Micro-Macro-Relations in the Kirk-Coleman-Model

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Abstract

The subject of this work is the analysis of level-transition and emergence problems. Philosophical argument is basically grounded on concepts of object identity and manipulation. It is furthermore extended to criticism of the methodology of bridge-hypotheses as proposed by methodological individualism in the social sciences. In order to provide an actual example, the proposed methodology is implemented: Area of application is the classical Kirk-Coleman-Model, a simulation of interaction behavior in a three-person group. In order to meet the requirements of the proposed methodology of level-transitory explanation the model is modified and implemented employing the powerful Bayesian-Network formalism.

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Chapter 1

Introduction

The question of level-transitory explanation lies on the very core of both individualistic and collectivistic social science.

The possibility of individualistic sociology depends on a solution to this question, and this question is commonly answered by the famous "Micro-Macro-Scheme," which goes back to McCLELLAND [36] and which is usually attributed to COLEMAN [12]. The core concept is to explain collective behavior from individual actions, to name the levels considered. The same question of level-transition is answered in a different way by collectivistic sociology: It is taken for granted that properties of collectives cannot be explained across levels, because of an irreducible process of "emergence". A classical presentation of the positions is given by ALEXANDER et al. [1].

The continuing actuality of the question is reflected by recent works which discuss the issues of reduction and emergence with reference to new theoretical developments from the field of Philosophy of Mind, compare for example HEINTZ [20] and SAWYER [51] [52].

Both level-transitory explanation and emergence are possible, and the intention of this work is to provide a coherent philosophical treatment of the problem which is based on new epistemological insights. Additionally, a further aim is to show an abstract but actual implementation of level-transitory explanation.

We will see that the answers to the philosophical question of emergence are not so easily obtained as proposed by the popular schools of individualism and collectivism. In fact, the problem of level transition mixes ontology and epistemology, namely at the particular definitions of *object* and *level*. Coherence of my treatment of this problem will come at the cost of loss of sincerity in knowledge, namely in a way that compromises both Logical Positivism and Critical Rationalism by the introduction of *decision* and consequently *meaning*[1] to Theory of Science. Employing this account, a criticism of

[1] I will rather relate meaning to the concept of *use* than to the concept of *truth*, to name my viewpoint.

common individualistic methodology will follow. (The topics are treated in sections [2,3 and 6].)

Since an actual implementation of the philosophic considerations is part of this work, employment of rather sophisticated mathematical methods is necessary. I will use and introduce the so called *Bayesian Network Formalism*, which is a variety of probability theory. Furthermore I will declare a procedure to define system models in the language of Bayesian Networks. (Sections [4 and 5];)

The actual modelling concentrates on reformulation of the classical *Kirk-Coleman-model* (KIRK / COLEMAN [29]) a simulation model of interaction in a three person group, developed in the late sixties. After introducing the original work, the theoretic circumstances of its modifications will be sketched. Subsequently, the modified model is declared formally as a Bayesian Network. (These topics can be found in sections [7, 8 and 9].) I should note that, because of its simplicity, the model should rather be considered as a toy application.

After a conduction of a sensitivity analysis of the modified Kirk-Coleman-model (section [10]) a level transitional explanation is finally realized in accordance to the proposed methodology (section [11]).

I hope the reader will accommodate to my rather packed style of writing. An apology in advance: My personal focus was rather on coherence and precision than on convenient readability.

Chapter 2

Level-Transitory Explanation and Emergence

Within this section I will introduce my general approach to these subjects.

In both cases, level-transitory explanation and emergence, the task consists in delivering conceptual coherent explanations when faced with a phenomenon which can be described on different levels of organization. (For instance on an individual and a societal level.)

I will not try to give a straight ontological solution: By definition, the problem poses that the different levels of the system assume the same status of reality. This raises the question of how their properties should relate to each other, which again instantly yields the general question on nature of properties and objects themselves.

2.1 Epistemic Account on Object Identity

Trying to solve the puzzle of relation between levels in purely ontological terms should be hopeless, but a trick may exist to bypass this question. A solution which is based on on epistemic argument would suffice for most purposes:

The first thing we have to start with, is the observation that objects (which might be for instance persons, electrons or social groups) can apparently be identified as such, regardless of their level of composition. The interesting question is now, what criteria are necessary for guaranteeing such an identification?

My answer originates in the ontological concept of *autonomy*[1] respectively *locality* of the mechanisms which are defining an objects identity. This concept has been proposed

[1] In my use of the word *autonomy* is closer to *isolatability* than to *self-regulation* which is a stronger assumption.

by PEARL [47] in his account on causality.[2]

It should furthermore be noted the the concept of locality of mechanisms implicitly defines the concept of *level*, since the lower level is exactly the set of objects which are (directly or indirectly) interacting by virtue of their defining mechanisms. This leads to the conclusion that level transition might necessarily violate the concept of level and therefore object identity.

Nevertheless, there exists the "trick" which was previously mentioned, namely the preservation of object identity in nested sets of objects (as demanded by the problem) is enabled by attributing autonomy of objects rather to *conception* than to reality. [3]

The results are immediate: This way, by employing *perceived autonomy*, the definition of identity of objects on higher levels does not interfere with a description in terms of the lower level (compare the sections [System] and [Macro States and Initial Conditions]). As long as the higher level behaves as if it were an object I can call it such, regardless of questions of composition. I will subsequently show that considerations regarding causality result in the claim that perceived autonomy may be one of the best ontological "estimators" one can get. (Compare section [Reduction and Manipulation].)

Consequently, because existence of elements on all levels and labels of their properties are granted by how the question is posed, the problem of level-transitory explanation reduces to finding an adequate map from one level to the other.

2.2 Realization of Macroscopic Properties

Because of the above arguments, if a higher and a lower level event occur together,I will say that the lower level event takes part in the *realization* of the higher level event. Furthermore, since a higher level entity is always defined on a set of lower level objects, all objects of this set will have to take part in an event in order to realize the macroscopic one.

Summarized, configurations of lower level events realize a higher level event. This can be thought of as an additional definition of a specific higher level property.

It is important to notice, that following this argumentation the higher level properties are not proposed to be *supervenient* (SAWYER [51]), which means that the higher level property is realized by the lower level objects and their relations, but is not necessarily "irreducible" to these. Perhaps the most important point is, that realization is something different from *causation*: The lower level objects are already *causally*

[2]PEARL defines the autonomy of an mechanism via considerations regarding conditional independence of certain variables in scope. Exact definition is given by his criterion of *Markov-Parentship*.

[3]A neat additional consequence is the reduction of ontological assumptions. Nevertheless, this does not hinder the existence of reality, it is just a more conservative point of view.

determined by its defining lower level mechanisms. Realization adds nothing to this. (Compare SCHLICK [54].)

2.3 Level-Transition

A direct result of the consideration of microscopic realization of macroscopic states is the definition of a macroscopic state as a function defined on the configuration-space of the lower level objects, resp. as a *function of the systems state-space.*

This function relates to the expressions on both levels and might typically be a kind of *classification* of the systems state-space. Given this exemplary case of classification, higher level statements are equal with collections (resp. aggregations) of lower level statements, which fits to the argumentation of the last section.[4]

Practically, now level-transitional explanation means usually to deduce dependencies between single *microscopic* objects and collections of them.[5], followed by aggregating (e.g. averaging) them over the defined classes of realization. Unfortunately, this will only be feasible for small systems, since the size of the state-space will explode otherwise[6]

As mentioned in the [Introduction], the section [Level-Transition Instantiated] provides an implementation of the proposed methodology.

2.3.1 Bridge Hypotheses and Violation of Object Identity

The functions connecting the different levels, as introduced in the previous section, are usually called *bridge hypotheses* (compare NAGEL [41] and OPP [44]).[7] It should be noteworthy to mention that above considerations expect bridge hypotheses to have *no* explanatory power. If they had, they would necessarily violate object identity on the lower level: Since this *level* is defined by the mechanisms of the objects it contains (as stated in the section [Epistemic Account on Object Identity]), a bridge-hypothesis mechanism would make the effecting macro-object a micro-object at the same time. A further result would be *causal over-determination*, since a bridge hypothesis would exceed the set of the lower level object's defining mechanisms.

In order to avoid these paradoxes, bridge-hypotheses are required to be mere definitions. The core explanation takes place in the system which is defined on a *single*

[4] Following the argumentation of lower level realization, the map from the lower to the higher level is requested to be a many-to-one map. Nevertheless it is possible for the higher level object to be defined by different attributes, with some of them being elementary (non-decomposable). These elementary attributes of the higher level will then necessarily be connected to the lower level via a one-to-one map. However, all maps together will still embody the concept of realization for level transition to make sense.

[5] I will use the Bayesian-Network formalism for reasoning on microscopic level.

[6] Sampling the state-space might be an initial solution.

[7] I will give additional discussion on this concept in the section [Semantics and Bridge-Hypotheses].

level which will be the lower one, because of its higher informational content. (Further discussion of the topic can be found in section [Explicit Application of the Micro-Macro-Scheme].)

Nevertheless, bridge hypotheses are very important concepts in social science since they code assumptions on processes which cannot be considered in every detail. Their inherent ontological paradoxy does not make them useless statements, as long it is controlled.

2.3.2 Proxy-Descriptions

Given that a description of the lower level process cannot be found, an aggregate description may certainly be used as a *proxy* for knowledge of the fundamental process. Practically, this will be necessary in most applications, typically for reasons of lacking data.

For this purpose an additional assumption will be necessary: Aggregate measures of the system (as they can be gained economically by sampling) usually need to be backed up with assumptions on structural homogeneity in order to serve as a proxy of the microscopic process. The assumption of homogeneity of unobserved structures can be justified by an argument similar to *OCKAHM's Razor* or LAPLACE's *Principle of Insufficient Reason*, which applies in this case as follows; Structural homogeneity is the simplest assumption as well as the expectation of random structures where every particular structure is assigned equal probability, because none can be preferred because of ignorance.

2.4 Reduction and Manipulation

By employing this methodology, level-transitional conclusions can be logically achieved, at the cost that the higher level seems merely more than a "useful manner of speech", how SCHLICK [54] called it.

But this may be true for everything. The definition of mechanisms (and therefore objects) seems to depend on *considerations on manipulation* as causality might be regarded generally (compare BISCHOF [5], PEARL [47] and VON WRIGHT in SOSA et al. [61]).

Emphasized: *Wether something is said to be an object might depend on our ability to manipulate it.*

Given this view, anything which can be attributed a use (a reason for manipulation) could be said to be possibly real. Usefulness seems to me to be the boundary of insight of reality: Given usefulness, logical coherence is just a means, depending on the features of the objects and mechanisms in focus.

This is the reason why I do not propose reductionism as a general ontology. It has the advantage of logical coherence, but intuition of both logic and object identity might be dependant on the needs of the human observer.

Assuming a level where it makes sense, while being reductionistically coherent only during level transition might be a more practical approach.[8]

2.4.1 Hypothesis or Definition?

The above result may certainly seem peculiar from the viewpoint of Theory of Science in its tradition of Logical Positivism and Critical Rationalism. (Compare for example SCHNELL et al. [55] or SEIFFERT / RADNITZKY [56].) The reason for this is that Theory of Science hides metaphysics (and thus ontology) in considerations on logical structures of theories.

For example, I might be expected to answer the serious question wether a bridge hypothesis is requested to be a "hypothesis", in form of an *implication*, or a "definition", in form of an *equivalence*.

Let me show the syntactic difference between those forms of propositions. In the case of *equivalence*, both antecedent and apodosis of the proposition are fixed. There cannot be any variation in it. In the case of *implication*, the antecedent is free and subject to variation. The point is that this difference fits nicely to the ideas of metaphysical *inertness* (for equivalence) and *productivity* (for implication). These concepts are *necessary* for the question to make sense.[9]

Returning to above question on bridge-hypotheses, it now seems that I am asked to answer an ontological question and no formal one, although this is not explicit anyhow. Needless to say that empirical methodology, being coherent to logicism, adheres to a different concept of observation (and thus intelligibility of reality) than the one proposed in the section on [Reduction and Manipulation]. Therefore the question wether bridge hypotheses are expected to be "definitions" or "hypotheses" wether can only be answered by regarding to the mentioned metaphysical considerations. The answer in my terms

[8] The pragmatic argument does not mean that I deny reality and truth or the need for ontology: it fits to the idea of manipulation being the substrate of causality and is furthermore epistemologically rather straightforward, as compared to the stronger criterion of correspondence between proposition and reality. More of this in the section on [Aspects of Modelling].

[9] I just want to remark that definitions and hypotheses do not necessarily need to be represented by the two mentioned constructs. Every computer program employs directed *assignment-operators* rather than equivalences for representation of definitions. Syntax is simply a question for the range of its coherence with semantics. As long as it works, it works. (Compare section [Aspects of Modelling].) Similar argument is employed by BUNGE [7] in order to attack RUSSEL's proposition of differential equations being the only possible formulation of natural laws.

is, that I request *inertness* being an attribute of bridge-hypotheses, but this does not necessarily deny *productivity* on a specific level. (Compare [Reduction and Manipulation].)

According to a metaphysics of correspondence between proposition and reality, for example Logical Positivism or its successor Critical Rationalism, this ambiguity would be senseless and contradictory. But it is not for a weaker metaphysics of Pragmatism as advocated in section [Aspects of Modelling].

2.5 Emergence

My preceding arguments concerning level-transitory explanation were based on an epistemic definition of the objects discussed. This now allows for an account on emergence, which is also indecisive regarding ontology.

The previous problem took the higher level for predefined, which is obviously not the case for emergence. Consequently, the task consists in defining its criterion in a way that it can coherently serve as *input* for my approach to the problem of level transition. This instantly yields in the following proposition: Collections of objects should be attributed emergent properties if they behave *autonomous* (see above) on at least a conceptual level.[10]

This is the case if the system or subsets of its elements exhibit *self-regulation* or *self-organization*. The first means that the system compensates for outside *disturbances* (BISCHOF [5]) while the second can bee seen as the tendency of a system to reach certain *steady-states* (BERTALANFFY [4]) given a certain range of environmental conditions. Both phenomena have in common that usually only a subset of the possible state-space is realized, and that this realized states are *functional* (partial probability-increasing) towards themselves (as mentioned) given a certain range of environmental conditions.

Now, by following the argumentation of the last section, I will define emergent properties as *classification*[11] *of the auto-functional subsets of the systems state-space.*

It is to be mentioned that this is no ontological definition of emergence, since it builds again on the concept of *perceived autonomy.* I just turned the direction of argumentation, compared to the approach to level transition: Self-regulation and self-organization are strong grounds for the attribution of autonomy.

In order to conclude the discussion of emergence, I should emphasize the fact that I am neither discussing properties of the auto-functional subsets of the systems state space nor the process of reaching them. The question of steady states and derivation of

[10] With regard to systems, autonomy is encoded by definition of the systems boundary. (Compare [System].)

[11] Or more generally as a function.

emergent properties is beyond the scope of this project[12], although I am very interested in it.[13]

2.6 Remarks on Complexity

An argument from the emergentist part of Philosophy of Mind, is denial of the possibility of microscopic modelling because of extremely complex patterns of "wild disjunction" of the possible instantiations of a macro-state, as advocated by SAWYER [51] [52] and mentioned by HEINTZ [20].

This argument is certainly challenging, but I will only give a short reply. Thanks to the possibility to cope with multiple dependencies granted by the use of probability theory, the "*complexity*" (e.g. difficult decomposability with respect to both structures and mechanisms)[14] of a system is not a problem anymore.[15] The problem remains its *size*.

If "wild disjunction" is interpreted before this background, a possible approach to solve this problem may be investigating the structure of the auto-functional subsets of the state-space, as defined in the emergence-section, for instance by considerations about the evolutionary advantage (e.g. *functionality*) of hierarchical structures defined on the systems components as done by SIMON. [59] [60]

Allow me a final remark: I guess that complexity, in a sense of exhausting human conception, may be responsible for occurrence of the question on levels. If we are faced with systems that are simple enough to be elementarily described, no one expresses his considerations in terms of levels. This becomes an option, when systems begin to refuse revealing information on their elementary processes and a gross-treatment becomes necessary for understanding. [16]

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[12] One could say, that a practical solution of this questions (or simply a definitory bypass) is a prerequisite for the epistemic approach on level transition.

[13] I only can recommend literature at the moment: the work of STEGMUELLER [62] provides an excellent general account on self-regulation, while the work of BISCHOF [5] is extraordinary, both with respect to teaching (it introduces time-discrete control-theory and information theory) and fundamental research (it describes a method for defining semantics on evolutionary systems without having to invoke intentionality). The works on General System Theory of BERTALANFFY [4] and Hierarchical Systems by SIMON [59] [60] can be considered classical. Right now, NICOLIS' and PRIGOGINE's book [43] on Dissipative Systems is on my bed table, but I will not finish it before delivering my thesis.

[14] This use of *complexity* is not necessarily isomorphic with the information-theoretic use of the word, where *Kolmogorov-Complexity* designates the informational content of a signal by referring to the length to the shortest program capable to produce that signal. This measure of complexity can be said to answer the question how complicated a possible "explanation" of the signal might be. Im not deep enough into this particular problem in order to treat it employing mathematical notions, but nevertheless the idea should become clear.

[15] This is discussed in the "Mathematics"-section

[16] This certainly does not mean that the lowest level of consideration is necessarily "real". Confounding of the conception of being with the conception of manipulation may be true for every level.

Chapter 3

Aspects of Modelling

Now I want to introduce the notions of *model* and *model-calculus*, followed by some considerations about modelling as a scientific methodology. [1]

3.1 Model

A *model* is essentially a map, but with certain characteristics: there exists 1. a so called *target set*[2] which is the system of interest, and 2. a so called *image-* resp. *model set*, into which the map proceeds. The discriminating point is the *intelligibility*, or more general *usefulness* of the image set compared to the target set. This is the reason for building models.

Furthermore, target- and image set can be compared with respect to their composition: the subset of elements of the target which is not represented in the image is called *preteriton-class* of the model-relation, while the subset of elements of the image which have no complement in the target is called *abundance-class* (see TROITZSCH [65]). This comparison is very interesting, because it shows the limitations of the model with regard to its "original" and therefore allows to investigate the actual relation between target- and model set.

3.2 Model-Calculus

Perhaps the most exciting and useful models are *mathematical models*. These are defined by the fact that the *image set is a subset of a domain of a calculus*. I will call this calculus operating on the image set of a model-relation *model-calculus*. A different way of articulating this is to say that the model-relation defines a *semantic* on the calculus (see CARNAP [9]).

[1] I apologize for introducing the method of modelling this late, although its omnipresence within this work. It is still abstract enough, I guess.

[2] The direct mathematical translation would be *preimage*, which is a little bit cryptic for our purposes.

The employment of a model-calculus allows 1. for "automatization" of the image set by providing rules of computation and 2. proving assumptions about the general behavior of the image set.

Certainly, a model-calculus may be hard to understand, but work invested here will pay off: large problems will not be resolved without automatization while complicated problems are longing for exactly defined semantics.[3]

3.3 Modelling as a Mindset

The advantages of modelling as an approach to science consist of flexibility and creativity. Since it is not imperative to generalize the applicability of the image set and its associated calculus beyond the scope of the target set, one is free to *choose* syntax and semantics (or resp. the kind of model) according to demands of the problem and the simplicity of the own mind. Certainly, gain of generality is not hindered by this approach, since generality is a hallmark of good theory. But nevertheless, *applicability* seems to be necessary criterion of a theory which corresponds to reality.

Applicability will be my central criterion for a good model, besides *generality* and *simplicity*. Generality as a result of abstraction might well be demanded from a model of complex processes. Therefore the claim for simplicity can be justified by considerations regarding the own mental capabilities, as implied above.[4]

This "ethical" argumentation may sound inadequate to the subject. But even if it looks differently, I am a believer with regard to synthetic truth. But I will not talk about justification of the genuine truth of my models, given they are of limited *use* for my purposes.

This criterion is weaker than for instance the most elaborate criterion of Critical Rationalism as posed by LAKATOS [30]. But it allows thought to flow more freely than the latter. I hold a similar attitude towards structuralism (see STEGMUELLER [63] or TROITZSCH [65]). Rational reconstruction of a domain of knowledge may be of greatest value, granted that there is already enough substance within this domain of knowledge. My claim regarding acquisition of this substance is: One needs to *dare* new theory.[5]

[3] This is often not the case with MABS.

[4] One could also say that the two lower criteria flow from applicability by invoking stupidity and laziness.

[5] Certainly there is much more to know in the field of Theory of Science. Nevertheless, one thing is never treated: the need for decision. The idea of truth contains too much ambiguity to give us specific orders.

Chapter 4

A Sketch of Bayesian Networks

During this section I will briefly introduce the employed mathematical apparatus.

4.1 Sketch of the Method

While I proposed in the philosophy-section that level-transitory explanation can be accomplished by aggregating dependencies between single microscopic objects and collections of them, it remained unclear how these dependencies should be modelled and deduced; especially, when the structure of dependency is rich but incomplete. This question has been approached with the technique of bayesian networks.

In principle, the bayesian network formalism is a special formulation of probability theory. The basic idea is as follows: By employment of the fundamental theorem of probability calculus, a joint probability distribution is decomposed into a graph, resp. a network. The nodes of this network represent the marginal distributions of the set of random variables in question, while the edges connecting the graphs nodes represent conditional probability distributions[1].[2]

It should be furthermore important to note, that although being named "bayesian", the formalism is not necessarily connected with a bayesian[3] interpretation of probability as provided by the Cox-Jaynes-Axioms (BALDI / BRUNAK [3]: p.50 and JAYNES [24]). [4] The name "bayesian network" rather stems from the use of Bayes Theorem for abductive[5] reasoning: While PEARL [47] advocates a bayesian standpoint, JENSEN [25] applies a classical frequentist definition of probability.

However, the application of probability theory and especially bayesian networks to

[1] Actually, this is often called a "Graphical Model".

[2] A good introduction to bayesian networks is JENSEN [25].

[3] Which reads *subjectivist, non-frequentist.*

[4] Although I am an enthusiast regarding bayesian theory, I will abstain from an introduction.

[5] Abduction is the inversion of deduction: $A \Rightarrow B$, B is there, therefore A is more plausible;

the problem is due to several advantages

- probability theory naturally allows inferring arbitrary dependencies between the variables considered, while taking account of multiple dependencies and even multicausality[6] (see JAYNES [24] and PEARL [47]). Usually, mathematical models which are deducing macroscopic statements from microscopic theory are ignoring structures of interdependence. Examples are the synergetic model of attitude formation by TROITZSCH [65], [67] and COLEMAN's [11]: pp.241 model of political activity.

- Bayesian Networks allow for a rather convenient representation of systems: since they can be modelled via local distributions (the nodes) and their probabilistic dependencies (the edges), only knowledge on the elementary level is necessary. In short, they naturally allow for microscopic modelling.[7]

- With respect to Multi Agent Based Simulations (MABS)[8] (see GILBERT / TROITZSCH [16], WEISS [69] and CONTE et.al. [13] for introduction), bayesian networks have the advantage that they operate on a carefully defined mathematical structure. This allows to clarify the relations between notions of the content-oriented theory and the mathematical one, which is very important for interpretation of models. I should add, that although lacking mathematical rigor (as mentioned) MABS easily allows for the modelling of structured dependencies.

- Bayesian Networks can be easily integrated with empirical data, as it is generally true for probability theory.

All these advantages certainly come at a cost. The cost is computer-memory and lack of speed.[9]Models easily become intractable, which makes simulation and MABS a better choice for investigating their middle and long term behavior.[10]

So much for introduction of the technique and back to the question of inferring dependencies between single objects and collections of them in systems with incomplete structure of dependency.

Bayesian networks allow to calculate and express these dependencies in form of a conditional probability distribution. What is left to do after computing the conditional

[6]The mode of inference in a probabilistic model is insensitive to hazards like structured independence and nonlinearity: it is basically summation, resp. integration.

[7]I hold the view that probability is no ontic "force": it rather stems from ignorance. And since I have got enough of this, I may add a flavor of it to all of my models.

[8]Swiftly spoken, MABS model a system by representing its lower level entities as interacting software objects.

[9]I experimented with a Dynamic Bayesian Network with twelve variables per time-slice, which could only be solved approximately by sampling techniques like Likelihood-Weighting in my case.

[10]MABS and bayesian networks could certainly be combined with the latter being an analysis tool operating on model generated data. During the course of this work this proved to be preferable compared to direct modelling. Compare [Model Sensitivity].

probabilities (which connect the realizations of a macro state with the single element state) is aggregating.

4.2 Definition of Bayesian Networks

After above informal introduction to bayesian networks, I will additionally provide a formal one.

First, I will briefly review some basic concepts of probability theory[11]. Then I will give an introduction to the concepts necessary for building bayesian network models. For this reason I will not talk about many details and especially the treatment of inference algorithms.[12]

4.2.1 Decomposition of Joint Probability Distributions

The first concept I want to mention is the concept of a *joint probability distribution*. This is a mathematical structure, where every joint occurrence of elements of a set of statements is attributed a probability.

In frequentist thought, the joint probability distribution is the structure which codes a probabilistic model and the most natural way of defining such a model is to generalize observed *frequencies* of the variables in focus towards probability. Given this view, the relationships between the elementary statements within the joint probability distribution are defined by the *Fundamental Theorem of Probability Theory*:

$$P(a|b) =_{def} \frac{P(a,b)}{P(b)} \tag{4.1}$$

The mentioned relationship is certainly a *conditional probability*. As can be seen, this conditional probability is defined in terms of joint- and marginal probabilities, which both can be easily gained by measurements of frequency. Certainly I can manipulate the formula in order to gain:

$$P(a,b) = P(a|b)P(b) \tag{4.2}$$

This equation shows the equivalence of the joint probability with a product of a conditional- and a marginal probability.[13] This formula can certainly be extended for a joint of more than two variables, which leads to the *Chain Rule*:

[11] For a complete introduction, the reader may be referred to the works of JAYNES [24] and LARSON [31], where the first is rich on philosophy and advocates a bayesian approach and the latter is a compact introduction to classical theory.

[12] For a sound contact to the topic the reader is referred to BALDI / BRUNAK [3], JENSEN [25] and PEARL [46] [47].

[13] The asymmetry in above definition (eq. 1) reflects a frequentist account to probability theory, demonstrated for convenience of the reader.

$$P(x_1, ..., x_n) = \prod_j P(x_j | x_1, ..., x_{j-1}) \tag{4.3}$$

Applying the Chain Rule allows for the decomposition of a joint probability distribution as a product of conditional- and marginal distributions.

This immediately results in the following semantic advantage: *Now the system of variables in scope can be described by their marginal distributions (as elementary properties) and their relationships in terms of conditional probabilities.* So to speak, global probabilistic propositions can be decomposed into local ones.

4.2.2 Graphs and Conditional Independence

Within the Chain Rule, indirect relationships between variables are represented explicitly. This prohibits the design of a "wiring scheme" (or network model) of the system, since it would contain unnecessary connections between the marginal distributions. This can be avoided by accounting for *conditional independence*[14] of the considered variables: Two variables X and Y are said to be conditionally independent given Z *if*

$$P(x|y, z) = P(x|z) \text{ whenever } P(y, z) > 0 \tag{4.4}$$

Given, that our network model should map the directions of the relations [15]and should furthermore contain no cycles[16], we can find the set of prior variables in this network which makes a certain variable x_j independent of all its other predecessors . This set is called *Parents of* x_j or pa_j. To eliminate all indirect connections towards x_j out of the directed and acyclic network, the *Parents of* x_j need to satisfy the following condition:

$$P(x_j | pa_j) = P(x_j | x_1, ..., x_{j-1}) \text{ for all } x_1, ..., x_{j-1} \text{ prior to } x_j \tag{4.5}$$

This is the *Markov-Parentship-Criterion*[17] for *directed acyclic graphs* or *DAGs*[18], how such a "wiring scheme" is called.

The Parentship-Criterion can easily be applied to the Chain Rule. This finally allows for the decomposition necessary for local representation of a joint probability distribution by a directed acyclic graph by invoking the *Chain Rule for Bayesian Networks*:

[14] More implications of conditional independence can be found at PEARL [47]:p.11, "Graphoid Axioms".

[15] Usually one has to decide on the ordering of the variables by causal intuition. Nevertheless there exist methods to extract causal orderings form data. PEARL [47]

[16] This is imperative for reasoning. Schematics which allow for cycles (like for instance block diagrams in Control Theory) are implicit with respect to the order of computations, resp. time.

[17] The Markov-Parentship-Criterion is a way to define the autonomy, resp. isolatability of an object with respect to certain,*a priori known* properties, as mentioned in the section [Epistemic Account on Object Identity].

[18] A more formal definition of a DAG will follow in the next section.

$$P(x_1, ..., x_n) = \prod_i P(x_i|pa(x_i)) \tag{4.6}$$

This equation, together with the prerequisite of representation of the conditional independence-relations between the marginal distributions via a DAG defines a bayesian network.

4.2.3 Inference in Bayesian Networks

Reasoning in Probability Calculus consists basically of *projecting* a joint probability distribution down to subsets of it: may that be joints, marginals or conditional probabilities.

So, the joint probability of two variables (Y, X) can be projected towards the probability of the occurrence of a certain value y_i of the variable Y by summing over the values of X:

$$P(y_i) = \sum_{j=1}^{m} P(y_i, x_j) \tag{4.7}$$

This is also called *marginalization* and is denoted the following way, if applied to distributions:

$$P(Y) = \sum_X P(Y, X) \tag{4.8}$$

conditional probabilities can be accessed by employing both fundamental theorem (eq. 1) and marginalization:

$$P(y|x) = \frac{\sum_s P(y, x, s)}{\sum_{y,s} P(y, x, s)} \tag{4.9}$$

A strength of Probability Calculus can be seen in the natural ability of performing *abductive reasoning*[19] efficiently. The inversion of a conditional probability is accomplished by *Bayes Theorem*:

$$P(y|x) = \frac{P(x|y)P(y)}{P(x)} = L(x|y) \tag{4.10}$$

The inverted conditional probability is often called *likelihood.*

Given that we gain *evidence* on the Values of some Variables within the network[20], we may wish to calculate the now unknown marginal resp. joint distributions on the remaining variables (which would yield conditional distributions in both cases). In short,

[19] As mentioned before, abduction is the inversion of deduction: $A \Rightarrow B$, B is there, therefore A is more plausible;

[20] "Evidence" means to select a category of some variable with probability p=1, respectively to look only at the part of the joint probability distribution which accords to what we learned about the actual state of this variables.

we might like to employ this knowledge in order to calculate the *total effect* of the evidence, considering the structure of interdependence of the system of hypotheses in scope . We could use above techniques to yield the results of interest.

But as mentioned, a necessary prerequisite for all computations but for abductive reasoning is access to the joint probability distribution. This may only be the case in the most seldom cases, since it grows exponentially with the number of variable values.

Thus the local representation by a bayesian network allows for the employment of local computations in order to gain results which may be intractable by common methods. This is accomplished by the various *inference algorithms* for bayesian networks. Since efficient calculation with probability theory is a highly complicated matter, the discussion of the several inference algorithms might exceed my capabilities. For more information the reader is referred to PEARL [46], JENSEN [25] and GILKS et.al. [17].

Chapter 5

Probabilistic System Model

Since it is necessary to feed the machinery of probability calculus in order to get a running model, it might be helpful to formally declare the actual model-relation.[1] This will be accomplished by introducing BUNGE's general definition of a *system*. Relating this definition to bayesian networks, I will gain the definition of a *probabilistic system model* which can be processed by probability calculus.

5.1 System

Usually, a system is conceived as a set of interacting elements[2], which might be in our case systems of individuals, collectives or institutions.

The eminence of the concept of system resides in the fact that it allows to represent a higher level entity in terms of its constituting lower level entities. It seems to be a natural container of the concept of perceived autonomy (see [Epistemic Account on Object Identity],[Level Transition] and [Semantics and Bridge Hypotheses]), whose domain is given by the systems boundary.[3]

In accordance to BUNGE [6] I will formally define a *system* $\mathcal{S}_t$ at time t by the triple of the sets of its components $\mathcal{C}_t$, the bonding relations $\mathcal{B}_t$ between its components and its environment $\mathcal{E}_t$.

$$\mathcal{S}_t = \langle \mathcal{C}_t, \mathcal{B}_t, \mathcal{E}_t \rangle \tag{5.1}$$

With the lists of the systems components, bonding relations

$$\mathcal{C}_t = \{c_{t,1}, c_{t,2}, ...c_{t,n}\} \tag{5.2}$$

[1] Another saying is that we declare a semantic on probability theory.

[2] A well known exception are the infernal works of LUHMANN, like [35].

[3] Nevertheless, for a complete level transition there is the need for translation of the notion defined on the respective levels.

$$\mathcal{B}_t = \{b_{t,1}, b_{t,2}, ...b_{t,m}\} \tag{5.3}$$

and the components of the environment[4], where the latter might often be condensed to e_{in}, the source of environmental *input* and e_{out}, the sink of the systems *output*.[5]

$$\mathcal{E}_t = \{e_{t,1}, e_{t,2}, ..., e_{t,o}\} \mapsto \mathcal{EC}_t = \{e_{t,in}, e_{t,out}\} \tag{5.4}$$

I just want to mention that these components may be as well elementary objects, unbounded sets of objects or systems itself. A *subsystem* $\mathcal{SS}$ may be defined by having subset relations of system $\mathcal{S}$ with regard to its composition set $\mathcal{C}$ and its bonding set $\mathcal{B}$ while having superset-relations with regard to the environment set $\mathcal{E}$.

$$\mathcal{SS}_t \text{ is a subsystem of } \mathcal{S}_t \text{ iff } (\mathcal{CS}_t \subseteq \mathcal{C}_t)\&(\mathcal{BS}_t \subseteq \mathcal{B}_t)\&(\mathcal{ES}_t \supseteq \mathcal{E}_t) \tag{5.5}$$

5.1.1 Graphical Representation

Systems can be modelled by a *graph*[6] $\mathcal{G}_t$ whose nodes $\mathcal{N}_t$ represent the union of components set $\mathcal{C}_t$ and the environment set $\mathcal{E}_t$[7] while its bonding set $\mathcal{B}_t$ maps[8] to the set of edges $\mathcal{EG}_t$.

$$\mathcal{G}_t = \langle \mathcal{N}_t, \mathcal{EG}_t \rangle \tag{5.6}$$

given

$$\mathcal{N}_t = \mathcal{C}_t \cup \mathcal{E}_t \tag{5.7}$$

and

$$\mathcal{B}_t \mapsto \mathcal{EG}_t \tag{5.8}$$

A natural representation of a graph is an illustration like [Figure 5.1].

5.1.2 Unfolding and Transition Representation

The representation of systems may be extended towards mapping the model in time. For this, I will define a *transition composition* $\mathcal{C}_{ts(t,t+1)}$ by uniting the composition sets $\mathcal{C}_t$ and $\mathcal{C}_{t+1}$, thus duplicating $\mathcal{C}$, given the special case that it is invariant in time.

[4] Note, that all that elements are indicated over time.

[5] Such a compression would certainly reduce the bonding set.

[6] The term "graph" does not necessarily refer to a picture, although network images are great visual representations of the mathematical structure of a graph.

[7] Environment influences are thus represented explicitly in the graph and thus the environment elements are united with the set of nodes.

[8] I leave this map unspecified, since it depends on the functional dependence of the respective coupled components. Directed edges and conditional probability tables defined on it will become handy in my example.

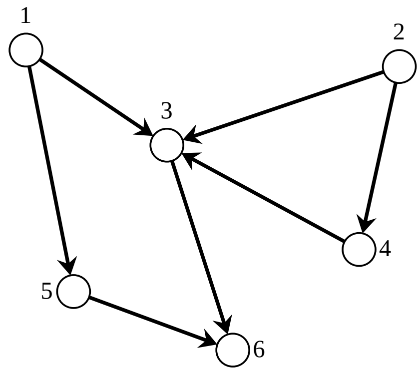

Figure 5.1: Example Drawing of a Graph

$$\mathcal{C}_{ts(t,t+1)} = \mathcal{C}_t \cup \mathcal{C}_{t+1} \tag{5.9}$$

The same way, a *transition environment* $\mathcal{E}_{ts(t,t+1)}$ is defined on the respective environment sets $\mathcal{E}_t$ and $\mathcal{E}_{t+1}$.

$$\mathcal{E}_{ts(t,t+1)} = \mathcal{E}_t \cup \mathcal{E}_{t+1} \tag{5.10}$$

This is completed by definition of a *transition bonding* $\mathcal{B}_{ts(t,t+1)}$ coupling the elements of both the transition composition $\mathcal{C}_{ts(t,t+1)}$ and the transition environment $\mathcal{E}_{ts(t,t+1)}$ via the bonding set $\mathcal{B}_{t+1}$ over time, given its element bonding relations have an origin in $\mathcal{C}_t$ or $\mathcal{E}_t$.

Coupling is achieved by locating the respective bonds root in the domain of time t and its target in the domain of time $t+1$. This is called *unfolding* of the bonding set.

$$\mathcal{B}_{t+1} \mapsto_{unfolding} \mathcal{B}_{ts(t,t+1)} \mid (\text{ origin components of } b_{i,t+1}) \in (\mathcal{C}_t \vee \mathcal{E}_t) \tag{5.11}$$

Now I am able to define the *transition representation* $\mathcal{S}_{ts(t,t+1)}$ of the system $\mathcal{S}$ between times t and $t+1$.

$$\mathcal{S}_{ts(t,t+1)} = \langle \mathcal{C}_{ts(t,t+1)}, \mathcal{E}_{ts(t,t+1)}, \mathcal{B}_{ts(t,t+1)} \rangle \tag{5.12}$$

5.1.3 Transition Graph

Equivalent to the procedure employed to yield the graphical representation of the system, I now define a *transition graph* $\mathcal{G}_{ts(t,t+1)}$ on the transition representation $\mathcal{S}_{ts(t,t+1)}$ by uniting the respective composition and environment sets $\mathcal{C}_{ts(t,t+1)}$ and $\mathcal{E}_{ts(t,t+1)}$ to the set of nodes $\mathcal{N}_{ts(t,t+1)}$while mapping the bonding set $\mathcal{B}_{ts(t,t+1)}$ on the set of edges $\mathcal{EG}_{ts(t,t+1)}$. This is expressed by:

$$\mathcal{G}_{ts(t,t+1)} = \langle \mathcal{N}_{ts(t,t+1)}, \mathcal{EG}_{ts(t,t+1)} \rangle \tag{5.13}$$

with

$$\mathcal{N}_{ts(t,t+1)} = \mathcal{C}_{ts(t,t+1)} \cup \mathcal{E}_{ts(t,t+1)} \tag{5.14}$$

and

$$\mathcal{B}_{ts(t,t+1)} \mapsto \mathcal{EG}_{ts(t,t+1)} \tag{5.15}$$

5.1.4 Representation in Time: Temporal Graph

Graphical representation of the system in time is gained by definition *temporal graph* $\mathcal{G}_{temp(t:t+n)}$ representing the system from time t to time $t+n$ by concatenation of the transition graphs $\mathcal{G}_{ts(t,t+1)}$ to $\mathcal{G}_{ts(t+(n-1),t+n)}$. Given indication of the systems sets over time, this is accomplished by uniting the respective transition graphs.

$$\mathcal{G}_{temp(t:t+n)} = \mathcal{G}_{ts(t,t+1)} \cup \mathcal{G}_{ts(t+1,t+2)} \cup ... \cup \mathcal{G}_{ts(t+(n-1),t+n)} \tag{5.16}$$

If the structure of the transition graph does not change over time, one could say that the transition graph is *unrolled* in time.

5.2 Bayesian Network Representation

Now I want to apply the results of the last subsections to the problem of defining a bayesian network on the system. The first conclusion is that the temporal graph $\mathcal{G}_{temp(t:t+n)}$ can be used to define the structure of a bayesian network modelling system over time, given that its edges are directed. Such a *temporal* bayesian network is usually called a *dynamic bayesian network*, although time is represented explicitly.

We need to define the conditional probability distributions attached to the edges. Recalling the chain rule for bayesian networks

$$P(n_1, ..., n_n) = \prod_k P(n_k | pa(n_k)) \tag{5.17}$$

We identify the term $n_k|pa(n_k)$ with the subset $\mathcal{EG}_{pa(nk)} \subseteq \mathcal{EG}_{temp(t:t+n)}$ of the set of edges of the temporal graph $\mathcal{G}_{temp(t:t+n)}$ which point towards the node n_k, given that the index k is a mapping of the time-explicit indication introduced in the [System]-section.

$$\mathcal{EG}_{pa(nk)} = \mathcal{EG}_{temp(t:t+n)} \; |(eg_j \text{ pointing towards } n_k) \in \mathcal{N}_{temp(t:t+n)}, n_{t,i} \mapsto n_k \tag{5.18}$$

What finally needs to be mapped is the bonding information BI exceeding the definition of the existence and direction of edges on the conditional probability distribution $P(n_k|pa(n_k)$ attached to $\mathcal{EG}_{pa(nk)}$.[9]

$$BI(\mathcal{EG}_{pa(nk)}) \mapsto P(n_k|pa(n_k) \tag{5.19}$$

This map can be arranged by either theoretic deduction of the probabilistic relation between the components in scope, or by learning the relationships from data (see BALDI/BRUNAK [3] and PEARL [47]).

Finally, if the joint probability distribution can be composed by invoking the chain rule for bayesian networks, the formulation of a *probabilistic system model* is achieved.

[9] As remarked in the section on [Graphical Representation], the map from elementary to graphical representation was left unspecified because of the implicitness of the actual functional dependence between the elements of the system. The bonding information BI is exactly the explication of this unmapped functional dependence.

Chapter 6

Methodological Individualism

Now, after having introduced the foundations of level transitory modelling, I will discuss my philosophical approach to the problem of level-transition in relation to its widespread equivalent in the social sciences, the *Coleman Micro-Macro-Scheme* (COLEMAN [12]). I should note that although it is named after James COLEMAN, its first occurrence can be found in McCLELLAND's "The Achieving Society" [36][1].

6.1 The Micro-Macro-Scheme and Methodological Individualism

In the "Aims"-section, I mentioned the Coleman Micro-Macro-Scheme as a central starting-point of my work and promised an introduction into the subject. This scheme is very important for individualistic social theory, because of its ability to connect individual behavior with collective dynamics. I will start by referring to [Figure 6.1] in order to provide an overview over its hypotheses. As mentioned, the Micro-Macro-

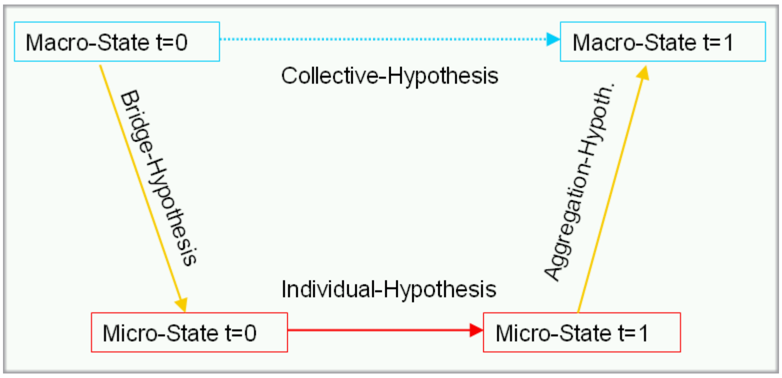

Figure 6.1: The Coleman-Micro-Macro-Scheme

[1] Thanks to Karl Dieter OPP for this hint.

Scheme can best be described as a typology of the hypotheses necessary to accomplish a level-transitory explanation. The hypotheses substituting the collective (or macro-) hypothesis during individual-based explanation can be classified as *Bridge-Hypothesis*[2], *Individual-Hypothesis* (resp. Action-Hypothesis) and *Aggregation-Hypothesis*.

Before examining these hypotheses in detail, I want to discuss the main assumptions of this scheme. The first assumption is certainly systemicity of the social realm, or in other words, that a social entity is a composite of lower level entities. The second assumption is, that the lower level entities are persons which are connected by their respective actions. This is the founding assumption of so called "Methodological Individualism".

6.2 Explicit Application of the Micro-Macro-Scheme

You wont be surprised to read that there is much discussion concerning the notion of Methodological Individualism. There exist several attempts to formulate it, or respectively its most usual formulation, the Micro-Macro-Scheme, in a way that allows for the occurrence of emergent properties.

A short excursion to the actual practice of level transitory explanation may be necessary to clarify the associated problems.

One frontline is the question how the Individual-Hypotheses should be instantiated, [3] which certainly shifts the focus from level-transition to microscopic theory and effectively relabels the problem while displacing it into the particular instances of the microscopic theories. Following this tradition, a level-transitory-explanation consists of determining the initial-values of the microscopic hypotheses, deducing their consequences and averaging the results. This way, the separate hypotheses of Micro-Macro-Scheme are applied one-to-one. A good introduction of this procedure can be found at OPP [44].

The interesting point is, that a microscopic approach to level transition, (as designed in the respective section) can certainly be interpreted as an implicit version of this procedure. The reason for this is that a systems state can certainly be interpreted as initial condition for the components mechanisms.

Nevertheless, a conventional approach which tries to instantiate the Micro-Macro-

[2] More on this type of hypothesis in the sections [Level Transition] and [Semantics and Bridge Hypotheses].

[3] This discussion can be followed in past issues of *"Koelner Zeitschrift fuer Soziologie und Sozialpsychologie" (KZFSS)*. The adversaries, all members of the "Rational-Choice" enterprise, consisted of LINDENBERG [33], [34] and ESSER [15], who advocate a theory-driven microeconomical approach, and on the other hand KELLE and LUEDEMANN [27], [28] and OPP and FRIEDRICHS [45] who are more prone to an empirico-statistical paradigm similar to those of empirical social research. A recent contribution to the subject is given by DAVIDOV / SCHMIDT [14].

Scheme by trying to explicitly conclude with its classes of hypotheses is logically or methodologically flawed. This is because of two reasons which already have been introduced in the section on [Level-Transitory Explanation and Emergence]:

First, an elementary (which reads "not composite") description of a macroscopic state is inconsistent with the idea of a composite system, which is constituting for the problem of level-transition. A elementary description of higher level entities can not be uninformative in the sense defined in the section on [Bridge Hypotheses and Violation of Object Identity]. Therefore coherence of object identities on both levels, which is imperative for reasoning, cannot be maintained in this case. Certainly making this problem implicit by shifting it into discussions about initializations of SEU-Theory will not solve it.[4] Discussion of this topic is given in the next section.

And second, a map from notions of composite system structure to notions of macroscopic states is many to one- and therefore not invertible map. Because of this syntactically elementary[5] description of a systems macroscopic state lacks the possibility to map its structure and is therefore only feasible for structureless phenomena. (These seemed to deliver a template for microscopic modelling until recently.[6]) A technique which copes with more complicated phenomena is required to map the relevant structure of the system. This further disqualifies the one-to-one application of the Micro-Macro-Scheme to many cases of interest.

6.3 Macro-States and Initial Conditions

It can be easily seen, that my arguments in the last section attacked merely a method of defining initial-values for an individual hypothesis, and not the individual hypothesis itself. This is not accidental since the research programme of the Sociology of Rational Choice identifies the Subjective Expected Utility (SEU) Hypothesis as its very core. (Compare for instance COLEMAN [12] and KELLE/LUEDEMANN [27].)

Shifting the question of macroscopic properties and emergence into the initial conditions allows to shield the individual hypotheses of the joint theory from doubt, resp.

[4]The discussion about theoretical "richness" or "abstinence" of bridge-hypotheses, as discussed in KZFSS, is an example for this. Focus of argumentation is the question how to gain valid instances of SEU, and not how to bridge levels.

[5]In the sense of CARNAP [9], who distinguishes between "elementary" and "complex" formulae and notions, an elementary notion is one that cannot be decomposed within a given system of axioms.

[6]Possible examples are so called "Mean-Field-Models". Since a mean is the "balance point" of a distribution, all its values can be understood as to be balanced against each other. The structure of interaction is treated as complete, which avoids the necessity for a detailed view on the lower level process. The models of TROITZSCH [65], [67] and COLEMAN [11]: pp.241 show such a characteristic, as mentioned. Although I am advocating structural modelling in this work, I have to confess that assumptions of structural homogeneity might be necessary for the most real problems. One needs to work with the data one can get and I would not trow away useful methods like HLM or Bayesian Hierarchical Models. Detailed discussion is given in section [Proxy Descriptions].

from the problems associated with level-transitory explanation.[7]

A striking example for the synonymous use of “Macroscopic State” and “Initial Condition” is OPP’s ([44]: p.90-105) introduction to the subject. He tries to explain the disintegration of a public audience by application of the macro-micro-scheme: Rain pouring on the audience is defined as “collective attribute” and employed to initialize the individual process of decision, wether to stay or to leave.

Certainly, “absence of rain” is no constituting property of a public audience. Situations are simply taken for collectives. Rightfully, I should note that OPP shares this error with his theoretical adversaries LINDENBERG [33] and ESSER [15], as well as eventually with DAVIDOV and SCHMIDT [14].

6.3.1 Semantics and Bridge-Hypotheses

Certainly the synonymous use of “bridge hypothesis” and “initial condition” avoids contact with the actual problem of level transition. I will now discuss that problem with regard to the definitory “delicateness” of bridge-hypotheses.

As far as I know, the term “bridge hypothesis” was coined by NAGEL before the background of explaining one theory via another, denoting a hypothesis which relates (or translates) notions of the different theories (Compare NAGEL [41] and SCHEIBE [53].). It should be furthermore noticed that NAGEL’s argumentation proceeds on a level of theoretic *statements* .[8]

In our case, the problem faced in the attempt of reduction consists in delivering definitions on the objects and properties defined on the different levels which are *semantically* consistent and allow for *conservation of object identity* on both levels. This implies the coherence of the respective definitory models on both image- (as postulated by NAGEL) *and* preimage / target - level.

This coherence can be gained by introducing a *third model*, which maps the relations between the preimage-sets on both levels. In my case the (rather implicit) model is a theory of knowledge and modelling which results in the assumption of *perceived autonomy* of objects. This assumption connects the “designata” on both levels.

I should mention, that in my view this discussion is usually no issue in sciences which employ the notion of a *system* rather than of *higher level entity.* The notion of system seems to be a natural container for the idea of perceived autonomy, as can be seen by the frequent discussions regarding systems boundaries (compare the section on

[7] I do not judge empirical correlates of the SEU-Hypothesis as being functionalities of social processes. Therefore this argument is not a serious attack on the Rational-Choice paradigm. Nevertheless, it could be such for a more “socialized” theory.

[8] The point is made very clear on [41]: p.364.

[System]).

As stated in the section [Bridge Hypotheses and Violation of Object Identity] I do not expect any explanatory content from bridge hypotheses: they are serving as mere definitions with the explanation taking place on a single (namely the lower) level. Above considerations are targeting a single result: Discussion on the logical structure of bridge-hypotheses will not save us from defining the objects in scope in a way that keeps their identity intact.

I will close the discussion with the following summary. Logical coherence on the model-level is not enough and the ability to initialize the proposed hypotheses is a necessity for any deduction. Nevertheless, this should not be attempted by employing a bridge-hypothesis due to the problem of violation of object identity.

6.4 Deduction of Micro-Macro-Hypotheses

In order to conclude this chapter I should relate my approach to the Coleman Micro-Macro-Scheme.

In a nutshell, aggregating over the respective microscopic deductions yields an instantiation of an arbitrary hypothesis taken out of the scheme, as discussed in the sections [Realization of Macroscopic Properties] and [Level Transition].

Furthermore, I might add that this approach is perfectly coherent with COLEMAN's own writings: In the meta-theory chapter of "Foundations of Social Theory", he argues that the hypotheses of the scheme should be best thought of "as macro-level generalizations which might be predicted as *deductions* of a theory." (COLEMAN [12]:p.20, no accentuation in the original;) A microscopic model is exactly such a theory.[9]

[9] Honestly, I should add that such a microscopic model should only be accessible in cases of small groups. For different applications the definition of entities on a larger scale could be attempted, thus "lifting" the micro-level as it is done by invocation of corporate actors (compare COLEMAN [12]).

Chapter 7

The Kirk-Coleman-Model

In order to be finally able to present an actual instance of the proposed methodology I chose the classical Kirk-Coleman model as exemplary application. Although it has been modified in order to meet modern theoretical standards it should be noted that it is a very simple example which could be (and has been) easily treated without the application of sophisticated methods like bayesian networks.[1] However, I will begin the discussion of the Kirk-Coleman model by sketching the original work.

The so called *Kirk-Coleman Model* originates in the late sixties and is part of an early attempt to explore the possibilities of both mathematical- and computer modelling in the social sciences by constructing various models of interaction behavior in a three-person group. Not surprisingly, the models discussed in the article (KIRK / COLEMAN [29]) differ in theoretical content due to the calculi applied. Namely, KIRK and COLEMAN construct two models: first, a microscopic differential equation system and second, a stochastic simulation model [2] of the group process. The differential equation system proceeds analogously to an earlier macroscopic model of SIMON.[3] The stochastic simulation model has been named "Kirk-Coleman Model" by several authors.

This work is similar to the original, where gaining *direct* sociological insights seemed to be only a secondary goal after the testing of freshly accessible methods.

[1]Nevertheless, the reader should bear in mind that the circumstances of this work did not allow for empirical modelling and exciting models are not so easily constructed out of the blue.

[2]It may be important to notice that a stochastic simulation is not necessarily equal with a numeric solution of a system of stochastic differential equations, since the former is not necessarily constrained to the mathematical notions of the latter. One can feel KIRK and COLEMAN's freshly gained "freedom" in the description of their simulation model.

[3]Since I was satisfied with KIRK's and COLEMAN's description of the model, I did not access the original work.

7.1 Simmel's Remark

The authors begin introducing the theoretic background of the model by citing (KIRK / COLEMAN [29]: pp. 171) a remark of the beginning of the 20th century sociologist SIMMEL [57].[4] According to his observations three-person-groups usually disintegrate into a pair and an isolated person. While the persons constituting the pair have relatively strong ties, their relationship to the third person, the isolated one, is substantially weaker.

They conclude, that according to SIMMEL's hypothesis a situation with balanced strength of the relationships between the three persons is immanently unstable. Furthermore they state, that the assumption of the specific equilibrium state of a pair and an isolated person is not trivial since different possibilities are thinkable.

Finally, KIRK and COLEMAN remark that the hypothesis demands only a mere "tendency" in the behavioral patterns of the triad (how the three-person-group may be called). Everyday experience and empirical research were both giving support and counterexamples to the Simmel-Hypothesis, while the tendency to decompose into the postulated pattern is supported by empirical findings of BALE and MILLS.[5]

7.2 The Homans-Hypotheses

KIRK's and COLEMAN's second theoretical starting point are psychological hypotheses concerning interaction behavior, developed by HOMANS [22] [23] [6], which had already been applied in the mentioned group-level model developed by SIMON.

The cental theoretical aim of the study of KIRK and COLEMAN was to investigate if the system-level predictions made by the Simmel-hypothesis could be explained by application of the psychological hypotheses of HOMANS.[7] I will now provide a short sketch of the hypotheses.

7.2.1 Social Behaviorism

HOMANS could be described as a "social behaviorist". His work exhibits the simplicity of elementary notions of behaviorism as well as its basic assumption of reinforcement learning.

Nevertheless, his hypotheses on interaction behavior seem to force him to employ

[4] I have not been able to locate the mentioned remark exactly, because the book is *very* unstructured. The reader may be referred to [57]: pp.106, where chances seem to be best.

[5] These are citations of KIRK and COLEMAN [29]. Results for a six-person group can be found in MILLS [37].

[6] KIRK and COLEMAN cite an earlier work of HOMANS, "The Human Group" (German translation: "Theorie der sozialen Gruppe" [22]). Nevertheless, I will refer to "Elementary Social Behavior" [23] which summarizes HOMANS' theoretic results.

[7] As you might have noticed, this is an exemplary case of level-transitory explanation: group-level phenomena are to be explained by hypotheses connecting *individual* properties.

behavioristically bended versions of such notions as "feelings" or "liking". I will come back to this later in the section on [Similarity and Attraction].

7.2.2 Mutual Reward and Interaction

HOMANS' ([23] p.181-184) fundamental argument is the possibility of people to act as mutual sources of reward. Since humans learn to show activities which maximize their reward, they will, ceteris paribus, begin to mutually reward themselves, as long one person (even accidentally) starts the process by showing behavior which is rewarding for the other.

Additionally, he assumes that reward and activity "targeting" this reward are somehow proportional on a not explicitly defined scale, thus allowing for a smooth incrementation of the intensities of mutual reward.

Interaction behavior is furthermore conceived to be only a special case of above argumentation, where the functioning of the process of mutual reward is enforced by, how HOMANS calls it, the *general reinforcer of liking* (or social approval, which is in HOMANS' view the according *operant*).[8]

7.2.3 Condensed Hypotheses

Usually, the reinforcement argumentation is only applied in condensed form, resp. in form of the following deduced hypotheses (see HOMANS [23], KIRK / COLEMAN [29], TROITZSCH [66] [9]):

$$\text{Hypothesis 1: Liking} \nearrow \text{Interaction}$$
$$\text{Hypothesis 2: Interaction} \nearrow \text{Kiking}$$

Or in words, liking increases interaction and interaction increases liking.

[8]HOMANS' original formulation is somewhat different and more complicated, although it should be equivalent with respect to the central issues. [23] p.181: "The more valuable to a Person a unit of activity Other gives him, the more often he will emit activity, including sentiment, rewarded by Others activity. ... Now one of the activities Person may give to Other is the generalized reinforcer called social approval; and we have seen in recent chapters that the more valuable to Person is the activity Other gives him, the more valuable is the approval or liking Person gives Other. ... One of the possible consequences of the first two propositions taken together is the following third proposition: the more valuable to Person the activity Other gives him, the more valuable the approval he gives Other *and* [originally emphasized] the more often he emits activity, including sentiment, to Other. To put the argument more crudely, if Other does Person a service, Person is apt to like him and interact with him often." [23] p.183: "If they interact at all, they emit activities to one another; and if no special factor is present that might bias systematically their values or their activities, the chances are that each one will find some of the other's activities valuable, if only because they may be obtained at less cost from him than from a third party at a greater distance: ... And to the extend that each finds the other's activity valuable, each is apt to express favorable sentiment toward the other. For this reason, an independent increase in interaction between persons is apt to be associated with an increase in liking between them."

[9]The formulations might differ somewhat.

These two hypotheses are the theoretical foundations of the various models invoked in the article of KIRK and COLEMAN for the attempt to instantiate the Simmel-Hypothesis.

Finally, I should again emphasize the fact that these formulations abstract from the more fundamental hypothesized process. The condensed Homans-hypotheses are only *projections* of iterative instantiation of the same hypothesis of reinforcement learning in the involved individuals.

7.3 The Kirk-Coleman-Model

After having introduced scope and theoretical foundations, I will informally describe the stochastic simulation model.

As mentioned before, the Kirk-Coleman-Model is a microscopic simulation of the assumed process of interaction in a three person group. The program instructions [10] modelling the individual behavior are mapping the condensed versions of the Homans-Hypotheses as they are declared in the last section. A single turn of the simulation proceeds as follows (KIRK / COLEMAN [29]: pp. 176).

- Every Individual $i = 1, 2, 3$ chooses one of the other individuals as preferred interaction partner. The probability of choosing an individual j is proportional to the *liking* for that individual.
- A single individual starts interaction with a probability proportional to a (at least semantically macroscopic) parameter of *dominance.* As a result, the choice needs not to be necessarily reciprocal.
- The realized interaction is counted and its initiator is given a *reward* K with a certain probability RA. If the choice has been reciprocal, the interaction partner is given a reward K with a equal probability RA. If the partner has originally chosen a different individual, he is given the reward K with a different probability RB.
- The reward K is added to i 's liking for the individual j it has just interacted with.
- Finally, all sympathies are lowered by multiplication with a shrinkage-parameter, thus ensuring that recent interactions have an greater effect than those further in the past.

This procedure is iterated until the desired number of interactions is reached. [11]

[10] The original program has been written in FORTRAN, but only the control flow is available in the article.

[11] The maximal number of iterations calculated by KIRK and COLEMAN was 1000.

7.4 Various Group-Interaction Models

As mentioned, there exist several models which are implementing the Homans' Hypotheses, three alone within the scope of the article of KIRK and COLEMAN (namely their two models and SIMON's group-level model). A fourth model has recently been developed by TROITZSCH [66] [67], which extends the Kirk-Coleman-Model with respect to the number of agents[12] considered.

It is interesting to observe how the models differ with respect to the constraints imposed on them by the different calculi and possibly by techniques of software-engineering. The latter are constraining the image set of a model in a similar way a calculus would do, because both are pre-structuring the image-set of the possible models.

An indicator for this effect is the "dominance" parameter in the actual Kirk-Coleman-Model. It seems to result from the necessity of *sequentialization* of the modelled process. There seems to be no theoretical justification for the notion of dominance, since it is first a macroscopic concept defined on more than a single individual, and second, the microscopic differential equations model of KIRK and COLEMAN ([29]: pp.174) is lacking this parameter. This is not surprising, since all equations in a differential equations system are solved simultaneously per definition.

Today sequentialization of simulation models is usually accomplished either by the software employed (by an interpreter like MATLAB for instance) or by making auxiliary assumptions in special program structures.[13] In simulation studies it is unnecessary to mix the semantics of theoretical notions with technical assumptions as it is the case with the mentioned parameter of dominance.

[12]By "agent" I will denote the model-representation of an actor.

[13]So, every MABS has structures where the interactions of the agent-objects are related to each other, i.e. are synchronized or a-synchronized. The common practice to assign "threads" to the agent-objects and thus assign them own processes makes no exception: It just passes the task to the operating system and to code hidden in the depth of the memory.

Chapter 8

Modification of the Kirk-Coleman-Model

Originally, the bayesian network-implementation of the Kirk-Coleman-model had been intended as a proof of the viability of the proposed method of level-transitory explanation. But when I started the actual modelling and tried to get rid of the semantically macroscopic parameter of dominance, I think I overshot by making the model "better", i.e. more coherent and fashionable.

Since experience is the thing which one gets only after having needed it, the now modified model can not directly be compared with its original. Nevertheless, it still shows the applicability of level-transitory explanation and resulted in interesting theoretical discussion, which shall be demonstrated within this section.

8.1 Homans-Hypotheses and Expected Utility

Within this section I will argue that the Homans-hypotheses (as a special case of reinforcement learning) along with the Subjective-Expected-Utility (SEU) hypothesis can be interpreted as special cases of considerations regarding *adaptation* of systems to environmental conditions.

My interest in Expected Utility is driven by the fact that it is rather explicitly formulated and widely welcomed in different disciplines.[1]

8.1.1 Subjective Expected Utility Theory

I will begin with a short sketch of subjective expected utility theory since the theory of HOMANS is already introduced.

[1] I would certainly resist the temptation to dogmatically propose it as standard template for models of human behavior.

SEU has its foundations in the Theory of Games developed by VON NEUMANN / MORGENSTERN [42], where maximization of the so called *expectation function* (or Neumann-Morgenstern-equation), which combines estimates of probability of events with estimates on the utility of these events, is shown to be the optimal guide for decisions under uncertainty.

These mathematical considerations were brought to a level of social science by SIMON [58] who introduced the notion of *bounded rationality*, emphasizing the fact that actual people do not act as desired by normative decision theory. A further milestone in the development of SEU-Theory were the studies of KAHNEMAN / TVERSKY [26] resulting in *prospect theory* which shows the cognitive biases in real-world decision behavior.

The subjective expected utility hypothesis can be formulated as follows:

$$Action = \sum_{j=1}^{n} U(c_j(a_i))P(c_j(a_i)) \rightarrow max_i! \tag{8.1}$$

This means that the action a_i is chosen which maximizes the sum of *subjective utilities* U defined on its consequences c_j, weighted by the *subjective probability* P of this consequence c_j. In other words, the action is chosen which maximizes the average utility expected by the acting individual.[2]

8.1.2 Adaptation

Now let me introduce *adaptation* as an argument which will allow me to compare the theories of reinforcement learning and expected utility.

According to the [Encyclopedia Britannica], adaptation means "*the process in which an animal or plant becomes fitted to its environment*". Extrapolated to systems, this definition will well serve my purposes, given I avoid the *trap of biologism* by introducing modes of selection which are adequate to the nature of the process considered. A biological mode of selection is by no means necessary for the application of evolutionary theory.[3].

Back to the concept of adaptation, it most importantly implies that there exists a certain criterion of *optimality* which should be approximated by a system while its state is disturbed by the environment (compare BISCHOF [5] on whom much of this

[2]Results of formulae of this *structure* are called "*expectations*" in probability theory. The notion of "expected utility" stems rather from this use of the word than from referring to subjective *prospects*, as used by KAHNEMAN / TVERSKY [26].

[3]Here an example: I can act in my shared flat in a variety of ways with every action having a certain potential probability to be repeated in the presence of my flatmates. Thus one may say that there exists a certain *fitness-function* on my actions given my flatmates, ceteris paribus. But certainly my chances of reproduction do not need to be mentioned if one analyzes this specific kind of evolutionary pressure. (Yet, my provocations have not been this extreme...)

argumentation is based). The existence of such a criterion of optimality is granted by evolutionary theory, for the reason that certain values of certain attributes may raise or lower a systems chance to exist in the future.

I will stop the considerations on evolution at this point since the question about evolutionary functionality of certain attributes is exactly the question about self-organization[4] and functionality which I avoided in the [Emergence] section. It will for now suffice to specify the criterion in the relevant context.

Returning to the question of modelling of individual level adaptation, I am now able to state the following: A constructive hypothesis which models adaptive behavior must necessarily combine elements representing assumptions on both

- on the *optimal state* of a system and
- the *mechanism of its approximation* given a particular environment.

8.1.3 Optimality and Reinforcement Learning

The behaviorist hypothesis of learning as invoked by HOMANS [22] [**?**] can be formulated as follows:

A individual may be exposed to environmental stimuli as a result of showing a particular behavior. These stimuli may either be experienced as rewarding or punishing by that individual. If the stimuli are experienced as rewarding, the probability of occurrence of the particular behavior will increase, which is called *reinforcement*. If they are experienced as punishing, the probability will decrease, which is called *extinction*.

The proposal of representation of the optimal state is fulfilled (in this case implicitly) by the notions of reward and punishment. Both can be understood as measures of the *gradient*[5] of the individual state which may be a complex function of individual behavior and environment. If the individual is punished, it descents the gradient away form the optimal state; if it is rewarded, it ascents the gradient toward the optimal state.

The mechanism of approximation is the rather explicitly formulated process of reinforcement and extinction. It is easy to see that iteration of the process will drive the individual to an area of its behavioral space where reward is maximized and punishment is minimized, namely somewhere near the optimal state.[6]

[4] Whereby in the case of evolution the system level is the population level.

[5] An rough but rather intuitive description of the mathematical notion of "gradient" could be the change of a state with respect to all its defining attributes.

[6] This procedure can be seen analogously to optimization by following the maximum gradient, together with the associated problem of finding only local optima.

8.1.4 Optimality and Expected Utility

The operation of subjective expected utility theory with regard to adaptation can be viewed analogously to the case of reinforcement learning. The difference between both theories consists in the fact that in the case of SEU there is the additional assumption of *representation* of some features of the environment *within the individual.*

This representation of environment features is indirectly accomplished via the subjective probability term of the SEU formula, since it states the feasibility of the respective actions.

Again, the optimal state is implicitly defined by a gradient-formulation, namely those of *utility.* States of the environment (action consequences) with positive utility push the individual up the gradient of the "well-being function", environmental states with negative utility push it down the gradient. Since the utilities are defined as being subjective estimates, they might be seen as "within-individual" representations, as well.

Given certain correspondence of the intra-individual representations with their real-world targets, iteratively maximizing the expected utility will finally result in approximating the optimal state.[7]

8.1.5 Evaluation of the Theories

In comparison with reinforcement learning, the representation of experience by a explicit set of subjective probabilities in SEU theory seems much more concise than summoning a vague "history of reinforcements". Needless to say that today one is not bound to "scientific" formulation which omits latent variables and processes.

Summarized, SEU can be seen as the more explicit theory and shows more coherence to "common sense" than reinforcement learning, besides the fact that is well tested empirically. Therefore I decided to replace the Homans-Hypotheses by a SEU formulation in my model, interpreting *liking* as a subjective assessment of *utility* and defining the action to be considered as the choice of an interaction-partner.[8]

But nevertheless, the relative simplicity of the behaviorist approach (including HOMANS' [Condensed Hypotheses]) is astonishing. Looking back, I would decide for the original Homans-hypotheses if I had to do the project again, for the sake of comparability and simplicity.

[7] It should make no sense to exaggerate the argument and talk about the "evolutionary advantage" of representation of the environment. Such a discussion would make it necessary to model the actual system in which evolution takes place.

[8] Certainly the decision was supported by important social factors: I wanted to do something *modern.* And furthermore it pleased the sociologists and social-psychologists I am working with.

8.2 Similarity and Attraction

By referring to "emission of feelings" [23]: p.181) or by invoking liking as "activity", namely the "reinforcer called social approval" [23]: p.181), HOMANS clearly highlights the limitations of methodological behaviorism. The formulations are very clumsy and contradictory to common sense, obviously in order to satisfy the demands of an obsolete theory of knowledge. Luckily, neither KIRK and COLEMAN [29] nor TROITZSCH [66], [67] are continuing these definitory acrobatics.

In order to bypass these problems and have a definition of liking which is both psychologically sound and ready for easy operationalization, I employed BYRNE's [8] theory of *attitude- similarity*. In essence, BYRNE proposes that attitude-similarity is the main determinant of interpersonal attraction, with the relationship between the two variables being linear positive. According to BAMBERG[9], this proposition has been confirmed in numerous studies.

In effect, I employed the notion of *attitude-similarity* as a proxy for *liking* since I did not incorporate factors to the model which could interfere with their relationship.

8.3 Feedback

In the original model of KIRK and COLEMAN the feedback-relation between liking and interaction is realized by proposing two counterdirected effects connecting these variables (see [Condensed Hypotheses]). The implementation proceeds analogously with explicit representation of the hypotheses (compare section [The Kirk-Coleman-Model]).

Now for the case of SEU-theory, the introduction of feedback to the process could have been accomplished as it is done implicitly by HOMANS' reinforcement argumentation. There he proposes that the change of attractivity is a function of the kind of behavior, assumed that the individuals can interact in more than one way.[10] This results in mutual dependency of liking and interaction. Thus, feedback might to be said an implicit attribute of the setup of the system of interaction, regardless of the kind of adaptation-hypothesis employed.

Needless to say that I discovered this fact after already having implemented the model. For sake of my reputation I have to emphasize that this account on feedback gets lost in the projection from theory towards the [Condensed Hypotheses].

As a consequence, I modelled the feedback-process with the introduction of Social

[9]The information was given to me during a personal conversation. SEBASTIAN BAMBERG is a senior lecturer of Social Psychology at the University of Giessen.

[10]This can be shown by the following passage: "...that every person evaluates some of the activities of the others as valuable..." ("...dass jede Person einige Aktiviteaten der anderen Person wertvoll findet..." [?]: p.155).

Impact Theory, which replaced in alliance with SEU the two original hypotheses of the Kirk-Coleman-model.

8.3.1 Social Impact Theory

LATANE's [32] *Social Impact Theory* (SIT) is a rather concise theory determining the impact of social influence on individual action and behavior. It is defined by the following two equations. The first formulates the "principle of social forces".

$$Imp = f(SIN) \tag{8.2}$$

Where the social impact Imp is defined as a multiplicative function f of the strength S, the immediacy I and the number N of sources of social influence.

The second equation formulates the "psychosocial law":

$$Imp = sN^t \mid 0 < t < 1 \tag{8.3}$$

Where the social impact Imp is defined by a power function of the number N of sources of social influence. Here s plays the role of a scaling constant, while the exponent t is required to be in the interval of $(0 : 1)$.

The equation has the following properties: it increases monotonous while its derivative decreases monotonous, which results in decreasing *marginal impact*.[11] Furthermore, if the exponent t converges to 0, the function will approximate a constant function while it will approximate a linear function if t converges to 1.

The name "psychosocial law" derives from the "psychophysical law" of STEVENS, which employs the same formulation to relate objective and experienced intensity of stimuli.

Social Impact Theory can be evaluated as a simple and well established *phenomenological* account on social influence. (compare TANFORD / PENROSE [64]).

8.4 Agent Level Theory: Action and Social Influence

The integration of both Subjective Expected Utility and Social Impact Theory is accomplished by invoking the previously introduced considerations on employment of attitude-similarity as a proxy for liking, resp. utility. This allows for attitudes being the basis for decisions on interaction and then being interaction being the basis for changes in attitudes:

SEU: Attitude-Similarity $\rightarrow$ Interaction

[11] This means that the more sources of influence are present, the less difference makes the appearance of a additional source.

$$\text{SIT: Interaction} \rightarrow \text{Attitude-Similarity}$$

This is the general theoretical setup of the Modified Kirk-Coleman-Model on the psychological- resp. agent-level.

Chapter 9

Declaration of the Modified Model

In this section I want to formally declare the Modified Kirk-Coleman-Model. For this I will use the theoretical results gained in the previous section, as well as the modelling procedure defined in the section [Probabilistic System Model]. The final aim is representation of the models theoretical assumptions in terms of a *dynamic bayesian network*.

Detailed description of the models implementation employing MATLAB Bayes Net Toolbox will be given in the respective appendix.

9.1 System Representation

The Modified Kirk-Coleman-Model is characterized by representing interaction behavior between three agents. Therefore a first system representation on the *social level* (as introduced in the [System] section) could be the following[1]:

$$\mathcal{S}_{pot} = \langle \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}, \mathcal{I} \subseteq \{I_{12}, I_{13}, I_{21}, I_{23}, I_{31}, I_{32}\}, \emptyset \rangle \tag{9.1}$$

Here A_i denotes the Agents (the composition-set) and $\mathcal{I}$ their (bonding-) set of realized interactions (excluding interactions with themselves)[2] while the environment is defined as empty. One should note that the set of realized interactions $\mathcal{I}$ is a central *explanandum* of the model and therefore unknown. The system is denoted $\mathcal{S}_{pot}$, a potential system, because of this.

Since the Agents are defined by SEU and SIT, they can be said to be systems themselves, consisting of the variables defined by the respective theories. Thus the models system representation needs to proceed on a *lower agent-level*.

In the last section I defined utility being equal with liking which was again defined by difference in attitudes, an agent A_i's composition C_{Ai} may be defined the following way:

[1] For now, I will ignore the aspect of time.
[2] The indices of interactions signify the respective source- and target-agents.

$$\mathcal{C}_{Ai} = \{Action, Attitude, Trust\} \tag{9.2}$$

The definition of the agents composition C_{Ai} becomes clear after looking at their respective bonding sets B_{Ai}, which are defined by the two agent-level theories and an *auxiliary theory of trust* which models the expectation-component of subjective expected utility theory.

$$\begin{aligned}
\mathcal{B}_{Ai} = \{&(SEU : \Delta Attitude_{ij}, Trust_i \rightarrow Action_i), && (9.3)\\
&(SIT : Attitude_{all}, Action_{all} \rightarrow Attitude_i), && (9.4)\\
&(AUX : Action_j, Trust_{ji} \rightarrow Trust_{ji})\} && (9.5)
\end{aligned}$$

This means in words that, according to SEU-theory, agent A_i's choice of an interaction partner $Action_i$ depends on both the difference in attitude $\Delta Attitude_{ij}$ between Agent A_i and the potential partner A_j (representing utility) and on his state of trust $Trust_i$ (representing prospect).

A_i's $Attitude_i$ is, according to SIT, dependent on the $Attitude_{all}$ of the other Agents, given they have interacted with A_i as signified by their actions $Action_{all}$.

The auxiliary theory defines A_i's state of trust $Trust_{ji}$ upon agent A_j's willingness to engage in interaction with himself as being dependent from its previous state $Trust_{ji}$[3] and the other agents interactions $Action_j$. (I will assume a variable of trust for every possible interaction.)

I will end the tiring explanation of the formulas by completing the definition of agents, accounting for the fact that they form their mutual environment. Invoking above definitions on composition and bonding-sets of agents together with the mutual environment consideration, an agent can be defined the following:

$$\mathcal{A}_i = \langle C_{Ai}, B_{Ai}, (\mathcal{A}_j, \mathcal{A}_k)\rangle \tag{9.6}$$

Finally, the connection of the two levels is achieved by defining the equivalence of interaction I_{ij} and $Action_i$:

$$I_{ij} \equiv Action_i \tag{9.7}$$

The two above definitions can be seen as *bridge hypotheses* (compare [Level Transition]and [Semantics and Bridge Hypotheses), because they are relating agent and social level. Nevertheless, there there will be the need for additional bridge hypotheses, translating configurations of individual actions into properties of the system of interaction.

Returning to model-declaration, inserting the agent definition into the social level

[3]Time is not introduced explicitly yet.

composition, given by the first term of equation (33), completes the definition of the models *qualitative* system representation:

$$\mathcal{S} = \langle\langle C_{A1}, B_{A1}, (\mathcal{A}_2, \mathcal{A}_3)\rangle, \langle C_{A2}, B_{A2}, (\mathcal{A}_1, \mathcal{A}_3)\rangle, \langle C_{A3}, B_{A3}, (\mathcal{A}_1, \mathcal{A}_2)\rangle\rangle \tag{9.8}$$

As we can see, the system exposes an obviously different structure on agent-level than on social level, namely because interactions are not explicitly specified as bonding relations but as components of the agents.[4] However, I will soon restore proper social level representation.

Since equation (39) contains all bonding relations on the lowest level of definition, it constrains the system in a way that allows for deduction of interactions, given specific functional definitions and starting values.

In conclusion: Specific instances of $\mathcal{S}$ allow for deduction of actual realizations of the potential system $\mathcal{S}_{pot}$ on social level.

9.2 Transition Graph

I will proceed by defining the transition representation of the Modified Kirk-Coleman-Model (see [Unfolding and Transition Representation]) and thus introduce time into the model.

$$\mathcal{S}_{ts(t,t+1)} = \langle \mathcal{C}_{ts(t,t+1)}, \mathcal{E}_{ts(t,t+1)}, \mathcal{B}_{ts(t,t+1)}\rangle \tag{9.9}$$

Its familiar look (as compared to equation (40)) is restored by separating composition and bonding sets.

$$\mathcal{C}_{ts(t,t+1)} = \{C_{A1t}, C_{A2t}, C_{A3t}, C_{A1t+1}, C_{A2t+1}, C_{A3t+1}\} \tag{9.10}$$

$$\mathcal{B}_{ts(t,t+1)} = \{B_{A1unfolded}, B_{A2unfolded}, B_{A2unfolded}\} \tag{9.11}$$

$$\mathcal{E}_{ts(t,t+1)} = \{\emptyset\} \tag{9.12}$$

In transition representation $\mathcal{S}_{ts(t,t+1)}$, the composition set is "duplicated" (as long it does not change), in order to represent the objects in time explicitly. The unfolded bonding sets now contain relations between the objects which originate in the domain of t and end in $t+1$.

[4] One should furthermore note that the environment $\mathcal{E}$ of the system $\mathcal{S}$ is still empty, regardless of the property that the agents form their mutual environment: The list of remaining agents $\mathcal{A}$ within the particular agent $(C_{Ai}, B_{Ai}, (\mathcal{A}_j, \mathcal{A}_k))$ can be interpreted as recursive pointer to their respective definitions. In fact, this represents properties of the bonding set, not the environment set.

Graphical representation follows naturally from above considerations: for the reason that the environment set $\mathcal{E}_{ts(t,t+1)}$ is empty, its merging with the composition set $\mathcal{C}_{ts(t,t+1)}$ (as proposed in [Transition Graph]) is unnecessary. The latter combined with the (still qualitatively defined) transition bonding $\mathcal{B}_{ts(t,t+1)}$ readily defines a *directed acyclic transition graph*[5] $\mathcal{DAG}_{ts(t,t+1)}$, the backbone of dynamic bayesian network representation:

$$\mathcal{DAG}_{ts(t,t+1)} = \langle \mathcal{C}_{ts(t,t+1)}, \mathcal{B}_{ts(t,t+1)} \rangle \tag{9.13}$$

In order to have a look at the actual transition graph, one now needs to explicate its sets. Nevertheless, the bonding set is to rich to understand the graph as a whole easily. Therefore I will decompose it into the respective *parentship subgraphs*.

9.2.1 Demonstration of Parentship Subgraphs

The parentship subgraphs are oriented on the theories employed in the model. This allows to interpret the graphs as templates since they exist three times, once for every agent. The agents are indexed i, j, k with i denoting the respective *ego* and j and k the respective *alteres*.

The illustration [Figure 9.1] shows the subjective expected utility theory template subgraph $\mathcal{DAG}_{ts(t,t+1)SEU,i}$ (compare equations (30),(35) and section on [Subjective Expected Utility Theory]): It incorporates all attitude variables in order to allow

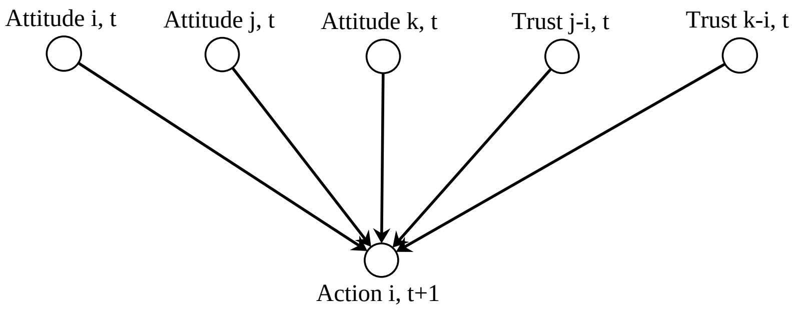

Figure 9.1: Parentship Subgraph of SEU-Template

computation of differences in attitudes (aka. utility or liking) as well as trust assigned to the willingness for interaction of the remaining agents, modelling expectation.

The second template subgraph $\mathcal{DAG}_{ts(t,t+1)SIT,i}$, as displayed in [Figure 9.2], correspondents to the assumptions of social impact theory (compare equation (36) and section on [Social Impact Theory]): The variables contained in this subgraph allow for determination of number and strength of social influence, with the latter defined via differences in attitudes and thus liking.

[5] Acyclicity is granted by introduction of time.

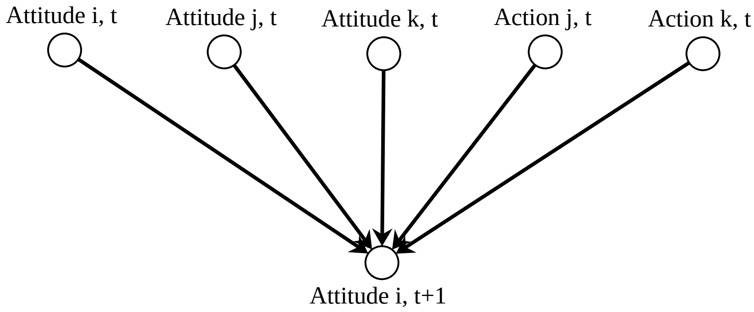

Figure 9.2: Parentship Subgraph of SIT-Template

The last template subgraph $\mathcal{DAG}_{ts(t,t+1)TRU,i}$ is the one assigned to the auxiliary theory of trust (compare equation (37)), which is displayed in [Figure 9.3] and will soon be explained in detail: This subgraph expresses the dependency of current trust on

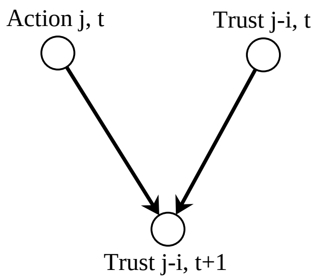

Figure 9.3: Parentship Subgraph of Trust-Template

previous trust and action of the trusted.

As stated, the transition graph $\mathcal{DAG}_{ts(t,t+1)}$ is the union of the template subgraphs for all agents.

$$\mathcal{DAG}_{ts(t,t+1)A,i} = \mathcal{DAG}_{ts(t,t+1)SEU,i} \cup \mathcal{DAG}_{ts(t,t+1)SIT,i} \cup \mathcal{DAG}_{ts(t,t+1)TRU,i} \tag{9.14}$$

$$\mathcal{DAG}_{ts(t,t+1)} = \mathcal{DAG}_{ts(t,t+1)A,1} \cup \mathcal{DAG}_{ts(t,t+1)A,2} \cup \mathcal{DAG}_{ts(t,t+1)A,3} \tag{9.15}$$

Equation (46) shows the aggregation of an individual agent transition subgraph from the theoretic template subgraphs, while equation (47) shows the aggregation of the systems transition graph from the individual agent subgraphs.

Finally, with [Figure 9.4], I also provide an illustration on the complete transition graph $\mathcal{DAG}_{ts(t,t+1)}$ which might look a little complicated compared to the previous illustrations:[6]

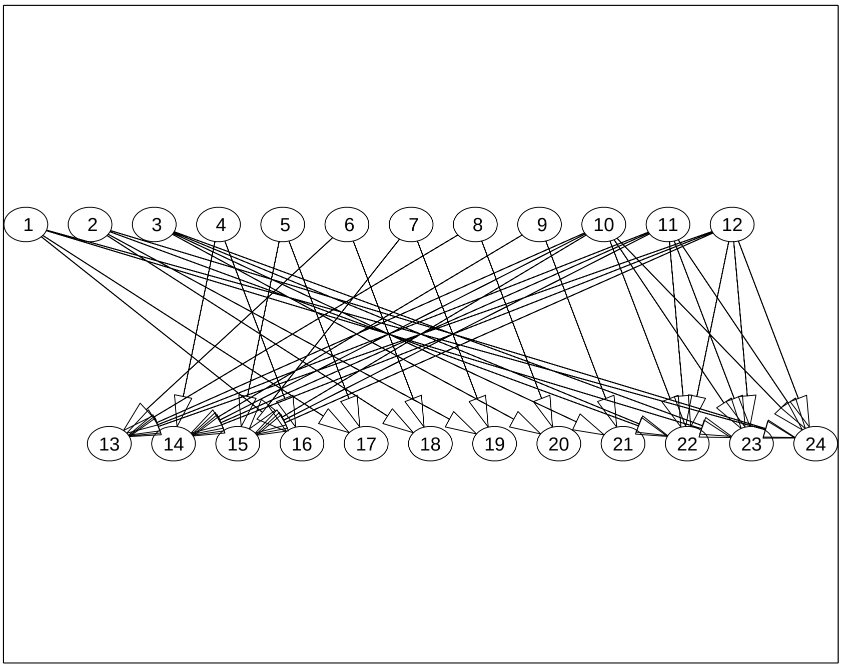

Figure 9.4: Complete Transition Graph

The top line of nodes represents the systems composition at time t, while the bottom line represents it at time $t+1$. The first three nodes in a line represent the action variables of the respective agents (indexed $i = \{1,2,3\}$), the following six the trust variables for every possible interaction (indexed $ij = \{12,13,21,23,31,32\}$) while the last three nodes in a line represent the agents attitude variables (indexed $i = \{1,2,3\}$). As mentioned, the transition graph is the union of all theoretic template graphs for all agents.

The definition of the structure of dependency in time is gained by unrolling the transition graph according to [Representation in Time: Temporal Graph]. This is accomplished automatically by the employed software. For the moment, the structure of the Modified Kirk Coleman-Model is readily defined.

[6]I used some automated software included in Matlab BNT to create the graph (I usually prefer Visone), so please excuse the absence of labels. Drawing graphs and outputting them in vector graphics is more complicated than it looks.

9.3 Functional Dependencies: Conditional Probability Tables

What still lacks in order to complete the model-declaration is the definition of the functional dependencies defined on the nodes of the transition graph. In other words: the formal implementation of the theories constituting the bonding set, as stated in section [System Representation], needs to be accomplished.

As introduced in [A Sketch of Bayesian Networks], I will use the bayesian network formalism for this. In order to achieve a bayesian network representation, the the already defined transition graph needs to be backed up with conditional probability functions representing the mentioned theoretic assumptions. Since those theories are defined on a domain represented by the template transition subgraphs introduced in the last section, I will attach the conditional probability functions on them.

Intended for my own convenience, I assumed discrete variables and thus tables as conditional probability functions.[7]. The tables are defined on variables with the following domains:

$$Action_i \in \{1,2\} \tag{9.16}$$

$$Attitude_i \in \{1,2,3,4,5,6,7\} \tag{9.17}$$

$$Trust_i \in \{1,2,3,4\} \tag{9.18}$$

The selection of domains is somewhat arbitrary, besides the proposal that the two latter should represent ordinal information. Attitude is represented by seven values since this is a usual size of Likert-attitude scales.

In the following section I will discuss the constraints laid on those conditional probability tables (CPD's) by the model-formulations of the respective theories.

9.3.1 Subjective Expected Utility Theory

As can be seen in the first illustration in [Demonstration of Parentship Subgraphs] and equation (35), the structure of probabilistic formulation of the subjective expected utility assumption within the modified Kirk-Coleman-model is the following:

$$P(Action_{i,t+1}) = P(Action_{i,t+1}|Attitude_{i,t}, Attitude_{j,t}, Attitude_{k,t}, Trust_{ji,t}, Trust_{ki,t}) \tag{9.19}$$

Formulation of agent i's utility $\mathcal{U}_{ij}$ of interaction with agent j is given by the following equation:

[7] It was *intended* for my convenience, but the decision did not pay out. Now I have got firmer knowledge regarding modelling with theoretical probability distributions and clearly see the work and complicatedness I could easily have avoided. More on this later.

$$\mathcal{U}_{ij} = \frac{5}{6} + \frac{1}{6}(1 - |Attitude_i - Attitude_j|) \tag{9.20}$$

This equation yields a definition of utility as being linearly decreasing with difference in attitudes and being normalized to the interval (1 : 0) for the defined domain of *Attitude*. Equality in attitudes results in an utility of 1 and maximum difference in an utility of 0.

Expected utility $\mathcal{EU}_{ij}$ of the interaction is given by multiplication of utility $\mathcal{U}_{ij}$ with the respective value of trust $Trust_{ji}$ in j's willingness to return the favor of interaction:

$$\mathcal{EU}_{ij} = \mathcal{U}_{ij}\ Trust_{ji} \tag{9.21}$$

This results in the possibility of an ordering of desirability of interaction choices.

Since deterministic maximization can not be modelled within an probabilistic framework, the agents are defined to choose interaction partners with a probability equal to their respective share of expected utility compared to the utility totally available at that time:

$$P(Action_{i,t+1} = j) = \frac{\mathcal{EU}_{ij}}{\sum_{j=1}^{n} \mathcal{EU}_{ij}}\ |i \neq j \tag{9.22}$$

Applying equation (54) to every possible configuration of its exogenous variables yields the desired conditional probability table attached to the SEU-template parentship subgraph. It should be mentioned that it is large, namely 5488*2 cells.

9.3.2 Social Impact Theory

The structure of probabilistic dependency of the implementation of social impact theory (as introduced in [Demonstration of Parentship Subgraphs] and equation (36)) is the following:

$$P(Attitude_{i,t+1}) =$$

$$P(Attitude_{i,t+1}|Attitude_{i,t}, Attitude_{j,t}, Attitude_{k,t}, Action_{j,t}, Action_{k,t}) \tag{9.23}$$

The implementation of social impact theory is somewhat more complicated than of SEU-theory. First, social impact is divided into three classes of influence, representing the behavioral alternatives of the agent: influence towards a more negative (smaller) attitude value $\mathcal{SI}_{neg}$, influence towards perpetuation of the current attitude value $\mathcal{SI}_{perp}$ and finally towards a more positive (bigger) attitude value $\mathcal{SI}_{pos}$. The particular impact for the respective class m is calculated according to following formula:

$$\mathcal{SI}_m = \bar{\mathcal{U}}_{agents \in m}\ sN(m)^r \mid 0 < r < 1 \tag{9.24}$$

This equation defines social impact $\mathcal{SI}_m$ for the classes via the power function defined on the number of agents in the respective class m, as proposed by the "psychosocial law", weighted by the mean utility $\bar{\mathcal{U}}_{agents \in m}$ of the agents in this class. The term s is a scaling constant, while the exponent r is required to be in the interval of (0:1) (compare section on [Social Impact Theory]).

It should be noted that the power function is not defined directly on the number of agents in the respective class n_m, but rather on a function of this number:

$$N(m) = n_m \mid m = neg \cap m = pos \tag{9.25}$$

$$N(m) = n_m + 1\ q \mid m = perp \tag{9.26}$$

For the case of influence towards change of attitude, the number of agents in the respective class enters the social influence equation (56), while for the case of perpetuation of attitude the number of agents in the class is incremented by one, accounting for the own attitude. The increment is furthermore multiplied by q, a factor representing the ego-agents resistance towards social influence. The higher q gets, the more important is the own attitude compared to the others.

The probability of obedience to a particular impact class $P(\mathcal{OBA}_m)$ is given by its proportion relative to the total amount of impact:

$$P(\mathcal{OBA}_m) = \frac{\mathcal{OBA}_m}{\mathcal{OBA}_{neg} + \mathcal{OBA}_{perp} + \mathcal{OBA}_{pos}} \tag{9.27}$$

Since a more positive or negative attitude may be defined on more than one attitude value, the mass of the probability of change $P(\mathcal{OBA}_m)$ may be required to be spread over those values. The assumption that the probability of attitude change is linearly decreasing with its size is expressed by the following equation:

$$P(Attitude_i = x) = P(\mathcal{OBA}_m)\ (1 - \frac{|x - margin_m|}{\sum_{o=1}^{X} |o - margin_m|}) \mid x \in m \tag{9.28}$$

The probability of showing a particular attitude-value x is defined by weighting the probability of obedience to a certain class of impact $P(\mathcal{OBA}_m)$ by the inverse of its share of the total distances towards the outer margin value of the particular category.

9.3.3 Auxiliary Assumptions on Trust

The structure of probabilistic dependency of the assumptions on trust, as defined in equation (37) and the last template subgraph in section [Demonstration of Parentship Subgraphs], is the following:

$$P(Trust_{ji,t+1}) =$$

$$P(Trust_{ji,t+1} | Trust_{ji,t}, Action_{j,t}) \tag{9.29}$$

The idea is simply that trust in ones willingness to interact is increased if an interaction occurs, while it is lowered when there is no interaction. In order to achieve probabilistic formulation I assume a certain probability p of erroneous change of the value of trust. Again I assume small deviances to be more probable than large ones:

$$P(Trust_{ji} = x) = (1 - p)\ e^{-\alpha\ |x - true|} \tag{9.30}$$

For application of the equation α needs to be determined. The following solutions were calculated using MAPLE, accounting for the fact that probabilities must sum to one:

$$\alpha = 1.60289 \mid true \in margin \tag{9.31}$$

$$\alpha = 2.13678 \mid true \notin margin \tag{9.32}$$

There exist two different solutions for the cases that the true value is on the margin of the domain of $Trust$ or not, resp. for a “sloped” and a “peaked” distribution.

Above equations result in a probability distribution of trust which assigns a probability $1 - p$ to the true value while the probabilities of realizing a different value are exponentially[8] decreasing with the size of this respective difference.

I should note that the implementations of the three theories share a common attribute, namely that events with zero probability are assigned a new probability, only slightly greater than zero. This way I tried to allow for all defined events to be possible.

This concludes the declaration of the Modified Kirk-Coleman-Model. Finally, it is far more complicated than the original, which was not really intended.

[8] If you compare this approach to the similar problem occurring in the implementation of SIT, you my wonder why I used both linear and exponential approaches. The honest answer is that these are ad hoc modelling solutions. In this case I was not able do guarantee positive probabilities by a linear approach and the $\sum_{i=1}^{X} p_i = 1$ constraint. Therefore I used an exponential equation, although Ockhams Razor would have requested a linear one.

Chapter 10

Model Sensitivity

During this section I will give a description of the results of the model. Please excuse me if the interpretation of the results is not too detailed, I just wanted to show that my philosophical convictions can be coded in a working method. This is, what the modified Kirk-Coleman model does.

10.1 Employed Soft- and Hardware

The model has been implemented employing MURPHY's [40] Bayes Net Toolbox for MATLAB. (BNT) BNT is a GPL licensed library of MATLAB functions for learning and inference in bayesian networks, while MATLAB is a very widespread software package for scientific and technical computing. I first experimented with readily compiled and comfortable software like NETICA and GENIE, but these proved to be unhandy for implementation of dynamic bayesian networks. On the other hand I did not want to write my implementation from scratch in a general language like C++, so the use of a sophisticated library in an interpreted language like MATLAB looked promising. A further advantage were its graphics features.

The calculations were executed on a computer with AMD Athlon XP-2500 processor and 512MB RAM running Windows XP operating system. A monte carlo parameter study of the model, running 100 times 100 time-steps took approximately seven hours of processor time.

This fact implies that it would have been a better choice to separate simulation and analysis of the model by means of probabilistic inference. This could have been accomplished by implementing the model employing standard multi agent based methodology and rather use bayesian networks on top of model -generated data.

10.2 Solving the Model

Because of the size of the model (12 times t variables) and its temporal structure[1], the application of exact algorithms for inference in bayesian networks has shown as being infeasible. After some bad experiences I finally arrived at *Likelihood Weighting* as an appropriate algorithm. I will give an introduction to it after a short summary of the trials.

For the case of the very popular *Junction Tree Algorithm* the software crashed at a model size of $t \approx 20$, exceeding an amount of 2GB central memory. Application of the so called *Boyen-Koller-Algorithm*, which is specially designed for approximate inference in temporal models, resulted in out-of-memory crashes at $t \approx 60$.

Experimentation with MATLAB BNT's implementation of *Gibbs- Sampling*, a simulation algorithm which is todays silver bullet for probabilistic inference (see GILKS et.al. [17]), yielded results for $t > 100$, but those were obviously flawed. The inferred distributions showed a strange pattern of entropy, switching between deterministic and uniform.[2]

10.2.1 Stochastic Sampling

Likelihood Weighting proved itself as a feasible approach. Being a sampling algorithm, it avoids computation with the actual rules of probability calculus but rather simulates the "random-experiments" which are modelled by it. Thus the concept of Joint Probability Distribution is not needed for computation, resulting in a minimal need for computer memory.

In order to introduce the algorithm I need to provide a glance at the common problems associated with stochastic sampling. I will start with the introduction of the most common method called *Rejection Sampling* (compare PRESS et.al. [48]). An example will demonstrate the procedure: If we want to sample a distribution say $P(X)$ with $P(X_i = x_1) = 0.2$ and $P(X_i = x_2) = 0.8$ we will employ a pseudo random generator which produces uniformly distributed sample values s_i in the interval $(0 : 1)$. If s_i is < 0.2 we will assign a realization of x_1 to it, if s_i is > 0.2 we will say that x_2 has been realized. This procedure is repeated until the counts of the sample values approximate the probability distribution $P(X)$ with desired accuracy.

The problematic issue is now that if the probability of a particular value of the distribution gets sufficiently small, we will have to wait a long time until a sample value realizes this value. And we will have to wait a accordingly longer in order to get a sufficient

[1] I will not discuss this problem here. The reader is referred to JENSEN's [25] treatment of the topic.

[2] A better software implementing Gibbs Sampling and other MCMC-algorithms is WinBUGS. Its designed for bayesian inference in graphical models, a method which extends this treatment of bayesian networks by the incorporation of knowledge known prior to data. Unfortunately, I discovered this software only after having finished my computations on the modified Kirk-Coleman-model.

number of samples realizing this particular value. For example, if $P(X_i = x_j)$ is 0.001 we will have to expect to draw thousand samples in order to get a single realization x_j and a million samples to get thousand counts of this specific value. One would have produced thousand times more samples than we wanted to.

In bayesian networks this problem has special significance since a sampling sweep through the net requires repeated sampling from multiple conditional distributions. This means that whole configurations of values of parent variables have to be hit by the sampler in order to *allow* for realization of a certain value of the child variable.

Now *Likelihood Weighting* is an approach to avoid the problem of improbability caused by multiple and nested conditions, as coded in the graphical structure of the model. The idea is not to wait until some improbable conditional sample is drawn, but instead to sample from an unconditional distribution and then weight the sample with the probability of its conditions, resp its parent distributions. This allows for significantly accelerated computation. A short description of the algorithm can be found at RIEDMILLER [49].

For completeness I should mention another, more powerful, approach, namely *Markov-Chain-Monte-Carlo* or MCMC, of which the mentioned Gibbs-Sampling is a special case: here a markov-chain is specified in a way that ensures that its stationary limiting distribution is equivalent to the distribution one wants to sample. The power of the method consists in the fact that no redundant samples are produced. It extends the advantages of likelihood weighting by the possibility to cope with improbabilities which do not stem from the structure of the bayesian network. Needless to say MCMC incorporates very sophisticated mathematics. Introduction can be found in GILKS et al. [17].

10.3 Monte-Carlo Parameter Study

In order get a proper idea of the models behavior I undertook a monte carlo parameter study of the model. The following parameters were instantiated with uniform random values:

- Initial values of the systems variables $\mathcal{C}_t$;
- Individual agents social impact theory parameters $\{q_i, r_i\}$ as previously introduced;
- Individual agents probabilities p_i of erroneous change of the value of $Trust_i$;

One hundred monte-carlo runs of the model over 100 time-steps[3] were calculated with the result that the model converged towards joint uniform distribution under all instantiated conditions. I will present some illustrations to clarify this result.

[3]The number of steps was this low due to the massive amount of computations needed. Since the process always converged within less than half of the steps this should not be considered as a problem.

10.3.1 Illustrations of the Monte-Carlo-Runs

Some introduction will be helpful for interpretation. The graphs show attributes of individual systems variables plotted against time. The first half of the plots shows the change of the variables expectation $\mu(v)$ while the second half shows its change of entropy $H(v)$. The latter can be interpreted as a measure of dispersion (besides the fact that it is one of the cental theoretical notions of information theory). An entropy of 0 signifies no dispersion at all, while a maximum value signifies uniform distribution. I abstained from calculating standardized measures since all distributions converge to uniform and thus maximum entropy. The respective maximum values $H_{max}(v)$ are:

$$H_{max}(Action) = 1$$
$$H_{max}(Trust) = 2$$
$$H_{max}(Attitude) = 2.807$$

[Figures 10.1 and 10.2] show the dynamics of agent $\mathcal{A}_1$'s interaction choice $Action_1$.

As can be seen, the expectation and entropy rapidly converge towards indifference[4], resp. maximum entropy which are functions of uniform distribution.

The same is true for the agents attitudes and trusts, where expectations converge towards indifference and entropies towards maximum. Subsequently, two exemplary characteristics of agent $\mathcal{A}_1$, namely $Attitude_1$ ([Figures 10.3 and 10.4]) and $Trust_{12}$ ([Figures 10.5 and 10.6]) are displayed. Please note that I am only showing illustrations for the first of the three agents, because the remaining ones look indistinguishable from these. (Compare [Appendixes] for complete results.)

10.3.2 Cluster-Analysis of the Monte-Carlo-Runs

A k-means cluster-analysis performed on the complete set of variables and runs supports the conclusion of convergence towards joint uniform distribution. According to its results it does not make sense to assume a partitioning of model runs. This is implicated by the low and rather constant characteristic of the Scree-plots of Mean-Silhouette- ([Figure 10.7]) and Gain-in-R^2-Values ([Figure 10.8]).

Silhouette-Values are coefficients in the range of $(-1 : 1)$, where -1 signifies a probably mistaken cluster-assignment of a single data entry (run). Accordingly, a value of 0 signifies an assignment to more than one cluster and a value of 1 a probably correct assignment. Gain-in-R^2 denotes the increase of explained variance over the data entries compared to a (k-1)-cluster solution and is sometimes called *Eta*-coefficient (compare BACHER [2]).

[4] $Action_i$ has a domain of $1, 2$, where a higher value corresponds to an agent with a higher index. Thus an expectation of 1.5 implies indifference. Compare [Functional Dependencies: Conditional Probability Tables] .

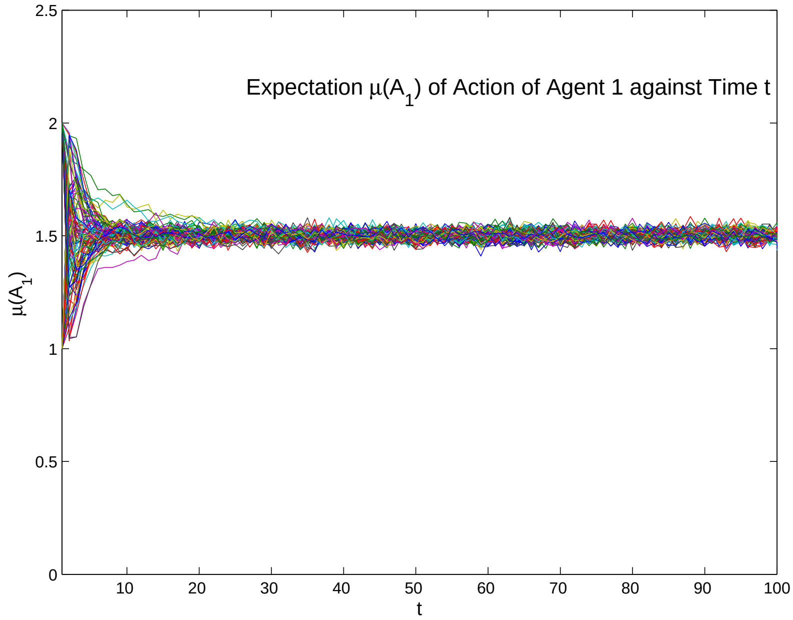

Figure 10.1: Expectation of Monte-Carlo runs of interaction choice $Action_1$ of agent $\mathcal{A}_1$ against time.

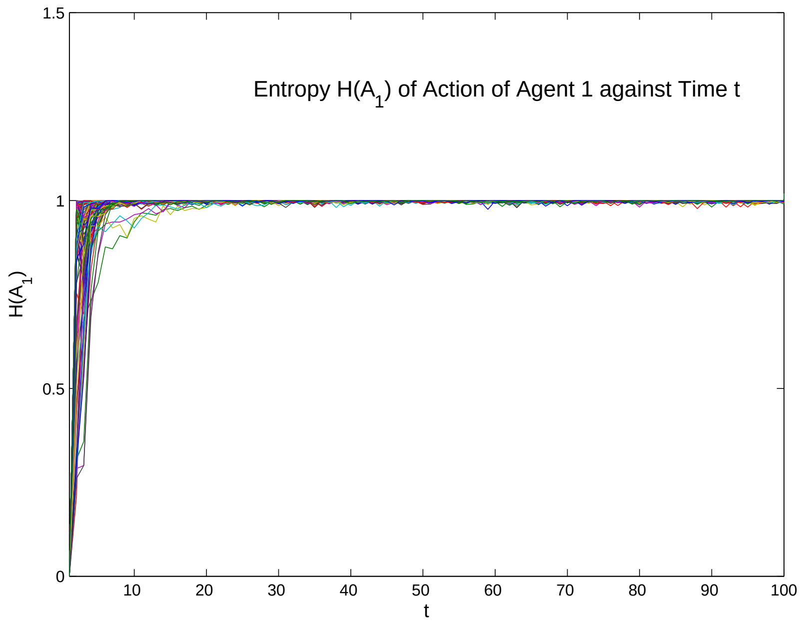

Figure 10.2: Entropy of Monte-Carlo runs of interaction choice $Action_1$ of agent $\mathcal{A}_1$ against time.

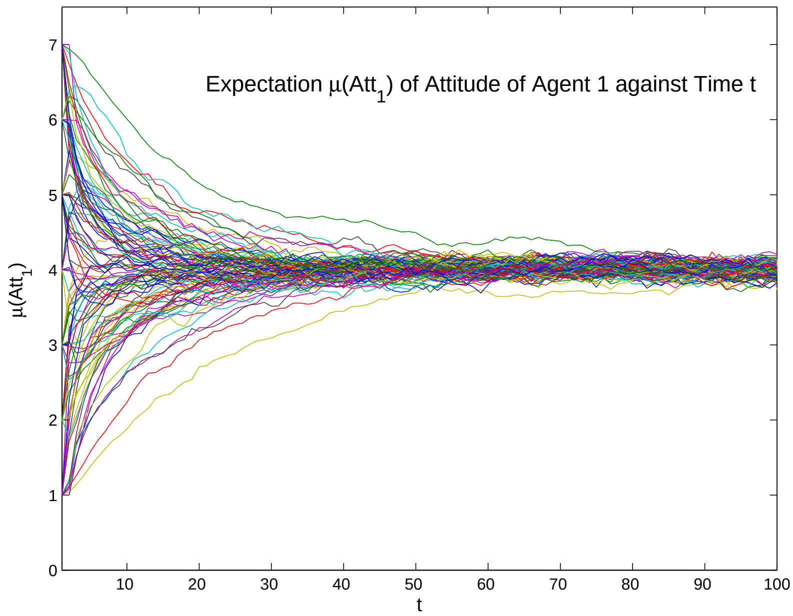

Figure 10.3: Expectation of Monte-Carlo runs of attitude $Attitude_1$ of agent $\mathcal{A}_1$ against time.

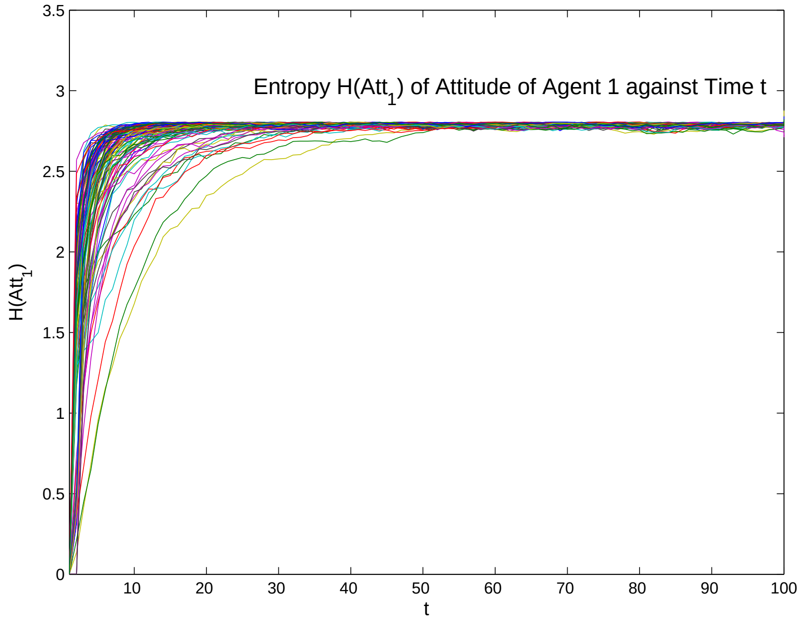

Figure 10.4: Entropy of Monte-Carlo runs of attitude $Attitude_1$ of agent $\mathcal{A}_1$ against time.

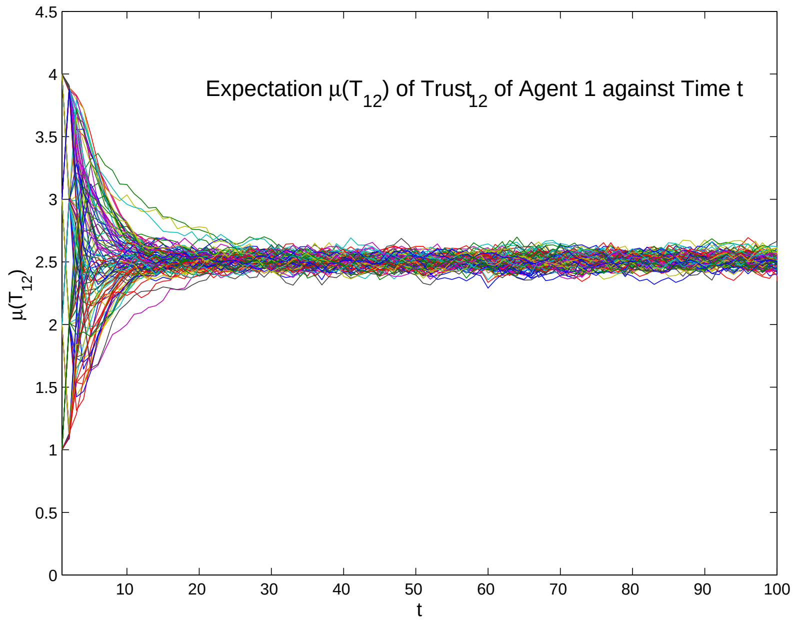

Figure 10.5: Expectation of Monte-Carlo runs of Trust $Trust_{1-2}$ of agent $\mathcal{A}_1$ against time.

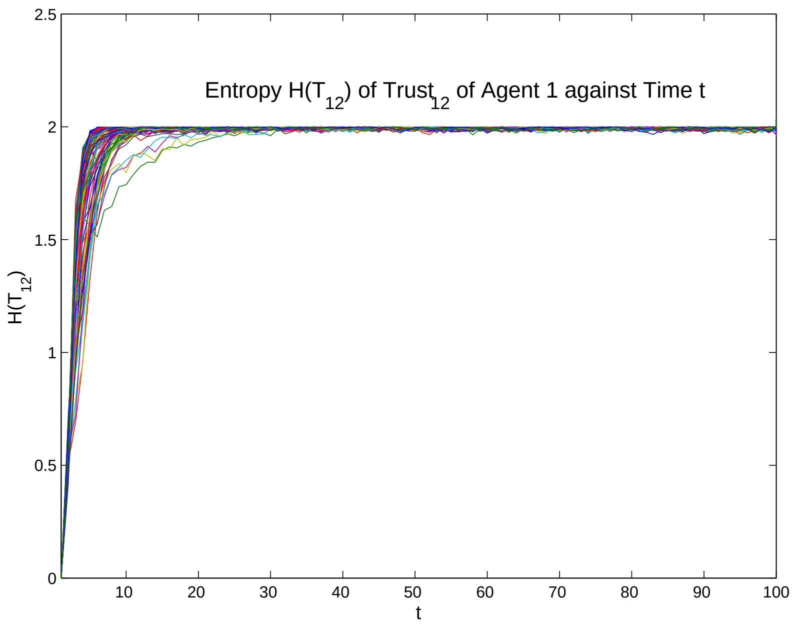

Figure 10.6: Entropy of Monte-Carlo runs of Trust $Trust_{1-2}$ of agent $\mathcal{A}_1$ against time.

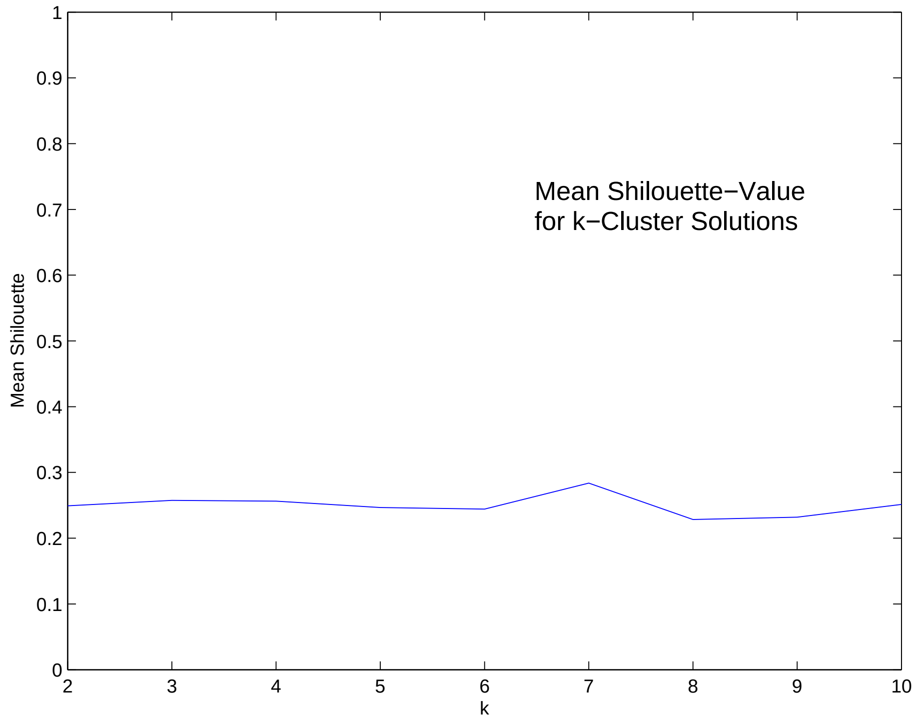

Figure 10.7: K-Means-Clustering of MC-parameter study results: Plot shows mean silhouette-values for k-cluster solution.

10.4 Interpretation of the Results

Intuitively, the results seem to be predictable from both employed theories and complete structure of possible interactions. Combination of Subjective Expected Utility - and Social Impact Theory implies an opportunistic view on actors: not only decisions but also preferences are flexible towards environmental conditions. Given homogeneous restrictions over the agents, what else than increasing similarity in attributes and thus mutual equality of opportunities could emerge from these theories?

Admittedly, this result is trivial and could have been easily predicted from the models assumptions with out employing complicated mathematical apparatus. As mentioned before, the choice of the Kirk-Coleman model has been a compromise. Certainly one is free to implement models which actually exploit the advantages of bayesian networks.

Maybe a more interesting (and realistic) result could be realized with a heterogenous structure of interaction. Nevertheless, this is beyond the scope of this particular

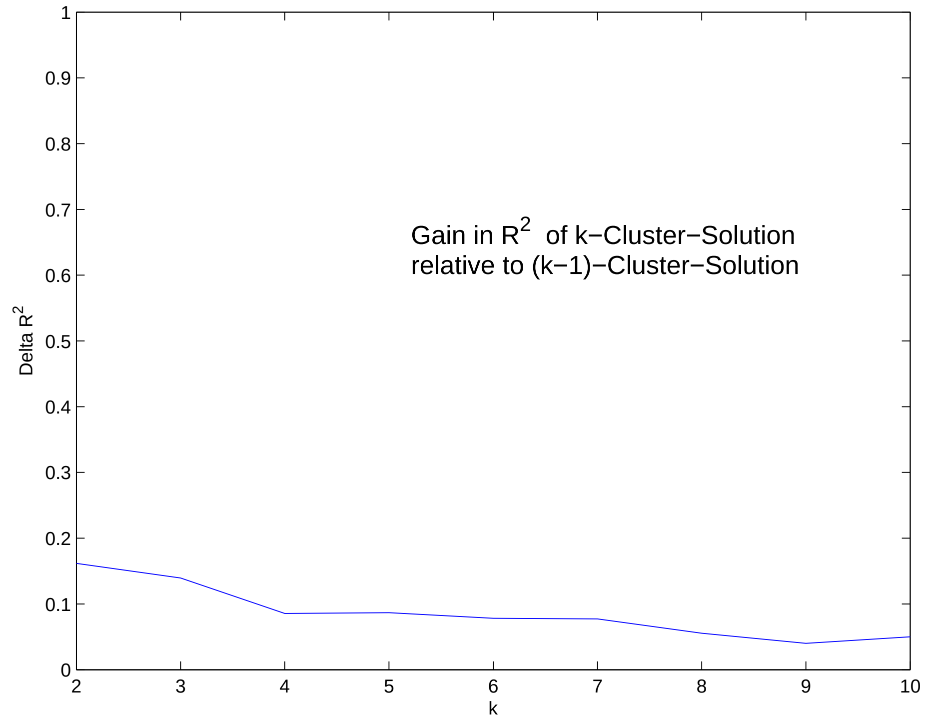

Figure 10.8: K-Means-Clustering of MC-parameter study results: Plot shows gain in R^2 for k-cluster solution as compared to $(k-1)$-cluster solution.

work.[5]

10.4.1 Comparison to the Original Model

KIRK and COLEMAN stated in their article ([29]: p.189) that the stochastic simulation model produced a wide array of patterns, among them the isolated-pair-configuration as proposed by SIMMEL.

Nevertheless, a view on their tables of simulation results shows a tendency towards lowering the differences between individual participation of interaction. This is always the case expect for extreme situations where one agent is forbidden to take initiative or initial liking-values differed by the magnitude of 100. TROITZSCH ([66]: p.222) reports for his implementation that asymmetries in interaction behavior seem to be dependant on initial values of the simulation. This is also reported by KIRK and COLEMAN ([29]: p.189) for their microscopic differential equation system implementation.

[5]TROITZSCH [66] [67] has experimented with sets of more of three agents and inhomogeneous structures of possible interactions. There exists a variety of possible realizations of the process, but there is no systematic review of the runs known, at least to me.

Sadly, KIRK and COLEMAN undertook no systematic parameter study and furthermore calculated maximally only 1000 iterations of the simulation. I assume that in the case of the modified model convergence has been accelerated by the more powerful feedback mechanism of Social Impact Theory as compared to the "reward" of interaction in the original model. For the case of extreme initial conditions the SIT feedback mechanism may have enabled convergence towards joint uniform distribution.

I finally arrived at the conclusion that the models behaviors do only differ with respect to the strength of the feedback modifying agents preferences and that the SIMMEL-hypothesis cannot usually be realized by assumption of opportunistic actors, given homogeneous restrictions on their possible actions. Exceptional realization may be enabled by initial conditions which exceed the impact of the particular feedback mechanism on the agents preferences.

Chapter 11

Level-Transition Instantiated

As promised in the [Introduction] and the section [Level Transition], I will finally show an implementation of an actual level-transitory explanation in order to prove the feasibility of both proposed methodology and method. Again, it should be understood rather as an demonstration than as a detailed analysis of the model.

In principle, the employed procedure (compare section on [Level Transition]) consists of the following operations:

- Definition of Bridge Hypotheses which translate properties on both levels;
- Performance of inference on the lower level: either exhausting the set of realizations of higher level properties or sampling it;
- Aggregation of the results of lower-level inference over the functions defined by Bridge Hypotheses, thus completing level transitory explanation;

This structure will organize the arrangement of the ongoing section. I will start by introducing the theoretical circumstances of the example.

11.1 Bridge Hypothesis: Balance Theory Classification

Theoretical background of the example is HEIDER's [19] *Theory of Structural Balance* which will deliver the bridge hypothesis to be considered. Balance Theory can be described as being similar to FESTINGER's *Theory of Cognitive Dissonance* and analyzes social situations with respect to the occurrence of dissonance. It should be mentioned that Balance Theory is not connected to the employed agent level theories. Nevertheless it allows a good exemplification of the method. I will now briefly describe its basic assumptions.

11.1.1 Balance Theory

HEIDER considers a undirected triad as an abstract scenario for Balance Theory, which is denoted P-O-X-triple: P stands for a person, O for a second one and X for some particular

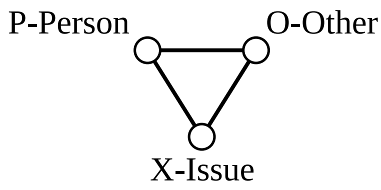

Figure 11.1: The P-O-X Triple.

issue or object, which can certainly be some third person[1]. In this case edges of the graphs represent mutual symmetric evaluations $Eval_{i,j}$ between two elements of the triad i, j and are defined to be either positive or negative. Negative evaluations $Eval_{i,j} = (-)$ will eventually be designated by 0 and positive evaluations $Eval_{i,j} = (+)$ by 1 in illustrations [2].

The core assumption of Balance Theory is now that there are configurations of evaluations which provoke cognitive dissonance. These are namely those in which a) the evaluations of two persons disagree while their mutual evaluation is positive and b) in which their evaluations agree while their mutual evaluation is negative. The situations which are provoking cognitive dissonance are called *unbalanced*, while the remaining are called *balanced*. Since cognitive dissonance implies a persons activity to eliminate it, unbalanced states are considered immanently unstable by HEIDER.

In the case of X being a person, the triad becomes symmetric, as mentioned. This leads to the possibility of modelling structural balance in analogy to multiplication of signs (compare WASSERMAN / FAUST [68]). An easy to remember example for this is the saying "the enemy of my enemy is my friend". The overall sign of a triad is computed the following way:

$$(+) * (+) = (+) \tag{11.1}$$
$$(-) * (-) = (+) \tag{11.2}$$
$$(+) * (-) = (-) \tag{11.3}$$

This can be generalized to the proposal that uneven numbers of negative signs (evaluations) result in a negative sign of the triad.

[1] A memory hook for my indication of the elements of the P-O-X triple is: It begins with P in the upper left and proceeds clockwise.

[2] This is because the employed software for drawing graphs, VISONE, does not support treatment of signed graphs.

Now, negatively signed triads are defined as unbalanced while positively signed triads are defined as balanced. The following illustrations shows the possible configurations of the P-O-X triple with 0 representing (-) and 1 representing (+):

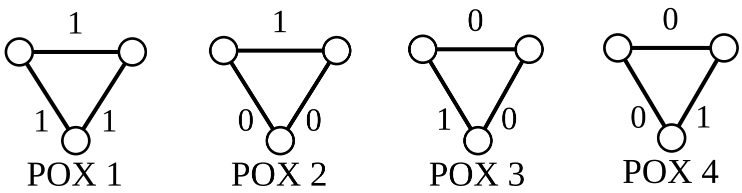

Figure 11.2: Balanced Triads, $(0 \equiv -, 1 \equiv +)$

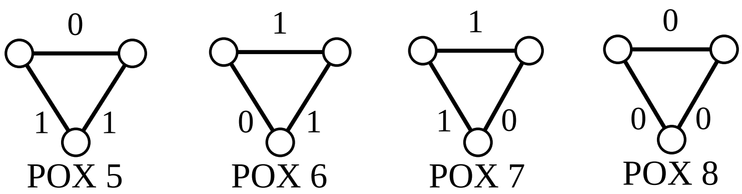

Figure 11.3: Unbalanced Triads, $(0 \equiv -, 1 \equiv +)$

It can easily be seen that the upper half of the eight possible P-O-X configurations is balanced because its even number of zeros, while the lower four are unbalanced due to the uneven number of zeros.

11.1.2 Assignment of Model-Realizations to P-O-X Triples

The core bridge hypothesis applied to the modified Kirk-Coleman model consists in mapping relations of agent variables on the set of P-O-X triples, resp. their balance-classes. This is achieved by application of the following criterion:

$$|Attitude_i - Attitude_j| < 3 \mapsto Eval_{i,j} = (+) \tag{11.4}$$

$$|Attitude_i - Attitude_j| \geq 3 \mapsto Eval_{i,j} = (-) \tag{11.5}$$

By application of above formulae, an evaluation $Eval_{i,j}$ is assigned to every two-digit difference-relation of attitudes $|Attitude_i - Attitude_j|$. This efficiently enables for assignment of the 343 attitude-configurations to the eight P-O-X triple classes, which can be further aggregated towards classes of balance and unbalance.

I should add that the criterion value of three is chosen completely arbitrary, since there is no concrete attitude object defined. Thus the attitude values lack a specific meaning. Furthermore the bridge hypothesis is quite uninformative, as proposed in the section [Bridge Hypotheses and Violation of Object Identity].

11.2 Aggregated Results of Lower Level Inference

The task of the computations was an arbitrarily chosen question for which interaction partner agent two would decide in the next step, given a specific P-O-X triple or balance state was instantiated.

As proposed at the beginning of the section, inference on the lower level was carried out, namely in an exhaustive manner for all 343 possible configurations. Thus the individual quantity to be computed via the Bayes Net was

$$P(Action_{2,t+1} = x | Attitude_{1,t} = w, Attitude_{2,t} = y, Attitude_{3,t} = z) \tag{11.6}$$

with w, x, y, z being configurations of values from the domains of the respective variables[3] For expression of the findings in balance theory notation I assumed agent 1 to be P, agent 2 to be O and agent 3 to be X.

After lower-level computation the results were aggregated over the P-O-X- and Balance-classes according to their respective definitions and the bridge-hypotheses of equation 68 and equation 69.

Mean probabilities of O's (resp. 2's) interaction choices $P_\mu(Action_O = i)$, given a realization of a particular class in the previous step are shown in the subsequent table. Additionally, the appropriate standard deviations over the element distributions are displayed:

Class	$P_\mu(Action_O = P)$	$P_\mu(Action_O = X)$	$SD_{P\mu}(Action_O)$
P-O-X 1	0.5000	0.5000	0.0490
P-O-X 2	0.7273	0.2727	0.1080
P-O-X 3	0.5000	0.5000	0.2031
P-O-X 4	0.2727	0.7273	0.1080
P-O-X 5	0.3954	0.6046	0.0344
P-O-X 6	0.5000	0.5000	0.0421
P-O-X 7	0.6046	0.3954	0.0344
P-O-X 8	0.5000	0.5000	0.3674
Balanced	0.5000	0.5000	0.1887
Unbalanced	0.5000	0.5000	0.0714

The interpretation of the table is quite straightforward. Since definition of membership to a particular P-O-X triple is dependent on configurations of attitudes and thus utility, the mean probability distributions express differences in opportunities for agent O (resp. 2) over the particular classes.

[3]These calculations called for 21 seconds of processor time. The amount of time is comparably low because of the minimal time-window considered.

In cases where the distribution is fifty/fifty the agents P and X are in average simply equally attractive. If a probability is greater than 0.5, this can be seen as a result of asymmetric attractiveness corresponding to the definition of the respective P-O-X triple. A further indicator for this is the symmetry over the asymmetric distributions within a single balance-class: the probabilities are simply twisted (compare P-O-X 2/P-O-X 4 and P-O-X 5/P-O-X 7, remembering that agent O is in the upper right corner). Differing probabilities over the triples of the two balance-classes are results of the asymmetric criterion value of three defined in equation 68 and equation 69.

Varying precision of aggregate predictions as given by the standard deviations $SD_{P\mu}(Action_O)$ cannot be so easily explained: detailed examination of the lower level realizations should be necessary.

This should finish the treatment of the actual implementation of level transitory explanation. A description of impact of macroscopic states to individual action has been achieved.

In the end, the actual process of computation might not seem too different from what one would expect after being reminded of the Coleman-micro-macro scheme. Nevertheless, the problem consisted in *proposing* a method, which avoids mistakes. And there are truly enough opportunities for making mistakes with this particular question, especially at concept formation. The fact that the actual procedure seems trivial is a great relief for me. The good ideas are always trivial with respect to something. Lets hope that this is true for this work, too!

Chapter 12

Conclusion

At last I want to summarize the problems and solutions faced during the course of this work and furthermore give an outlook towards future research.

12.1 Summary: Problems and Solutions

The basic problem faced in an account on level-transitory explanation and thus emergence is the identification of an appropriate criterion of object identity. This has been found in the definition of objects by the set of causal mechanisms attached to these. As being dependent on the conceivability of manipulation causality provides a criterion which assures object identity on all levels considered. Some might be disappointed by this subjectivist approach, but I guess that proposition of restricted independence of object and observer is the only firm ground reachable during discussion of this matter. Furthermore the employed philosophical approach allowed for well founded criticism of methodology of both methodological individualism and bridge hypotheses.

For implementation of a proposed methodology of lower level inference, appropriate methods of computation were necessary in order to cope with complexity of system structures. Exemplary hazards of complexity are multicausality and nonlinearity of interactions. An appropriate method has been found with probability theory in its formulation of bayesian networks: It allows for modelling of global processes by means of structures of local dependencies, while employing a mode of inference (marginalization, resp. integration) which is insensitive to the form of the actual functional dependencies.

The Kirk-Coleman-model proved to be a grateful toy for application of the methodology and was modified in order to fit its requirements and include modern theories as Subjective Expected Utility and Social Impact Theory. Furthermore a method for translation of System Representation into bayesian network Representation has been proposed.

Two types of calculations were undertaken with the modified Kirk-Coleman-model: First, a monte-carlo parameter study which led to the conclusion of convergence of the process towards joint uniform distribution under all instantiated conditions; and second, an actual realization of level-transitory explanation. A classification of states of the model according to Balance-theory has been introduced and effects of macroscopic state-classes on a particular agent property (one time-step later in the model) have been computed.

Problematic issues concerning the modelling task were associated with the employed algorithms and software. It turned out that even a "toy-application" like the modified Kirk-Coleman model easily puts bayesian networks inference algorithms to their limits. Only sampling algorithms seemed to deliver appropriate performance. Due to problems with the employed implementation of Gibbs-Sampling I finally used the Likelihood-Weighting algorithm. It turned out that application of standard simulation methods for modelling combined with separate bayesian network analysis of generated model data would have been a more practical approach. Reasons for this would be greater ease of implementation and enhanced speed of computation.

12.2 Outlook

There are some possible directions of future research which emanate from this work.

One direction would be to deepen its considerations regarding stability and emergence. I will track this path for personal interest, but the issue is complicated and has not yet revealed a promising starting point. What seems more promising is the application of the methodology to empirical problems. Either to structural analysis of rather small and completely surveyed social systems, or large scale data, utilizing structural proxy-assumptions. The latter enterprise would certainly demand modified methods of computation, like Synergetic or Bayesian Hierarchical Models , to name examples. Experimenting with this is in fact part of a plan about what to do this summer.

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I have invested a lot time in considerations about the methodological foundations of social theory, probably years if I start counting when the question first arose. Since a kind of result is achieved I will not complain about passed opportunities but instead engage in something tangible in my future. I have to thank many people, with no particular order:

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Appendix A

Online Provision

Appendixes can be accessed online at the following location:

http://www.stud.uni-giessen.de/ st1334/MicroMacro/append.html

The reason for online provision is to enable direct code-access.
Since its amount adds up to 65 pages the reader might excuse its absence.

Appendix B

Complete Monte-Carlo Results

Nevertheless, the reader should not be deprived of a plot of the dynamics of the complete set variables in the model:

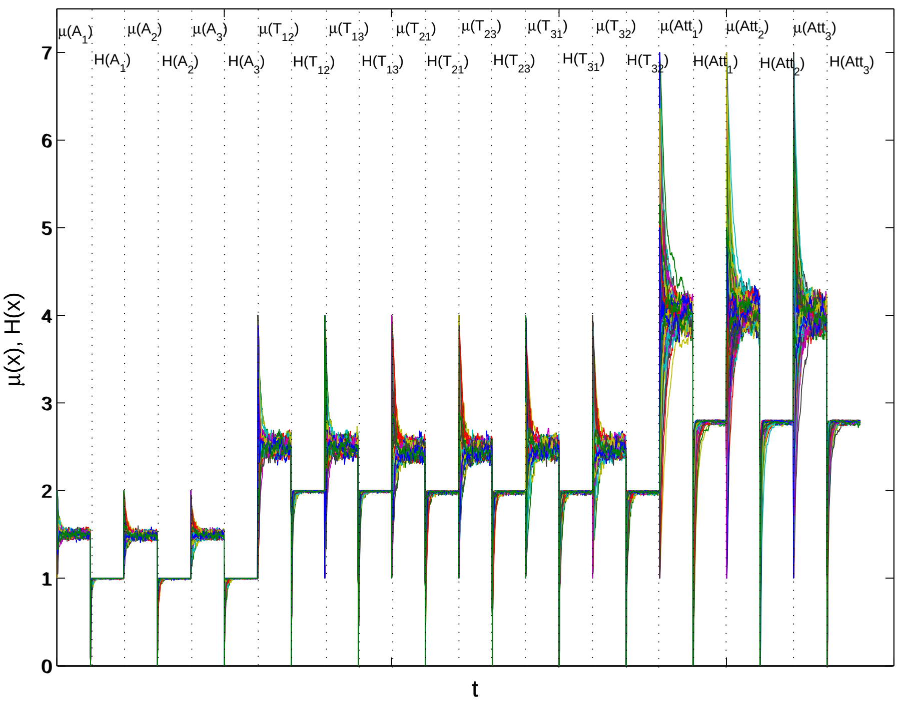

Figure B.1: Plot displaying dynamics of expectations and entropies of all variables simultaneously.