem with hysteresis as part of an entire macroeconomic model. There the exchange rate (as the forcing variable for foreign trade) is endogenously determined by the whole macroeconomic model/system.

4.2 Completely inelastic demand and endogenous price

In a market with a perfectly inelastic demand the equilibrium quantity is determined by the fixed demand quantity \bar{D} . Thus, in this special case the level of the output variable of the hysteretic subsystem is exogenously given, while the equilibrium level of the price (as the hysteretic input variable) is determined endogenously. The demand curves now are horizontal if the prices are on the horizontal axis (see Fig. 5). In an initial situation on a spurt line (point A), the quantity is determined by exogenous demand quantity ($y_0 = \bar{D}_0$). An increase of demand by $\Delta \bar{D}_1 = (\bar{D}_1 - \bar{D}_0)$ results in an identical increase in y . If this change takes place on a spurt line (as for trajectory $A \rightarrow B$ in Fig. 5), the resulting endogenous price effect $\Delta p_{(\text{spurt})} = p_1 - p_0$ is relatively small. In comparison, for $\Delta \bar{D}_2 = (\bar{D}_2 - \bar{D}_0)$, if the reaction at first passes play (as for trajectory $A \rightarrow C \rightarrow G$), the resulting price effect $\Delta p_{(\text{pass})} = p_2 - p_0$ is relatively large in size.

Fig. 5 – Supply with Play and inelastic demand (i.e. exogenous quantity)



The supply quantity y is described by the play&spurt equation (index j is omitted for reasons of simplicity):

$$(5) \quad y = C + \alpha \cdot p + \beta \cdot s$$

The demand (quantity) function D in case of an inelastic demand is: $D = \bar{D}$. Thus, market equilibrium is:

$$(6) \quad y = D \Rightarrow \bar{D} = C + \alpha \cdot p + \beta \cdot s$$

A change in the exogenous demand quantity $\Delta\bar{D}$ leads to an endogenous price reaction Δp . This price reaction is different if it takes place inside play or on a spurt line. If the change is continuing a movement on the current spurt line (as e.g. for trajectory $A \rightarrow B$), the large spurt-slope $(\alpha + \beta)$ is relevant. In this case the change in the price Δp is equivalent to the change in the spurt variable Δs :

(7) if continuation on spurt-line: $\Delta p = \Delta s$

$$\Rightarrow \Delta y = \Delta\bar{D} = \alpha \cdot \Delta p_{(\text{spurt})} + \beta \cdot \Delta s = (\alpha + \beta) \cdot \Delta p_{(\text{spurt})} \Rightarrow \Delta p_{(\text{spurt})} = \frac{\Delta\bar{D}}{\alpha + \beta}$$

If the movement appears only inside the play, no change of the spurt variable occurs, and the low play-slope α is relevant:

(8) if inside play area: $\Delta s = 0 \Rightarrow \Delta y = \Delta\bar{D} = \alpha \cdot \Delta p_{(\text{play})} \Rightarrow \Delta p_{(\text{play})} = \frac{\Delta\bar{D}}{\alpha}$

Due to the lower slope α , representing a weak reaction of supply on price changes, the price effect of an exogenous demand change in the play area is stronger than on a spurt line (with slope $\alpha + \beta$).

If the movement starts with entering the play area, according to eq. (1) price changes first appear inside play ('a'), and if play is passed [if $a = \text{sgn}(\Delta p) \cdot \gamma$] going further on the opposite spurt-line ($\Delta s \neq 0$):

(9) if play is passed (starting from spurt): $\Delta p = \text{sgn}(\Delta p) \cdot \gamma + \Delta s$

$$\Rightarrow \Delta s = \Delta p - \text{sgn}(\Delta p) \cdot \gamma = \Delta p - \text{sgn}(\Delta\bar{D}) \cdot \gamma$$

$$\Rightarrow \Delta y = \Delta\bar{D} = \alpha \cdot \Delta p + \beta \cdot \Delta s = \alpha \cdot \Delta p + \beta \cdot [\Delta p - \text{sgn}(\Delta\bar{D}) \cdot \gamma] = (\alpha + \beta) \cdot \Delta p - \beta \cdot \text{sgn}(\Delta\bar{D}) \cdot \gamma$$

$$\Rightarrow \Delta p_{(\text{pass})} = \frac{\Delta\bar{D}}{\alpha + \beta} + \frac{\beta}{\alpha + \beta} \cdot \text{sgn}(\Delta\bar{D}) \cdot \gamma = \text{sgn}(\Delta\bar{D}) \cdot \gamma + \frac{\Delta\bar{D} - \alpha \cdot \text{sgn}(\bar{D}) \cdot \gamma}{\alpha + \beta}$$

$$\Rightarrow \Delta s = \frac{\Delta\bar{D} - \alpha \cdot \text{sgn}(\bar{D}) \cdot \gamma}{\alpha + \beta}$$

In Fig. 5 this is illustrated by trajectory $A \rightarrow C \rightarrow G$ with a decrease in demand of $\Delta\bar{D}_2 = (\bar{D}_2 - \bar{D}_0 < 0)$: If a past upward spurt movement has led to point A on the spurt-up line, now changing the direction means entering the play area. With the price decrease ($p_3 - p_0 = -\gamma < 0$) the play is passed ($A \rightarrow C$). After reaching the opposite spurt-down line the rest of the price effect ($C \rightarrow G$) is captured by a decrease in the spurt variable (with $\Delta s = p_2 - p_3 < 0$). The entire

price effect of the decrease in demand $\Delta\bar{D}_2$ is $\Delta p_{(pass)} = -\gamma + \Delta s = p_2 - p_0 (< 0)$. This entire price effect of demand changes is the bigger the larger is the width γ of the play area.

If the movement starts inside play and passes the rest of the play area, calculation is analogous to eq. (9), however instead of the entire width of play (γ) the remaining distance to the spurt-line must be applied.

4.3 Endogenous price in a situation with “normal” price elasticity of demand

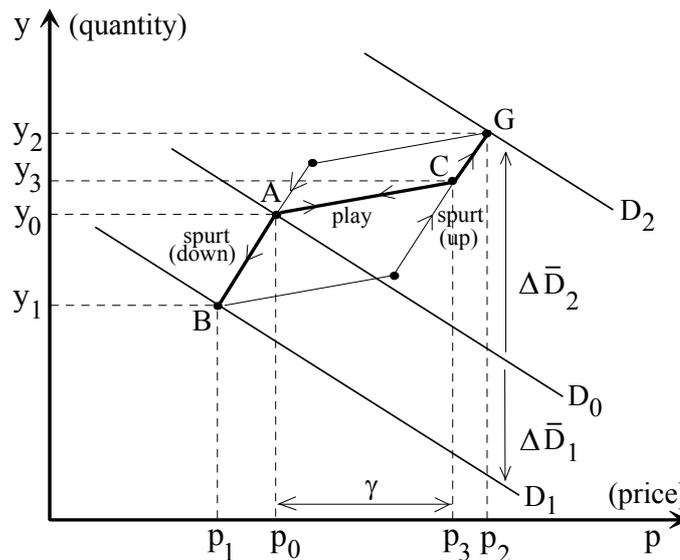
For a more general situation without perfect (in)elasticity the general demand function is:

$$(10) \quad D = \bar{D} - \delta \cdot p$$

Market equilibrium for this general case is:

$$(11) \quad y = D \Rightarrow \bar{D} - \delta \cdot p = C + \alpha \cdot p + \beta \cdot s$$

Fig. 6 – Supply with play and “normal” price elasticity of demand



A change in the exogenous demand quantity $\Delta\bar{D}$ again leads to an endogenous price reaction Δp . If this occurs on a spurt-line (e.g. in Fig. 6 trajectory A → B on the spurt-down for a decreasing demand $\Delta\bar{D}_1 < 0$) this is:

(12) if continuation on spurt-line: $\Delta p = \Delta s$

$$\Rightarrow \Delta y = \Delta \bar{D} - \delta \cdot \Delta p_{(\text{spurt})} = \alpha \cdot \Delta p_{(\text{spurt})} + \beta \cdot \Delta s = (\alpha + \beta) \cdot \Delta p_{(\text{spurt})}$$

$$\Rightarrow \Delta p_{(\text{spurt})} = \frac{\Delta \bar{D}}{\alpha + \beta + \delta}$$

$$\Rightarrow \Delta y_{(\text{spurt})} = \Delta \bar{D} - \delta \cdot \Delta p_{(\text{spurt})} = \Delta \bar{D} - \frac{\delta \cdot \Delta \bar{D}}{\alpha + \beta + \delta} = \frac{\alpha + \beta}{\alpha + \beta + \delta} \cdot \Delta \bar{D}$$

If the movement is only inside the play, there is no change in s :

(13) if inside play area: $\Delta s = 0$

$$\Rightarrow \Delta y = \Delta \bar{D} - \delta \cdot \Delta p_{(\text{play})} = \alpha \cdot \Delta p_{(\text{play})} \Rightarrow \Delta p_{(\text{play})} = \frac{\Delta \bar{D}}{\alpha + \delta}$$

$$\Rightarrow \Delta y_{(\text{play})} = \Delta \bar{D} - \delta \cdot \Delta p_{(\text{play})} = \Delta \bar{D} - \frac{\delta \cdot \Delta \bar{D}}{\alpha + \delta} = \frac{\alpha}{\alpha + \delta} \cdot \Delta \bar{D}$$

If the movement starts with entering the play area and if play is passed and goes ahead on the opposite spurt line (in Fig. 6: e.g. trajectory $A \rightarrow C \rightarrow G$ for an increase in demand of $\Delta \bar{D}_2$) the following results:

(14) if play is passed (starting from spurt): $\Delta s = \Delta p - \text{sgn}(\Delta \bar{D}) \cdot \gamma$

$$\Rightarrow \Delta y = \Delta \bar{D} - \delta \cdot \Delta p = \alpha \cdot \Delta p + \beta \cdot \Delta s = (\alpha + \beta) \cdot \Delta p - \beta \cdot \text{sgn}(\Delta \bar{D}) \cdot \gamma$$

$$\Rightarrow \Delta p_{(\text{pass})} = \frac{\Delta \bar{D}}{\alpha + \beta + \delta} + \frac{\beta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta \bar{D}) \cdot \gamma \quad \Rightarrow \Delta s = \frac{\Delta \bar{D} - (\alpha + \delta) \cdot \text{sgn}(\Delta \bar{D}) \cdot \gamma}{\alpha + \beta + \delta}$$

$$\Rightarrow \Delta y_{(\text{pass})} = \Delta \bar{D} - \delta \cdot \Delta p_{(\text{pass})} = \frac{\alpha + \beta}{\alpha + \beta + \delta} \cdot \Delta \bar{D} - \frac{\delta \cdot \beta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta \bar{D}) \cdot \gamma$$

If a past downward spurt movement has led to point A, now a rising demand, by changing the direction, leads to entering the play area. With the resulting price increase ($p_3 - p_0 = \gamma > 0$) the play is passed ($A \rightarrow C$). After reaching the opposite spurt-up line the rest of the price effect ($C \rightarrow G$) is captured by an increase in the spurt variable (with $\Delta s = p_2 - p_3 > 0$). The entire price effect of the increase in demand $\Delta \bar{D}_2$ is $\Delta p_{(\text{pass})} = \gamma + \Delta s = p_2 - p_0 (> 0)$.

Compared to the simple case with perfectly inelastic demand, the price reactions are now smaller in size (in the play as well as in the spurt area), since a part of the adjustment in a price elastic demand situation is done via adaptation of the demand quantity to changing prices, which is represented by the additional parameter δ in the denominators in eq. (14).

4.4 A “demand cycle” in a situation with normal price elasticity of demand

Now we look at a “cycle”, i.e. a temporary change in demand which is later on exactly/completely compensated. I.e. Starting with an autonomous demand level \bar{D}_0 , a change of $\Delta\bar{D}_1$ results in the level \bar{D}_1 . Later on, the initial level \bar{D}_0 is regained by a change of the same size, but the opposite sign: $\Delta\bar{D}_2 = (-\Delta\bar{D}_1)$. For simplicity, in the following a starting point of a cycle is assumed to be on a spurt line.

4.4.1 A cycle starting with passing play

A cycle which starts with entering the play area, takes place along the play line, as long as the opposite spurt line is not reached. E.g., in Fig. 7, if starting from point A, a demand cycle which does not reach further than point B will just look like a linear forth and back reaction on the play line with a slope of α . However, a cycle which leads to a “full loop” (as depicted in Fig. 7 by the trajectory $A \rightarrow B \rightarrow C \rightarrow G \rightarrow A$), which is caused by a demand cycle of $\Delta\bar{D}_1$ (< 0) followed later by a compensating change of $\Delta\bar{D}_2 = (-\Delta\bar{D}_1)$ is characterized by a path-dependent reaction of both endogenous variables, price p and quantity y .

Fig. 7 – Demand cycle, starting with passing play

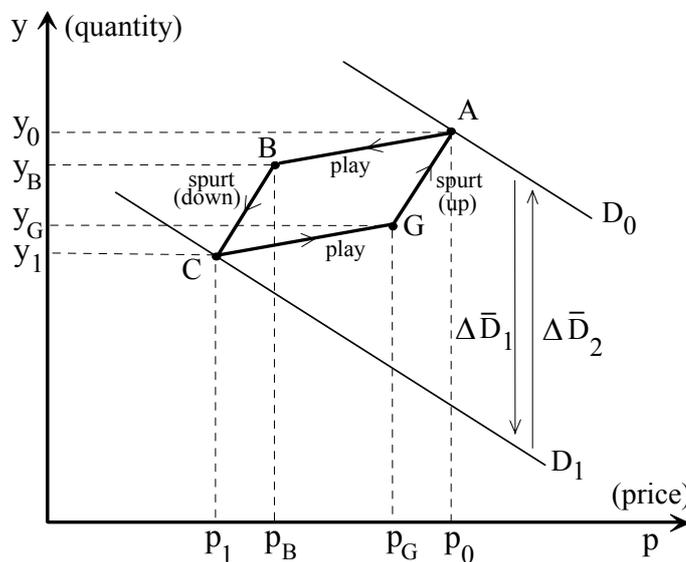


Fig. 8 illustrates in a stylized way the relative dynamics of autonomous demand \bar{D} as well as the resulting endogenous price and quantity reactions on the trajectory $A \rightarrow B \rightarrow C \rightarrow G \rightarrow A$. Starting (at time t_0) from point A, the play reaction from $A \rightarrow B$ on an initial decrease ($\Delta\bar{D}_1 < 0$) is based on a weak under-proportional quantity reaction ($y_0 \rightarrow y_B$) but an over-proportional price decrease ($p_0 \rightarrow p_B$). On the subsequent spurt-down line ($B \rightarrow C$) we see a

weak price ($p_B \rightarrow p_1$) but a strong quantity effect ($y_B \rightarrow y_1$). Later (in t_1), starting the movement back to the initial level, at first passing play ($C \rightarrow G$) results in a weak quantity ($y_1 \rightarrow y_G$) and a strong price effect ($p_1 \rightarrow p_G$), which is followed by a spurt reaction with a weak price ($p_G \rightarrow p_0$) and a strong quantity effect ($y_G \rightarrow y_0$). After finishing the cycle (in t_2) we regain the initial situation in point A and *no remanence* effect remains.

Fig. 8 – Stylized time-path of autonomous demand (\bar{D}), price (p) and quantity (y)

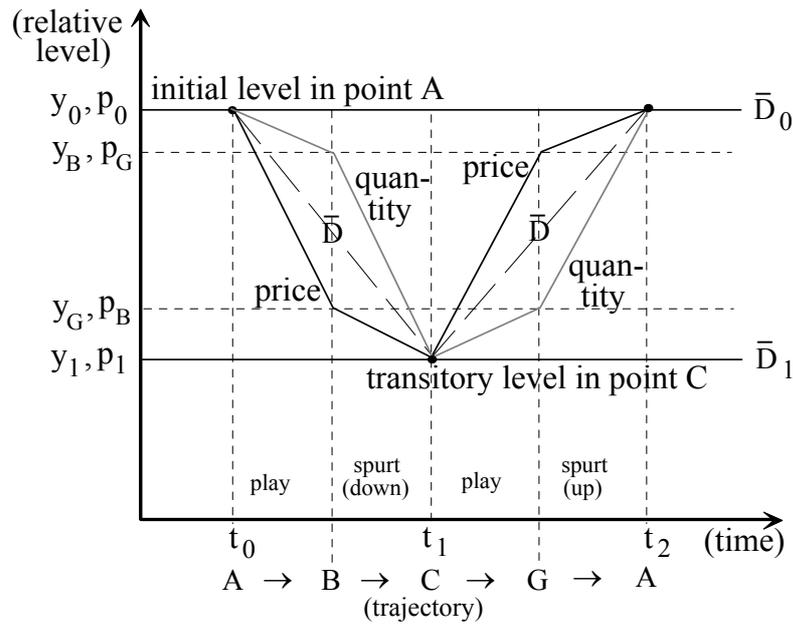
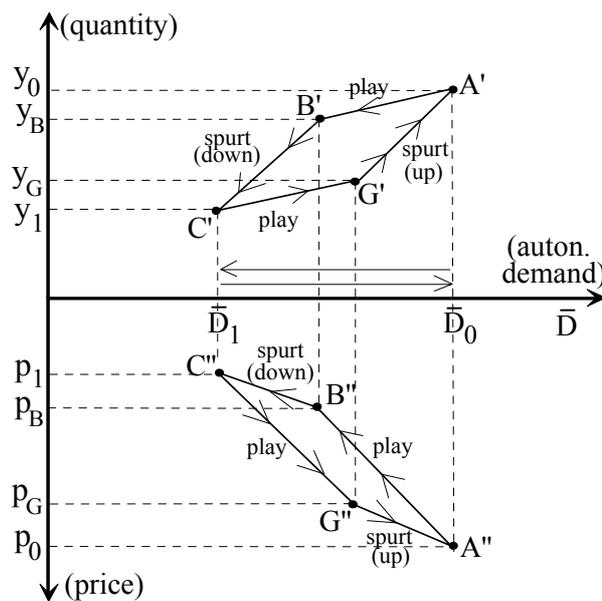


Fig. 9 – Play-loops of quantity (y) and price (p) for an autonomous demand (\bar{D}) cycle



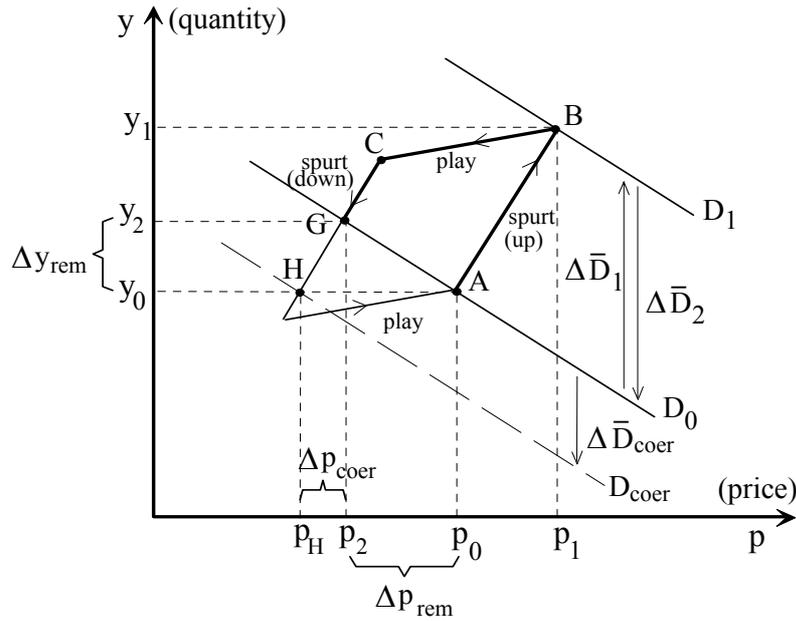
The reaction of price and quantity to changes of the autonomous demand \bar{D} can be illustrated in a (\bar{D}, y) - and a (\bar{D}, p) -diagram, which is done simultaneously in Fig. 9. The resulting loops

look, at first sight, as the play-loops in the (p,y)-diagram. For the (\bar{D} ,y)-loop the assignment is analogous: the flat-slope parts correspond to the play area and the steep parts to spurt lines. However, the (\bar{D} ,p)-loop is different: The strong price reaction corresponds to the play area in the original (p,y)-loop.

4.4.2 A cycle starting with a continuation on the spurt-line

A movement on the spurt line (in Fig. 10, e.g. from point A \rightarrow B, caused by $\Delta\bar{D}_1 > 0$), where the first change is exactly compensated later on (by $\Delta\bar{D}_2 = (-\Delta\bar{D}_1) < 0$, trajectory B \rightarrow C \rightarrow G), results – although the initial demand curve D_0 is valid again – in a permanent after effect of both endogenous variables: as a result of a *temporary increase* of demand a *negative remanence* in the price level ($\Delta p_{rem} = p_2 - p_0 < 0$) and a positive remanence effect in the equilibrium quantity ($\Delta y_{rem} = y_2 - y_0 > 0$) results.

Fig. 10 – Demand cycle, starting with continuation on spurt line



The remanence effects can be calculated using the result of eqs. (12) to (14). According to eq. (12) the price and quantity effect of $\Delta\bar{D}_1$ on the spurt line (point A \rightarrow B) is:

$$(15) \quad \text{on spurt-line (point A} \rightarrow \text{B): } \Delta p_1 = \frac{\Delta\bar{D}_1}{\alpha + \beta + \delta} \quad \text{and} \quad \Delta y_1 = \frac{\alpha + \beta}{\alpha + \beta + \delta} \cdot \Delta\bar{D}_1$$

The price effect of the move back $\Delta\bar{D}_2 = (-\Delta\bar{D}_1)$ on trajectory B \rightarrow C \rightarrow G is at first inside the play area (B \rightarrow C), and according to eq. (13):

(16) inside/passing play line (point B → C):

$$\text{sgn}(-\Delta\bar{D}_1) \cdot \gamma = \Delta p_{(B \rightarrow C)} = \frac{\Delta\bar{D}_{(B \rightarrow C)}}{\alpha + \delta} \quad \text{and} \quad \Delta y_{(B \rightarrow C)} = \frac{\alpha}{\alpha + \delta} \cdot \Delta\bar{D}_{(B \rightarrow C)}$$

$$\Rightarrow \gamma = \text{sgn}(-\Delta\bar{D}_1) \cdot \frac{\Delta\bar{D}_{(B \rightarrow C)}}{\alpha + \delta} \Leftrightarrow \Delta\bar{D}_{(B \rightarrow C)} = \text{sgn}(-\Delta\bar{D}_1) \cdot (\alpha + \delta) \cdot \gamma$$

Only if a cycle is big enough in size, the movement back will pass the whole play area (as it is depicted in Fig. 10). This passing of play (with point C) requires a size of $|\Delta\bar{D}_1| = |\Delta\bar{D}_2| > |\Delta\bar{D}_{(B \rightarrow C)}| = (\alpha + \delta) \cdot \gamma$. If the entire play is passed, the remaining reaction (C → G) takes place on the opposite spurt line:

(17) ‘rest’ on opposite spurt line (point C → G):

$$\text{with} \quad \Delta\bar{D}_{(C \rightarrow G)} = \Delta\bar{D}_2 - \Delta\bar{D}_{(B \rightarrow C)} = -\Delta\bar{D}_1 + \text{sgn}(\Delta\bar{D}_1) \cdot (\alpha + \delta) \cdot \gamma$$

$$\Rightarrow \Delta p_{(C \rightarrow G)} = \frac{-\Delta\bar{D}_1 + \text{sgn}(\Delta\bar{D}_1) \cdot (\alpha + \delta) \cdot \gamma}{\alpha + \beta + \delta}$$

The entire effects of the movement back $\Delta\bar{D}_2$ are – if play is completely passed – analogous to eq. (14):

$$(18) \quad \Delta p_2 = \Delta p_{(B \rightarrow C)} + \Delta p_{(C \rightarrow G)} = -\text{sgn}(\Delta\bar{D}_1) \cdot \gamma + \frac{-\Delta\bar{D}_1 + \text{sgn}(\Delta\bar{D}_1) \cdot (\alpha + \delta) \cdot \gamma}{\alpha + \beta + \delta}$$

$$\Rightarrow \Delta p_2 = \frac{-\Delta\bar{D}_1}{\alpha + \beta + \delta} + \frac{\beta}{\alpha + \beta + \delta} \cdot \text{sgn}(-\Delta\bar{D}_1) \cdot \gamma$$

$$\Rightarrow \Delta y_2 = -\Delta\bar{D}_1 - \delta \cdot \Delta p_2 = \frac{\alpha + \beta}{\alpha + \beta + \delta} \cdot (-\Delta\bar{D}_1) - \frac{\beta \cdot \delta}{\alpha + \beta + \delta} \cdot \text{sgn}(-\Delta\bar{D}_1) \cdot \gamma$$

The resulting endogenous permanent/remanence effects of the entire temporary $\Delta\bar{D}_1 - \Delta\bar{D}_2$ -cycle (A → B → C → G) on price and quantity now are [if play is passed on the move back, i.e. if $|\Delta\bar{D}_1| = |\Delta\bar{D}_2| > |\Delta\bar{D}_{(B \rightarrow C)}| = (\alpha + \delta) \cdot \gamma$]:

$$(19) \quad \Delta p_{\text{rem}} = \Delta p_1 + \Delta p_2 = \frac{-\beta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma$$

$$\Delta y_{\text{rem}} = \Delta y_1 + \Delta y_2 = -\delta \cdot \Delta p_{\text{rem}} = \frac{\beta \cdot \delta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma$$

These results demonstrate, that the hysteretic after-effects are the more severe, the larger is the difference in slope (β) between play and spurt lines, and the larger is the play distance (γ) between both spurt lines, i.e. the more ‘kinked’ and ‘blown-up’ the loop looks like.

If the size of a cycle is not big enough to pass the play area on the move back [i.e. $\Delta\bar{D}_2$ is so small that point C is not passed, this is relevant for $|\Delta\bar{D}_1| = |\Delta\bar{D}_2| < |\Delta\bar{D}_{(B \rightarrow C)}| = (\alpha + \delta) \cdot \gamma$], the remanence effects are:

$$(20) \quad \Delta p_{\text{rem,play}} = \Delta p_1 + \Delta p_{(\text{play})} = \frac{\Delta\bar{D}_1}{\alpha + \beta + \delta} + \frac{-\Delta\bar{D}_1}{\alpha + \delta} = \frac{-\beta}{(\alpha + \beta + \delta) \cdot (\alpha + \delta)} \cdot \Delta\bar{D}_1$$

$$\Delta y_{\text{rem,play}} = -\delta \cdot \Delta p_{\text{rem,play}} = \frac{\beta \cdot \delta}{(\alpha + \beta + \delta) \cdot (\alpha + \delta)} \cdot \Delta\bar{D}_1$$

In order to regain the initial quantity level of the dependent variable quantity (y_0 , in point H) the demand has to ‘overshoot’ its initial level: An additional counter move of the exogenous variable which is overcompensating the initial shock ($\Delta\bar{D}_1$) is necessary. In the case of a “big” cycle (passing the play, $|\Delta\bar{D}_1| > (\alpha + \delta) \cdot \gamma$), the *coercive demand force* $\Delta\bar{D}_{\text{coerz}}$ takes place on the opposite spurt-line and must correct for the quantity remanence Δy_{rem} . Moreover, this extra demand change will induce an additional *coercive* price change Δp_{coer} ($= p_H - p_2$, in Fig. 10), with the same direction as the price remanence effect ($\Delta p_{\text{rem}} = p_2 - p_0$). Since the coercivity change occurs on the opposite spurt-line, the following condition must hold:

$$(21) \quad \text{cond. (I), reaction on spurt-line: } (-\Delta y_{\text{rem}}) = (\alpha + \beta) \cdot \Delta p_{\text{coer}}$$

$$\Rightarrow \frac{-\beta \cdot \delta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma = (\alpha + \beta) \cdot \Delta p_{\text{coer}} \Rightarrow \Delta p_{\text{coer}} = \frac{-\beta \cdot \delta}{(\alpha + \beta + \delta) \cdot (\alpha + \beta)} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma$$

Thus, in order to regain the initial quantity level (y_0), e.g. after a temporary demand increase [$\text{sgn}(\Delta\bar{D}_1) > 0$], there must be a compensation by a persistent price remanence effect Δp_{rem} plus an additional price coercive effect Δp_{coer} . The sum of both price effects ($\Delta p_{\text{rem}} + \Delta p_{\text{coer}} = p_H - p_0$) which is necessary for regaining the initial quantity y_0 (in point H) is:

$$(22) \quad \Delta p_{\text{rem}} + \Delta p_{\text{coer}} = \frac{-\beta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma + \frac{-\beta \cdot \delta}{(\alpha + \beta + \delta) \cdot (\alpha + \beta)} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma$$

$$= \frac{-\beta}{\alpha + \beta} \cdot \text{sgn}(\Delta\bar{D}_1) \cdot \gamma$$

The adequate exogenous coercivity demand change $\Delta\bar{D}_{\text{coer}}$ can be calculated using the condition that the demand curve must be valid in the new path-dependent equilibrium:

$$\begin{aligned}
 (23) \quad \text{cond. (II), reaction on demand curve: } & (-\Delta y_{\text{rem}}) = \Delta \bar{D}_{\text{coer}} - \delta \cdot \Delta p_{\text{coer}} \\
 \Rightarrow \frac{-\beta \cdot \delta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta \bar{D}_1) \cdot \gamma &= \Delta \bar{D}_{\text{coer}} - \delta \cdot \frac{-\beta \cdot \delta}{(\alpha + \beta + \delta) \cdot (\alpha + \beta)} \cdot \text{sgn}(\Delta \bar{D}_1) \cdot \gamma \\
 \Rightarrow \Delta \bar{D}_{\text{coer}} &= \frac{-\beta \cdot \delta}{\alpha + \beta + \delta} \cdot \text{sgn}(\Delta \bar{D}_1) \cdot \gamma + \delta \cdot \frac{-\beta \cdot \delta}{(\alpha + \beta + \delta) \cdot (\alpha + \beta)} \cdot \text{sgn}(\Delta \bar{D}_1) \cdot \gamma \\
 &= \frac{-\beta \cdot \delta}{\alpha + \beta} \cdot \text{sgn}(\Delta \bar{D}_1) \cdot \gamma
 \end{aligned}$$

Thus, as with the remanence effects in eq. (20), the additional coercive demand change $\Delta \bar{D}_{\text{coer}}$ must be the larger the larger is the difference in slope (β) between play and spurt lines, and the larger is the play distance (γ) between both spurt lines. However, the lower is the reaction (δ) of the demand on price changes, the smaller is this coercive demand force $\Delta \bar{D}_{\text{coer}}$; and for perfectly inelastic demand ($\delta=0$) no coercive demand change is necessary to regain the initial quantity (as illustrated by Fig. 5).

In markets with factors inducing path-dependent hysteretic behaviour – which is an implication in the case of sunk adjustment costs and thus should be very realistic – merely transient exogenous disturbances (as e.g. by $\Delta \bar{D}_1$) can have permanent effects on the equilibrium level of the endogenous variables, i.e. prices and quantities. In order to overcome these after effects on the dependent variable (supply quantity) the exogenous disturbance must be overcompensated by an extra/coercive change (by $\Delta \bar{D}_{\text{coer}}$ with the opposite direction of the initial shock $\Delta \bar{D}_1$). However, the consequence of the exogenous coercive (demand) force is even an additional effect (Δp_{coer}) on the equilibrium of the price level.

5. Conclusion

In this paper the consequences of aggregate hysteresis on the supply side on the market equilibrium were illustrated. The path-dependent sub-system supply was only a part of the entire market model, while the forcing variable of the hysteric supply (the price) and the dependent variable (supply quantity) were both endogenous from the perspective of the entire market. This results in feedback effects on the equilibrium levels of both endogenous variables, price and quantity, if merely temporary exogenous disturbances affect the market for some time. Modelling of hysteresis was performed in a non-standard way: Not micro-level discontinuous-switching (between activity and inactivity) type path-dependence (so called “non-ideal relay”) was applied, but – more adequate if *aggregate* market supply is addressed

– hysteresis was modelled by a continuously switching type of aggregate/macro hysteresis which shows similarities to *mechanical play*. This allows capturing quite complex path-dependent dynamics in a relatively simple way, just by two additional parameters leading to a linear supply equation: (1) the width of the inaction/play and (2) a difference of slope between play (inaction) zones and spurt (strong reaction) areas. Play-hysteresis is formally captured by a linear equation, extended by an additional variable (“spurt”), which is just the forcing variable where small changes (play) are filtered out. Due to this simple structure, the utilisation of play-hysteresis as part of more complex theoretic models is straightforward. Furthermore due to this formally simple linearized structure it is directly applicable to econometric estimation (as it was done for a single equation / partial equilibrium model by *Belke/Göcke/Günther*, 2012, and *Mota/Varejão/Vasconcelos*, 2012).

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