

Justus-Liebig-University Gießen

# Play-Hysteresis and Okun's Law

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**Doctoral Thesis**

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*Author:*  
Pedro Polli Scheidemantel

*Supervisors:*  
Prof. Dr. Matthias Göcke  
Chair of Transition and  
Integration Economics  
Prof. Dr. Peter Tillmann  
Chair of Monetary Economics

*This thesis is dedicated to my family,  
with appreciation for their support along the way.*

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# Abstract

We investigate the role of economic hysteresis in the relationship between output and unemployment, as originally described by Okun's law. Drawing on concepts from hysteresis theory in physics, we model how firms face sunk costs when adjusting employment in response to output shocks, leading to persistent effects. At the micro level, we formalise firm behaviour using a non-ideal relay framework and aggregate these dynamics using the Preisach model. The resulting macroeconomic hysteresis is approximated using a play algorithm, enabling empirical estimation. Applying this framework to data from Germany and other EU countries, we identify significant non-linear and asymmetric responses in labour markets. Our results suggest that temporary output shocks can have long-lasting effects on employment, underlining the importance of incorporating hysteresis into economic policy design. By extending Okun's law with a hysteresis component, we contribute theoretically and empirically to the understanding of labour market dynamics in the face of economic fluctuations.

# Chapter 1

## Introduction

Throughout history major events have shaped the economy to its present state, each time challenging the minds of the brightest economists, eager to understand the influence of these developments on the market dynamics and how they impact critical macroeconomic indicators, such as the total output of goods and services (GDP), unemployment, and inflation (to account for some of them). These macroeconomic events have a massive and direct impact on the well-being of most of us. Thus, the ideal goal of policy makers, by undertaking macroeconomic policies, is to achieve low unemployment and inflation, and steady economic growth, conciliating them the best as possible. On the contrary, mismanagement of the economy could have disastrous consequences for a country's citizens. Unfortunately, in the last decades, the world has faced several economic downturns. Perhaps the most iconic moment was the bankruptcy of Lehmann brothers, symbolizing the world financial crisis of 2008, which impacted the labour market worldwide. Regarding specifically the European labour market, the intensity of its effects varied from country to country. One of the best examples of robustness is Germany. The unemployment rate was on the decline since the labour market reform between 2003 and 2005, and the financial crisis did not stop this trend. Also, the aftermath of the financial crisis differed radically among European countries. It led to the European debt crisis, which had high social costs for countries like Greece, Portugal, and Spain. They faced fiscal problems, private and public debt accumulation, and difficulties in borrowing from private capital markets so the interest rate spreads on government bonds soared, while other countries experienced a drop in interest rates on government debt. The joint effect of the subsequent crisis in the 2000s skyrocketed unemployment rates to levels that were not recovered until the present day in Spain and especially Greece, while Portugal and Italy are on similar unemployment levels of the pre-crisis. Despite the efforts of policy makers to mitigate the social and economic damage, companies made use of their option of waiting and delayed the decision of hiring new employees or laying off the current ones in such uncertain times, since it can be seen as an investment that incurs costs.

Given the importance of the subject, economists approach it by simplifying complex economic dynamics to their core, establishing relations between economic factors to understand their mechanics.

Over the course of time, these models evolve due to structural changes caused by economic shocks or societal shifts. And new approaches are developed to explain the new state of the economy, as Blanchard pointed out: “Macroeconomics is not an exact science but an applied one where ideas, theories, and models are constantly evaluated against the facts, and often modified or rejected”. We employ a rather uncommon framework in the mainstream economy, a hysteresis approach, to model the negative relation between GDP and unemployment, which was first pointed out by Okun, as an empirical finding. Okun’s law as it is known, has suffered criticism despite being largely accepted by the scientific community. However, the criticism is not directed at its essence, but rather at the complexity of the relationship, due to its asymmetry and time-varying features.<sup>1</sup> Hysteresis tackles these issues to some extent, enabling more accurate modelling of the output-unemployment relationship. To include hysteresis, a new microeconomic model was developed and through an aggregation method used to study the employment dynamics. In short, the microeconomic model is based on the following reasoning: Companies decide to hire or fire employees based on the potential turnover. In a case where the production capacity reaches its limit, a company hires a new employee to augment its production. This action bears costs (e.g. hiring and training costs), that cannot be recovered, even if the employee gets fired. These kind of costs are called sunk costs. If, after hiring an employee the economic situation returns to its initial state, the newly hired employee may not get fired. Thus temporary changes can have permanent effects. We extend the original version of Okun’s law by adding a hysteretic coefficient to a simple statistical Ordinary Least Squares (OLS) framework in order to assure the comparability of the results of each country assessed in this work, and to some extent enable a comparison with those established by other scientists.

The motivation behind this manuscript is to enhance our comprehension of the European labour markets in the face of changes in the economy using the lens of hysteresis. And due to its time-variant characteristic, we restate the relationship for the sake of economic policies, caused by world-shaking events, as we previously mentioned. Such events may have shifted Okun’s coefficient, possibly growing the sensitivity of unemployment to output shocks. Understanding the role hysteresis plays in the relationship between economic growth and unemployment could lead to more effective strategies for policy makers in their endeavour to stabilize the economy, taking into consideration the long-term effects of economic events. Thus, we aim to contribute to the development of economic theory and

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<sup>1</sup> Non-linearity of Okun coefficients can be found in the following papers: Zanin and Marra, 2011; Huang and Lin, 2008; Bod’a and P Považanová, 2019, while for asymmetry over the business cycle it can be found in Cevik et al., 2013; P Bod’a et al., 2015; Silvapulle et al., 2004; Grant, 2018; Nebot et al., 2019.

policy. The work has a total 7 chapters. Chapter 2 comprises an introduction to the subject of hysteresis, explaining its origins, fundamental characteristics, and its simplest form, the non-ideal relay. Moreover, we present the application of hysteresis in another economic context than labour market, as well as in other fields of study. Chapter 3 briefly explains Okun's law in its different forms. It also highlights the version of Okun's law used in this work. Chapter 4 builds the theoretical frame for the work. It starts with the non-ideal relay for firm-level capacity utilisation. Then, uncertainty is discussed and integrated into the microeconomic model. Next, the non-ideal relay of the employment dynamics is derived from the capacity utilisation model and employed as the microeconomic foundation (based on sunk cost hysteresis) of this work. In the next step, the non-ideal relays are aggregated using the method of Preisach 1935, leading to the continuous macroeconomic hysteresis loop. In order to use it for empirical research, the macroeconomic hysteresis loop is linearized, so that its core dynamics can be defined in simpler mathematical equations. The linearized version is called play hysteresis and contains two parts, "play" and "spurt", which respectively represent a weak and a strong reaction of employment dependent on past development. In the next section, we present the algorithm developed by Belke and Göcke (Belke & Göcke, 2001), which can compute the play hysteresis. Chapters 5 and 6 are the main contributions of the work. In both chapters, we test Okun's law for several European countries using the play algorithm described in the previous chapter. This is the novelty of the work since it is the first time that the play algorithm has been used in the context of Okun's law. Chapter 5 starts with the definition of the variables and data used. Then a full analysis of Germany is conducted. We find out that the threshold for GDP to start having a strong impact on unemployment is roughly 3 percent growth over the potential output, which is coincidentally the ratio 3-to-1 found by Okun for the United States. Also, we tested different filters, and estimated a model under uncertainty. The model showed that uncertainty did not play a decisive role in the relationship between economic growth and unemployment. The last section of the chapter is dedicated to the detailed analysis of the estimation for Netherlands, Italy, and Greece, and ends with an overview of the estimations for all 10 European countries. Chapter 6 deals with a variant of Okun's law, in which gross value added (GVA) and employment, respectively as independent and dependent variables, are considered. First, the data used is described. Then, the main findings for Germany are presented in detail. Following, the estimations and outcomes for the Netherlands, Greece, and Italy are discussed. We then summarized all results, including the remaining 6 EU countries. In the last part, a sectoral analysis of the German data is conducted. The data used is presented, followed by the estimations for all sectors of the German economy. Chapter 8 presents a summary of the findings of the work.

# Chapter 2

## Hysteresis in economics

### 2.1 Origin of hysteresis

Hysteresis can be described as “The phenomenon in which the value of a physical property lags behind changes in the effect causing it, for instance when magnetic induction lags behind the magnetizing force.”(lexico.com, 2022/2022). As the quote alludes, hysteresis has its origins in the field of physics, more specifically in (ferro-)magnetism. One of the first works in this field was conducted by Wilhelm E. Weber and published in 1852, suggesting that ferromagnetic hysteresis was caused by the interaction between individual magnetic moments. Decades later, in 1881, Maxwell modified the former hypothesis and suggested that an array of magnetic moments causes hysteresis (Jiles & Atherton, 1986, p. 49). The confirmation of his hypothesis came in 1890 when James Alfred Ewing, in his paper “Contributions to the Molecular Theory of Induced Magnetism”, showed through simple calculations that hysteresis was caused by strongly interacting moments (Jiles & Atherton, 1986, p. 49). He also coined the term hysteresis years before in his paper “On the Production of Transient Electric Currents in Iron and Steel Conductors”, which means in ancient Greek “to be behind” (Encyclopaedia Britannica, 2022; Ewing, 1883, p. 123). It was the Hungarian physicist Ferenc Preisach in 1935 who developed a mathematical hysteresis model utilising a rectangular loop (Preisach, 1935, pp. 295–296). His proposed model for the treatment of hysteresis became well-established and applied to the study of other physical phenomena, like porous media filtration (Bagagiolo & Visintin, 2000; Mualem, 1974). Although the Preisach model was conceived in the field of physics, it was taken under the scrutiny of a Russian mathematician called Krasnosel’skiĭ and a physicist named Pokrovskii, who together developed general mathematical methods to study systems with hysteresis non-linearities by dismantling the systems into smaller parts (hysterons) (Krasnosel'skiĭ & Pokrovskii, 1989). Although this new mathematical tool generalized the hysteretic effect to be applied in any context, and not only for physical phenomena, it does not determine the conditions for the occurrence of hysteresis non-linearities in the Preisach model (Mayergoyz, 1986, p. 603). These were formulated by Mayergoyz in 1986 and enable the use of the Preisach model independently of the (physical) causes of hysteresis (Mayergoyz, 1986, p. 603).

To count for some applications of hysteresis in different fields: In Biology e.g. for protein folding (Andrews et al., 2013) and lung deformations (Escolar & Escolar, 2004, p. 159). In the realm of engineering, studies have been conducted utilising the Preisach model for lithium-iron-phosphate batteries (Baronti et al., 2015), and modelling properties of materials like piezoceramic or alloys (D. Hughes & J. T. Wen, 1995). Another field where hysteresis plays a role is in chemistry, ranging from the use of molecular compounds in electronic devices, which exhibit the characteristic of bistability contained in hysteresis (Krober et al., 1993, p. 9810), to physisorption of gases by pore structures, which pattern also follows a hysteresis loop (Sing & Williams, 2004). In the field of economics, hysteresis could also establish itself (this topic will be extensively discussed in section 2.3).

Next, the general idea of hysteresis will be drafted, as well as, its mathematical properties will be presented.

## 2.2 Hysteresis characteristics

Generally, hysteresis can be described as a characteristic of an input–output system, which is usually determined by an operator. Normally the system has the shape of a loop (although the loop itself is not an essential feature of hysteresis (Bertotti & Mayergoyz, 2006, p. 6)). Hysteresis can be defined as a rate independent memory effect. This effect is a result of the operator being a hysteresis operator. In order to be considered one, it has to have some properties: The dependent variable  $w$  depends on the initial positioning of the system and the evolution of the independent variable  $v$ , thus the system has a memory (Bertotti & Mayergoyz, 2006, p. 5; Visintin, 1994, p. 13). Second, we assume that  $v$  and  $w$  are time dependent but, time does not influence directly the state of the system, so the topology of the path of  $(v, w)$  does not change because of time for any fixed initial state of the system. Taking these assumptions into consideration, one can say that the graphical representation does not change according to changes in time itself, but only with regard to changes in the input-output variables relation. Therefore, a static shape of hysteresis can be drawn and it will be forcibly invariant over the derivative of the input variable  $v$  (Visintin, 1994, p. 13). That is the definition of “rate-independence”, introduced by Truesdell and Noll (Truesdell & Noll, 2004, 402 sect. 99) when describing hypo-elastic materials. It infers that the path of  $(v, w)$  does not depend on its velocity and is a main feature of hysteresis (Visintin, 1994, p. 13). Thus, hysteresis systems have a *rate independent memory effect*.

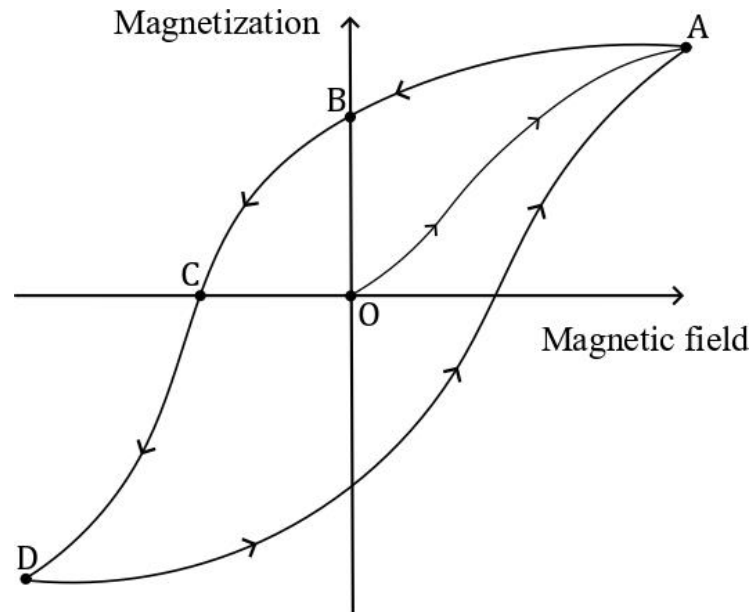
This main characteristic is the reason for the system to be built with non-linear multi-branches, and the transition between the branches happens due to input extremum, more specifically, the past

## 2.2 Hysteresis characteristics

values of the input variable (Mayergoyz, 1986, p. 604). Multi-branch non-linearity implies the following characteristics: Remanence, coercivity, and path-dependency.

Since hysteresis is well defined in physics, we explain these characteristics using a simplified example of the magnetization of an iron body found in (Göcke, 1993, pp. 12–13). Figure 1 shows the relation between the magnetization and the external magnetic field of the iron body. Let us start at point  $O$ , where there is no magnetic field force applied, and neither is the iron body magnetized. Then a magnetic field force is applied. The stronger it acts upon the object, the higher the magnetization, until saturation is reached in point  $A$  (curve  $OA$ ), i.e. a higher magnetization level than in point  $A$  is not possible. If the magnetic field strength decreases until it is no longer existent (as at the initial state), the magnetization of the iron body is only partially reversed (point  $B$ ), with a residual magnetism amounting to the value of  $OB$ . Thus temporary changes have a permanent effect. This is the definition of remanence: A remaining after-effect. Coercivity is the force needed to bring the iron body back to the demagnetized state. In this case, a magnetic field force in the opposite direction of the initial force, with a magnitude of the line  $OC$  would be needed to reach point  $C$ . Enhancing the strength of this (negative) magnetic force would lead to point  $D$ , where the iron body is magnetized to saturation again, but with a different polarity. If the direction of the magnetic field is once more reversed, and the force is strong enough, point  $A$  will be reached, closing the “external” hysteresis loop. Path-dependency is the result of remanence: The fact that the iron body kept to some extent magnetized at point  $B$ , although solely the initial magnetic field is again applied, shows a new relationship between them, which allows more than one output value possible for a single input value, depending on past output realizations of the input variable.

Figure 1: “External” hysteresis loop of an iron body



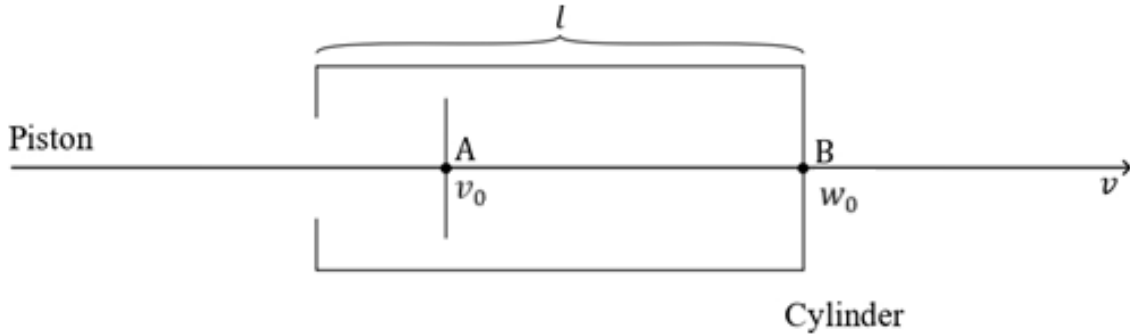
Source: Own representation based on (Göcke, 1993, p. 12).

There are various types of hysteresis operators that fulfill these properties. One of the first was proposed and studied by a PhD student named R. Bouc in the 1960s at the *École Polytechnique de Paris*. He created a mathematical model to apply to several problems in the field of engineering (Bertotti & Mayergoyz, 2006, 7). Another early developer of hysteresis models was Pierre Duhem, who created a related model for hysteresis systems (Bertotti & Mayergoyz, 2006, p. 13). There is also a class of models called Prandtl-Ishlinskii. They were introduced by Krasnoselskii and co-workers (Bertotti & Mayergoyz, 2006, p. 12), and are built from two elementary operators called stop and play (Bertotti & Mayergoyz, 2006, p. 30). The play operator is an essential part of the Preisach model used in this work, therefore only it will be described.

Krasnoselskii and Pokrovskii give in their books an example to understand how the play operator functions: Considering a system composed of two elements, a cylinder with length  $l$  and a piston, which can move along the  $v$ -axis (see figure 2). The initial position of the piston is in point  $A(= v_0)$  and for the cylinder is in point  $B(= w_0)$  and the state of the system is given by the coordinate  $(v, w)$ . The piston is the independent variable, and therefore, its variable location on the Cartesian plane is the input of the system. The output is the location of the cylinder (Krasnosel'skiĭ & Pokrovskiĭ, 1989, pp. 6–7). The piston can move back and forth, but it will only have an impact on the relative location of the cylinder if  $v$  moves in the positive direction of the  $v$ -axis and surpasses  $w_0$ , or moves backward and crosses the left boundary of the cylinder so that it automatically moves backward. This system is called *play*.

## 2.2 Hysteresis characteristics

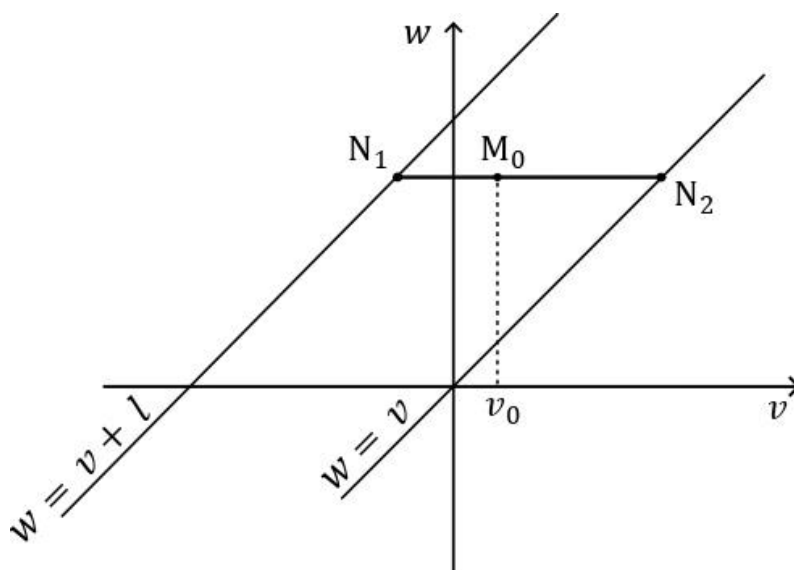
Figure 2: System with cylinder and piston



Source: Own representation based on (Krasnosel'skiĭ & Pokrovskiĭ, 1989, p. 7).

Figure 3 illustrates the possible movements of the cylinder caused by the piston: Starting from the arbitrary initial position  $M_0 = (v_0, w_0)$  on the horizontal segment with end-points  $N_1$  and  $N_2$ , which are on the lines  $w = v$  and  $w = v + l$ . The cylinder's relative position stays unaltered, despite any changes of the relative location of the piston, as long as the defined boundary is not trespassed. In this case, only a horizontal movement in figure 3 can be observed along the line  $N_1N_2$ . If the piston moves forward and exceeds the upper threshold ( $N_2$ ), the cylinder and piston will have the same position (in the graphic both positions are represented over the line  $w = v$ ). The last possibility outlines that when the piston moves in the negative direction of the  $v$ -axis trespassing the lower boundary ( $N_1$ ).

Figure 3: Movements of the cylinder



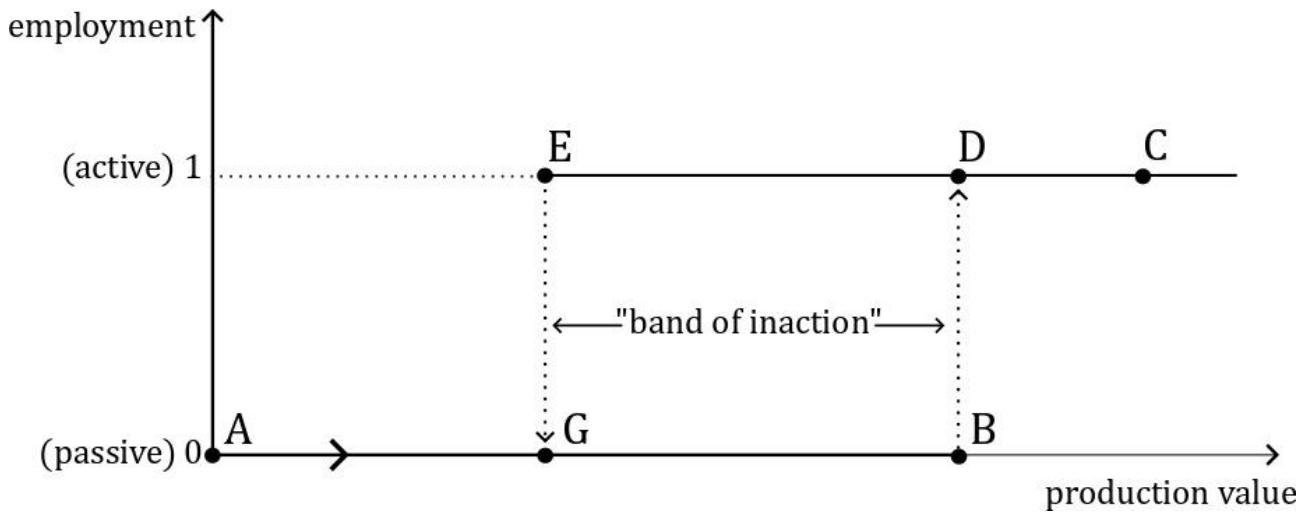
Source: Own representation based on (Krasnosel'skiĭ & Pokrovskiĭ, 1989, p. 8).

We utilise the “non-ideal relay”, which mimics the hysteretic behaviour of the play operator, but with a different movement pattern. It is also one of the simplest forms of hysteresis operators (illustrated in figure 4). The system’s state is described by the variables  $v$  and  $w$  with a one-way causality. The general representation of figure 4 can be applied to any pair of variables that constitute a hysteretic system with a play operator. An extensive theoretical explanation of the non-ideal relay follows in section 4.1.1. For now, we briefly explain the mechanics of the system for the reader using the microeconomic example of the introduction, namely, the decision of a company to hire or fire an employee. For that, the hysteretic variable  $v$  could be viewed as the value of the production output and the dependent variable  $w$  as employment. More about the non-ideal relay of employment dynamics will be discussed in 4.1.3.

The ordinate represents the state of activity, “active” in case of having hired a person (upper branch) or “passive” deciding not to hire a new employee/fire an employee (lower branch). These are the two potential equilibria. The abscissa reflects the production of the company. The behaviour of the firm also depends on its previous state of activity. Let us say that the company just opened, and thus does not produce a single good (point  $A$ ). Between points  $A$  and  $B$ , it is operating at its full capacity, and there is no need for a new employee. Only when  $B$  is surpassed, the company needs a new employee to augment its production. Therefore, point  $B$  is a hiring threshold value, previously calculated, for a change in equilibrium to the upper branch. If production reaches such a level, there will be a “jump” from  $B$  to  $D$ , and from that moment on production-employment-relation will correspond to the line  $EDC$ . A movement towards point  $C$  and beyond does not have any implication in the system, but if the production decreases over  $E$ , the exit/firing trigger is activated, and another change in the branches from point  $E$  to  $G$  happens, and the employee is fired. The initial equilibrium of the system is reached again. The space between the two triggers is the “band of inaction” since only a movement outside this band causes a change in the state of the system.

## 2.2 Hysteresis characteristics

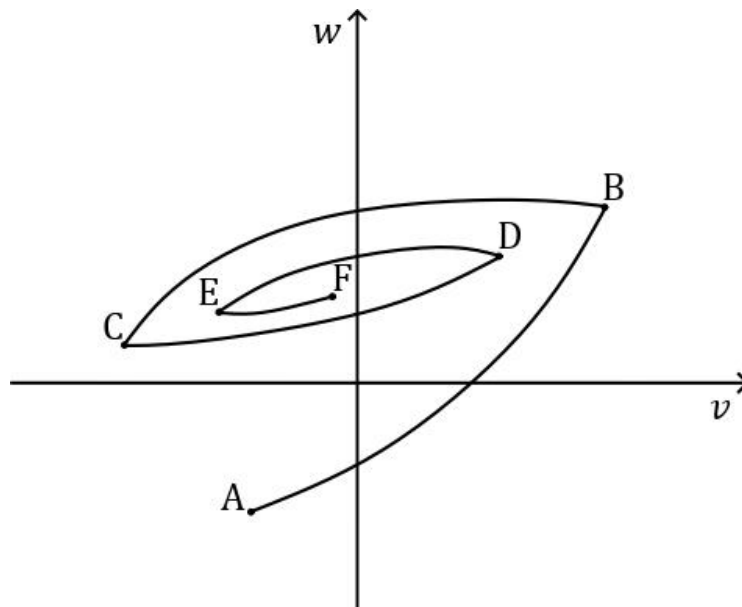
Figure 4: The non-ideal relay



Source: Own representation based on (Belke & Göcke, 2001, p. 183).

The aggregation of a large number of these non-ideal relays (rectangular loops) on the level of heterogeneous micro-elements with idiosyncratic thresholds, i.e. each company has its own threshold values, results in a macroscopic continuous loop depicted in figure 5, with the non-linear multi-branches ( $AB$ ,  $BC$ ,  $DE$ ,  $EF$ ). We use the Mayergoyz (1986)-Preisach (1935)-procedure, which will be explained in detail in section 4.2.

Figure 5: The continuous macroeconomic hysteresis loop



Source: Own representation based on (Mayergoyz, 1986, p. 604).

## 2.3 The concept of hysteresis applied to economics

Although the concept of hysteresis is well-defined in physics, there is some vagueness about its meaning when applied to economics. As stated in the previous section, hysteresis is a phenomenon of open dynamic systems<sup>2</sup> (Göcke, 2002, p. 180), and can be described as a permanent effect of transitory actions. In economics, such a description leaves room for structurally different concepts of hysteresis systems (Amable et al., 1992, p. 1). I.e. there are different kinds of hysteresis and it is possible to categorize them into two main types. Although, for economists, they carry the same name and are commonly used to refer to persistence phenomena in this discipline (Göcke, 2002, p. 167). The first main type (“genuine hysteresis”) is grounded on the works presented in the last chapter ((Krasnosel'skiĭ & Pokrovskiĭ, 1989);(Mayergoyz, 1986)), and is known as “microeconomic hysteresis”, which can be aggregated to a macroeconomic level, changing/strengthening the persistence properties, and thus forming the so called (genuine) “strong” hysteresis (Amable et al., 1992, pp. 2–3). The second main type is based on linear difference equations<sup>3</sup> with unit roots, and possesses persistence characteristics as well, but is structurally different. To enlighten the roots of this difference, the following example based on Göcke (Franz, 1990, p. 2; Göcke, 2002, pp. 181–182) is given with a discrete autoregressive time series of order 1,  $\varepsilon_t$  as the error term i.i.d. and

$$X_t: X_t = \phi X_t + \varepsilon_t, \quad (2.1)$$

utilising the lag operator  $L$ , we have:  $X_t - \phi L X_t = \varepsilon_t \Rightarrow (1 - \phi L) X_t = \varepsilon_t$ , if  $0 < |\phi| < 1$  is valid, the roots would be outside of the complex unit circle and the steady-state values would be:  $X^* = \frac{1}{1-\phi} \varepsilon^*$ . In the case of  $-1 < \phi < 1$ , the process is stationary,  $X^*$  is a constant non-zero steady-state and does not depend either on its past values ( $X_t$ ) or  $\varepsilon_t$ , but on the steady-state  $\varepsilon^*$  (with  $\varepsilon^* \neq 0$ ). In the case of  $1 < \phi$ , the time series is not stationary and the AR(1) process is considered an explosive process resulting in instability. But in our example the premise is the presence of unity root (with  $\phi = 1$ ), thus the variable  $X_t$  is determined by:

$$X_t = X_0 + \sum_{\tau=1}^t \varepsilon_\tau. \quad (2.2)$$

<sup>2</sup> Systems that contain an exogenous control variable (Göcke (2002, p. 180).

<sup>3</sup> For everything that follows, the same is valid for differential equations with zero roots (Göcke (2002, p. 167).

### 2.3 The concept of hysteresis applied to economics

With a unit root, the only possibility to achieve a constant  $X_t$  is if the exogenous (distortion) variable equals zero (*i. e.*  $\varepsilon_t = 0$ ) at any time  $t$ . Otherwise,  $X_t$  depends on the initial state  $X_0$  and on the past realisations of  $\varepsilon_t$ . Thus, the AR(1) process has the same two characteristics above mentioned of hysteresis: Path dependence and the permanent effect of a temporary stimulus.

The structural difference between true hysteresis and unit root dynamics is rooted in the terms “non-linearity” and “multi-branch” found in dynamic systems. I.e. for true hysteresis, a structural change alters the input-output relation (Amable et al., 1992, p. 2), which renders the local structural instability of the system (Amable et al., 1992, p. 14). In this case, there are two stationary states, each of them in one distinct branch, and the jump from one branch to another, by trespassing a certain threshold, is the modification of the structure of the system (Amable et al., 1994, p. 46), enabling a remanence effect. In the example above, the case of unit root<sup>4</sup>, a persistent effect emerges due to the continuum of the equilibrium space (Amable et al., 1992, p. 12, 1994, p. 46) and thus the system has a global structural stability (Amable et al., 1992, p. 14). Therefore, both systems cannot be equalized because a clear difference between the two types of hysteresis causes can be outlined. To highlight the difference, the following example is given: An exogenous shock that is applied to a unit root system, followed by another shock with the same intensity but opposite direction would lead the system to the initial state, while in the case of “genuine” hysteresis there could be a residual effect left from the first shock, which is the remanence. Nonetheless, the confusion between persistency and remanence appears due to the terminology used in economic papers. The use of the word “structural” when referring to economic variables in a unit root system is misleading because it suggests that a shock on this variable would change the structure of the system (set of equilibria and stability properties), while it actually only affects (permanently) its state (Amable et al., 1992, p. 8). Further authors also highlighted the fundamental differences between hysteresis’ concepts. Some of the literature include (Belke et al., 2014; Cross, 1993, 2014). This work utilises the “genuine” hysteresis, based on structural changes in the dynamics of the system.

## 2.4 Hysteresis in different economic fields

Since hysteresis comprises a set of properties of a model, there can be different economic reasons for it to emerge. The major economic fields, where it has been applied are international economics, specifically international trade, and labour economics when associated with

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<sup>4</sup> For discrete time systems. For continuous time systems, the same would apply to a zero-root process.

unemployment. Thus, this work deals with one of the classical applications of hysteresis. Nevertheless, hysteresis can also be applied to other economic fields, especially in economic fluctuation theory, where growth is not steady, thus business fluctuations may be hysteretic, when effective demand shocks, either positive or negative are modelled asymmetrically (Goodwin & Punzo, 1987, 106, 117). For example, an economic expansion leads to an augmentation of consumers' and firms' expenditures, while a contraction of the economy leads to a decrease. When both effects have the same magnitude but opposite signs, they may not have the same exact outcome in absolute values. I.e. a contraction of the economy on a certain level may reduce the expenditures less than proportionally, compared to economic growth of the same magnitude (Franz, 1990, p. 6). The hysteresis phenomena can also be found in other areas of economic science, such as consumer behaviour theory (Brown, 1952; Georgescu-Roegen, 1950; Kennedy & Thirkell, 1988), institutional economics (Setterfield, 1993), investments (Brennan & Schwartz, 1985; Dixit, 1992; Pindyck, 1991), capital formation (Dumas, 1989), and public finance (Engler & Tervala, 2018; Galí, 2020; Stockhammer & Sturn, 2012). The first connection between hysteresis and unemployment is credited to Phelps. He assessed the role of several factors that affect natural unemployment and claimed that the switch between two natural unemployment rate equilibria is affected by the development of those social factors, e.g. minimum wage, labour unions, social pressure, and racial discrimination, among others. He finished his thought with the phrase "Such a property is sometimes called hysteresis" (Phelps, 1972, p. xxiii). A further significant contribution to the field was made by Hargreaves Heap. His approach was to define a natural rate of unemployment dependent on the difference between the previous rate of unemployment and the natural rate of unemployment. Therefore, he addressed a path dependence (Heap, 1980, p. 612). Sachs gives some reasons that drive this relationship. When the economy produces below the potential production level, new factories are not built, while old ones are not re-opened, and workers lose their abilities. Leading to a new natural unemployment rate (Sachs, 1986, p. 3).

The next landmark was set by Blanchard and Summers. They found in hysteresis a possible explanation for the persistent rise in unemployment recorded from the beginning of 1970 until the mid-1980s for most Western European economies. According to them, the forecasts of that time did not predict a downturn of the trend aiming at levels before 1970 (O. J. Blanchard & Summers, 1986a, p. 15). Indeed, for countries like Germany, France, and the UK, those unemployment levels were only after the 90s reached again. On the other hand, the United States did achieve such low unemployment levels timewise again. Blanchard and Summers concentrated their efforts on explaining the phenomenon in Western Europe and argued that the theories until that time could not account for the persistence of the unemployment rate, thus a new approach should be undertaken. They created a

## *2.4 Hysteresis in different economic fields*

model that contains the idea presented by Hargreaves Heap, namely, that unemployment influences the natural rate of unemployment and the insider-outsider hypothesis of wage determination created by Lindbeck and Snower (Lindbeck & Snower, 1986). These two essential parts are embedded in the model through the following reasoning: Employees are members of unions, and as members (insiders), they exert influence on the wage-setting process. While the share of people looking for a job (outsiders) cannot determine to any degree the wage-setting process. This asymmetry leaves the insiders with an advantage and they are inclined to set the wages with the intention to secure their own jobs, instead of taking the interest of outsiders into consideration. Within this framework, any negative shock on employment would raise unemployment, inducing a decrease in the number of insiders. Also, the unemployment group would rise. This result would influence the wage-setting process, since the new smaller circle of union members bargains for their permanence in the job, which leads to another equilibrium comparably to the former situation. The negative transient exogenous disturbance caused a permanent change in the insider-outsider-ratio and led to a new unemployment level, with no tendency to go back to the original value, which represents the hysteretic component of the model.

In the same year, Blanchard and Summers released another paper pointing two theories that attempted to explain the persistent unemployment rate in Europe at that time (O. J. Blanchard & Summers, 1986b). The first belongs to the membership theories and was already mentioned previously, namely the insider-outsider-theory. The second is called duration theory, and argues that only short-term unemployed workers exert influence over the wage-setting process, while long-term unemployed people do not. Both, together or separately, are employed to explain the hysteretic behaviour of unemployment. The latter suggests that the duration of the unemployment, influences directly the employability of a person, and thus, its bargaining power: In the eyes of recruiters, applicants that have not worked for a long time are considered less capable compared to others, because long-term unemployment kept them outside of the labour market for a significant time of period, so that the set of skills required for the job may be compromised (“deskilling” effect) (P. R. Hughes & Hutchinson, 1988, p. 33). Second, recruiters may consider the unemployment history of applicants to evaluate their productivity, and base their decision of hiring only on this feature. This effect is called the “stigma” effect (P. R. Hughes & Hutchinson, 1988, p. 33). Both hypotheses for long-term unemployment are rooted in the demand side. On the supply side of labour, work-seeking individuals tend to be demotivated and slow down the intensity of their efforts to get a new job, after a long and unfruitful search for work (Franz, 1990, p. 11).

In both papers, Blanchard and Summers made compelling arguments to explain hysteresis in the labour market. Nevertheless, they made use of the unit root hysteresis, although some authors, like

Dosi, argued that models based on first order difference equations with unit root do not describe the phenomena well, e.g. (Dosi et al., 2018). We do share the same view as Dosi and chose the option to model “genuine” hysteresis using the generalized Preisach model (see section 4.2). The application of the model in the economic sciences can be done with solely minor modifications. One can replace magnetism by “unemployment” and the magnetizing force by a negative supply shock (Franz, 1990, p. 5). In this work, the magnetizing force will be substituted by the utilisation of production capacity of firms (see section 4.1.3). The premise is that workers are needed to produce goods/services and their wages are not tied to the firm’s production, but rather to the invested capital (see chapter 4 for more details). Therefore, the production of a company is linked to the act of hiring/firing employees. Moreover, a company does hire/fire a worker according to the sales and production value.

To explain this behaviour, we give the following example: Let us depict a phase of economic growth, a company wants to enter the market, therefore, it must bear the sunk costs pegged to the employment of a worker. E.g. screening process, marketing, etc. The firm will only hire an employee and start producing if its capacity can be fully utilised and the marginal revenue also covers the interest rate on capital and sunk entry costs. In a scenario of recession, the severance payment is an example of sunk exit cost, because it cannot be retrieved if the employee is fired due to underutilisation of the production or even bankruptcy. Therefore, not only wage is a relevant cost factor for firing the employee, but also the exit costs tied to the dismissal. In this case, the company does not fire an employee if its revenue does not only cover the wage but also if the operating loss is not as high as the sunk exit costs. The sunk costs create a gap between the thresholds for hiring and firing a person and an it is an essential part of the model. Since unemployment was the first economic subject brought in connection to hysteresis, there is a vast body of literature on the subject, to cite some recent contributions (Mota & Vasconcelos, 2021, 2022b), (Ball & Onken, 2022), and (Cross et al., 2006), as well as an overview (O. Blanchard, 2006) and country/continent specific studies (Ayala et al., 2012), (León-Ledesma, 2002), (Brunello, 1990).

In international trade, hysteresis has also played a major role. One of the first researchers to explicitly examine this phenomenon by taking into consideration firms’ adjustment costs were Murray C. Kemp and Henry Y. Wan Jr. (Kemp & Wan, 1974). About 15 years later, this line of research was once again pursued by three important contributors Richard Baldwin, Avinash Dixit, and Paul Krugman. They released a series of important papers on this matter (Baldwin, 1988, 1989, 1990; Baldwin & Krugman, 1989; Baldwin & Lyons, 1989; Dixit, 1989, 1990). Baldwin demonstrated that when sunk costs are taken into account, temporary exchange rate shocks do have a permanent effect on the domestic market, implying hysteresis (Baldwin, 1988). He also argued in his paper “Hysteresis

## 2.4 Hysteresis in different economic fields

in Trade” that the overvaluation of the dollar in the 1980s caused a permanent rise in net imports in the United States due to the lower marginal costs of foreign firms compared to local competitors’ costs (Baldwin, 1990, p. 129). This effect shifted the equilibrium of the economy permanently, thus characterizing a hysteretic behaviour. The permanent effect was contrary to the beliefs of the economists at that time, which predicted a lagged decline of the net imports in the US, once the dollar devaluated. Therefore, his novelty was a model that accounts for sunk market-entry costs (Baldwin, 1990, pp. 127–128). Examples of sunk entry costs are research and development, brand development and marketing activities, logistic infrastructure, legal affairs, market research, personal capacitation, adapting products, meeting sanitation requirements, and many more costs that are necessary to start operations but cannot be retrieved in the case of exiting the market (Baldwin & Krugman, 1989, p. 635; Prince & Kannebley Junior, 2013, p. 529). The relationship between exchange rate and imports can be depicted by the following example: In the case of a large shock, such as the persistent overvaluation of the US dollar, either local firms can leave the market or foreign firms venture to enter the US market by paying the firm specific one-time sunk entry costs. These sunk costs can be considered irreversible investments and firm specific, thus they can be partially or not at all recouped if the company disinvests the capital. Therefore, firms are cautious about entering new markets and only do so, when high profits are expected. Once stabilized in the new market, business can be profitable even when the currency exchange rate becomes less attractive for foreign companies (Baldwin, 1990, p. 129; Baldwin & Krugman, 1989, p. 635). It illustrates the hysteretic reasoning behind the model of Baldwin, which was first presented in 1986 and extended by several other papers. Baldwin and Krugman used a stochastic shock for stationarity purposes, compared the aggregated behaviour of imports with single sector behaviour to assess if the hysteresis effect would be dissolved by the aggregation of different industries, and how the entry-exit movements of firms affect the exchange rate (Baldwin & Krugman, 1989, pp. 636–637). Dixit introduced uncertainty in the band of inaction (Dixit, 1990). All these papers constitute the first attempts to organize the idea of sunk costs in the frame of international trade and serve as inspiration for more founded microeconomic models with applicability in reality, either based on the Preisach model (Belke & Göcke, 2001; Piscitelli et al., 2000) or unit root (Ljungqvist, 1994). The former is also used in this work and consists of adding to an artificial variable the regression that filters the original variable capturing its hysteretic essence. Some more recent applications of hysteresis in international trade can be found in (Belke et al., 2013; Belke et al., 2015; Belke & Ptok, 2018; Campa, 2004; Campbell, 2017; Kannebley Júnior et al., 2015; Mota & Vasconcelos, 2022a; Prince & Kannebley Junior, 2013).

Nevertheless, sunk costs are not the only determinant of hysteresis. Other effects that influence the entry/exit decision of firms are learning-by-doing, and demand carry over. The former was introduced by Arrow (Arrow, 1971) and explains the increase in productivity of firms due to practice. It implies that a company can permanently decrease its production costs through small amelioration of the production process. E.g. we assume that a firm enters the market because its costs of production are below the revenue price. Initiating production leads to a cost reduction due to the leaning-by-doing effect. Thus, if the market price falls, the company does not necessarily exit the market, since its cost reduction lowered the threshold price for the company to leave the market. Learning-by-doing effect can be implemented in a model where hysteresis is caused by sunk entry and exit costs by extending the boundaries of market exit. It extends the width of the market price (the difference between the entry and exit market prices), tolerated by the company to stay active.

Hysteresis may occur on the supply side, e.g. caused by a learning-by-doing effect, and on the demand side. Carry-over effect is an example of a determinant for hysteresis due to lack of information. Although a company that produces a new good or service is very well aware of its quality, uninformed customers do not know the quality of not yet purchased good or service. Therefore, when a new product is introduced, demand tends to be low until the lack of information vanishes by slowly penetrating the market. E.g., assuming a company wants to enter a foreign market and thus sunk costs apply, the initial unwillingness of customers to try the new product causes a lower initial demand. Thus, the company anticipates a lower profit in the early phase, and later the corresponding profit of when the brand has established itself in the market. So, by entering the foreign market the condition must be met that the sum of all future profits must cover the production and sunk costs. If the company is currently not active in a foreign country, its situation may change due to external factors such as the depreciation of the home currency.

In this case, exporting would become more profitable and the company would be in a position to cover its costs and enter a foreign market, becoming “active”. After a while, in the event that the currency devaluates to the initial level, the profit margin falls again, but the company had time to penetrate the foreign market. Customers would have tested the product eliminating the lack of information about it, in turn, augmenting their willingness to pay for it. This would raise the demand for the new product and therefore, the firm would not necessarily leave the market, leading to a hysteretic effect (Baldwin, 1990, p. 130).

As depicted in this chapter, the hysteretic effect can emerge from the demand side (carry-out effect) or the supply side (sunk costs). Werner and Göcke created a market model in which hysteresis appears on both market sides (Göcke & Werner, 2015).

#### *2.4 Hysteresis in different economic fields*

Until this point, we presented the development of hysteresis in the last decades in an economic context. But innovations on this matter did not cease. One of the most recent novelties regarding the use of hysteresis is called “hysteresis losses” and was developed by Adamonis and Göcke (Adamonis & Göcke, 2019). They modelled hysteresis losses and subsequently created an indicator using sunk costs. The calculation is based on sunk costs caused by entering and exiting a market. Hysteresis loss requires that the hysteresis loop is complete, in analogy to energy dissipation in the form of heat in the classic physical sense, when a material is magnetized until saturation and then demagnetized. Since it can be geometrically calculated, the size of the loss is proportional to the area within the hysteresis loop. Their model was built using sunk costs so that the loss is calculated in monetary units. Its application in other spheres is not straightforward, for example, if unemployment is the hysteretic variable, the model must be adapted to account for another dimension. The model was first based on the play model of hysteresis developed by (Belke & Göcke, 2001). Later on, a new hysteresis indicator was developed by Werner (Werner, 2020).

Overall we demonstrated that hysteresis can have many causes, such as sunk costs, learning-by-doing, carry-over effects, loss of skills, and bargaining power of employees. But there are even more determinants, e.g. uncertainty and price elasticity of exports (Belke et al., 2013).

# Chapter 3

## Okun's law

### 3.1 The original idea of Okun's law

Arthur Okun in 1962 wrote a prominent paper titled "Potential GNP: Its measurement and significance" where he pointed out the relationship between Gross National Product (GNP) and unemployment. Using data from 1954 to 1963 he applied three different estimations to find a relationship between the two variables and concluded that "The uniformity that emerged from these various techniques was the approximate 3-to-1 link between percent of output change and unemployment rate" (Okun, 1962, p. 3). Due to the notoriety of his findings, it is nowadays known as Okun's law. Although it is called a law, the 3-1 ratio consistency is somewhat flawed, because the author claimed that the estimates hold only for the considered time period when the unemployment range was between 3% and 7,5% and a specific country. Therefore, it is not a surprise that the ratio varies across different countries. Additionally, different time periods can influence Okun's coefficient (Lee, 2000, p. 352).

As the title suggests, Okun attempted to quantify potential GNP to answer the question "How much output can the economy produce under conditions of full employment". GNP was chosen as a proxy to output because it englobes many important economic factors, such as technological progress, capital stock, natural and human resources. By that time 4% unemployment rate was among economists a unanimous target for maximum production without inflationary pressure so that GNP would achieve its highest. Therefore, GNP is only under its potential if the unemployment rate is over 4% for the time frame, creating a so called "gap" between actual and potential output. To connect GNP and unemployment Okun assumed that all the information regarding production (hours worked, labour force participation, productivity) is comprised in unemployment. In this way, unemployment is a proxy for any underutilisation of resources. Three statistical estimates to quantify the results were used: a) First Difference, b) Trial gaps, and c) Fitted trend and elasticity. The paper states that all the methods converge to the same result: On average, a 1% increase (decrease) in unemployment is associated with an approximately 3% decrease (increase) in real GNP.

### 3.1 The original idea of Okun's law

In this work, we utilise the second method, and the following regression explains the gap version:

$$u = u^* + m * (gap). \quad (3.1)$$

$u$  stands for unemployment rate.  $u^*$  is the natural unemployment rate, although the terminology was developed only after 1962. Therefore, Okun labelled it as the rate of unemployment for which potential and actual GNP are equal (by that time the consensus was 4%).  $m$  is the slope of the equation, and  $gap$  represents the deviation of GNP from the defined trend. He presented two trends and advocates for the use of a line that goes through the real GNP in mid-1955 moving upwards at a 3,5% annual rate as a benchmark for potential GNP.

The empirical regression reported by the Council of Economic Advisers by that time was  $u = 3.72 + .36 gap$ , where 3.72 is the unemployment expected when GNP equals its potential, which is very close to the estimation of 4% for full employment. The “gap” version can be interpreted as “an increment of unemployment of one percent is associated with an output loss equal to 2.8 percent of potential” (Okun, 1962, p. 3).

The first and third methods are not discussed, because they are not relevant to this work. Therefore no further thoughts are dedicated to them. Moreover, the third suggested method is the least used method among economists to describe Okun's law.

On a note about the formulation of his words in the selected passages, as they are somehow fuzzy regarding the causality direction. In fact, Okun is not consistent throughout the paper concerning causality.

Many slight variations of Okun's law emerged over time (Schorderet, 2001), e.g.  $\Delta u = \beta_{Okun}(\Delta y - \Delta y^*)$  for the first difference approach, where the term on the left is the change in unemployment rate.  $\beta_{Okun}$  is the so called Okun's coefficient,  $\Delta y$  is the change of the actual output, and  $\Delta y^*$  is the trend growth of output (change of output gap). For the “gap” version Schorderet writes the following equation, which is also going to be used in this work:

$$(u - u^*) = \beta_{Okun}(y - y^*). \quad (3.2)$$

## 3.2 Link between hysteresis and Okun's law

A strong body of literature has shown evidence of asymmetry in the aggregated unemployment rate (Rothman, 1991) and (Neftçi, 1984), which provided statistical evidence that employment-related

series behave differently during ups and downs of the business cycle. Their attempt to account for asymmetry raised the question of whether a non-linear relationship better explains the empirical link between unemployment rate and growth rate proposed by Okun. Since then Okun's law has been analysed through the lens of hysteresis by many researchers. One of the first attempts to take account of asymmetry in the business cycle was made by Palley (Palley, 1993). He re-specified Okun's law equation to account for asymmetric responses and implemented, for the first time, the variable GDP to calculate the output growth rate instead of using the variable GNP, as Okun did in his paper. The present work follows the insight of Palley and uses real GDP as the basis to calculate the explanatory variable, which is the percentage change of real GDP relative to its potential. Using GDP better explains changes in unemployment because GNP accounts for income generated by its citizens outside the countries' borders, which does not directly influence unemployment in the domestic country. Moreover, foreigners within the domestic country are not accounted for, although they have a direct impact on unemployment (Palley, 1993, p. 145).

Further studies use other econometric models to study the matter. In his extensive work, Lee (Lee, 2000, p. 346) tested data from 16 OECD<sup>5</sup> countries for the gap and first-difference variants of Okun's law and included as a novelty the possibility of asymmetric effects for negative and non-negative unemployment data. For the "gap" version he estimates an Okun coefficient for the unemployment rate series, for which the value is greater or equal to the natural unemployment rate ( $u_t - u_t^* \geq 0$ ) and a separate Okun coefficient estimate for cases where the values are negative ( $u_t - u_t^* < 0$ ). The "first-difference" version was also extended to Okun's equation. Here, the unemployment rate data is divided into non-negative and negative changes. Specifically, non-negative changes indicate periods where the unemployment rate increased or remained constant  $\Delta u_t \geq 0$ , while negative values correspond to periods where the unemployment rate decreased  $\Delta u_t < 0$ .

His findings validate the relationship, but are not conclusive regarding the presence of asymmetry, since it varies across countries for both forms of Okun's law. The results also vary between the "first-difference" and the "gap" form. For the latter, Lee found that Okun's coefficients are higher for negative unemployment data than non-negative, suggesting that the effect is more pronounced in economic decline than expansion (Lee, 2000, p. 348). A broader approach was undertaken by Harris and Silverstone (2001). They argue that Lee only depicts the short-run equilibrium between unemployment and real output. Their argument is that the relationship may be non-stationary and should be estimated using cointegration. The model includes a time trend to allow

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<sup>5</sup> Organisation for Economic Co-operation and Development.

### *3.2 Link between hysteresis and Okun's law*

for long-run linear growth and is associated with a short-run threshold error-correction model (TECM), which is the asymmetric version of the error-correction model (ECM), allowing two error-correction terms.

The authors present a model that captures the long- and short-run relationships between growth and unemployment with an asymmetric response to short-run adjustment to disequilibrium in the long-run relationship, depending on the phase of the business cycle.

Under the assumption that asymmetry is due to labour market dynamics Kosfeld and Dreger studied the role that capacity reserves of German firms play in Okun's law relationship, arguing that output growth needs to exceed a certain threshold level to lower the unemployment rate. They also found that the response of unemployment due to changes in output growth is unstable over time (Kosfeld & Dreger, 2006).

Lang and Peretti follow another theoretical foundation, in their version of Okun's law: The cause of asymmetry lies in the discontinuity of adjustment of heterogeneous firms. They used a simulation-based approach to build a model of "strong hysteresis" and compare their results with the non-hysteretic benchmark model. Their results for the three examined countries were mixed, yet expected. For France, a rather hysteretic labour market, their new model better fitted the data than the non-hysteretic model, while for the US labour market, the contrary is true. While the results for the UK were not clear. Their findings are a hint that a country specific analysis should be preferred over a panel data approach when testing for asymmetries in the data (Lang & Peretti, 2009).

It is well known that Okun's law may vary across countries depending on the structure of the home labour market. Nevertheless, the presented studies have established that the overall sign of the link between output growth and unemployment is negative. As well as the non-linear nature of the relationship between these two variables.

# Chapter 4

## Sunk cost hysteresis and utilisation of production capacities

Die Abschnitte 4.1 und 4.2 basieren auf einem gemeinsamen Working Paper mit Matthias Göcke mit dem Titel "Utilization of Production Capacities and Hysteresis".

### 4.1 Sunk cost hysteresis and non-ideal relays

#### 4.1.1 Sunk cost hysteresis in situations with no uncertainty

We start with presenting a microeconomic model resulting in a (generalized) non-ideal relay type reaction of the production and employment activity of a single firm as a reaction to potential sales: A price-taking firm  $j$  decides in period  $t$  whether or not to produce and supply. For producing a quantity  $x_{j,t} \in ]0,1]$  some investment in capital  $K_j (\geq 0)$  is required. This results in interest cost  $(i \cdot K_j)$  and additional operation cost  $c_j$  per unit of  $K_j$ , e.g. for workers operating the machines or for the capital depreciation. The maximum production quantity is 1, i.e. a utilisation of the production capacity by 100 percent. For production activity, some intermediates and labour are required, resulting in variable costs  $k_j$  per unit of production. If the production is sold, a price  $p_t$  per unit is received. Selling the product, the firm receives the market price  $p_t$  as a unit revenue. In addition to the “running costs” of ongoing production, if the firm has not produced in the preceding period ( $x_{j,t-1} = 0$ ), it has to pay the starting/entry costs  $H_j (\geq 0)$ .  $H_j$  represents sunk adjustment costs and is completely firm specific and decays immediately if the firm does not produce/sell. If the firm has been active in the preceding period ( $0 < x_{j,t-1} \leq 1$ ) only the “running costs” components are relevant.

In the current period  $t$  the contribution margin  $e_{j,t}$  as a kind of “gross profit”, disregarding capital costs related to interest payments on the capital stock  $(i \cdot K_j)$  and sunk entry costs  $H_j$ , is:

$$e_{j,t} = (p_t - k_j) \cdot x_{j,t} - c_j \cdot K_j \quad (4.1)$$

#### 4.1 Sunk cost hysteresis and non-ideal relays

As a simple example, we assume the firm is expecting a constant level of the production quantity, of the price level, and for the interest rate for the whole *infinite future* in subsequent periods subsequent periods  $t + \varsigma$  ( $x_{j,t} = x_{j,t+\varsigma}, p_t = p_{t+\varsigma}, i = i_{t+\varsigma}, \text{ for all } \varsigma > 0$ ). The expectation is based on an anticipation with certainty. Thus, in the case of activity the (expected) present value of future contribution margins as the annuity is:

$$V_t = \frac{e_{j,t}}{i} \quad (4.2)$$

This present value  $V_t$  of gross profits has to cover (at least) the value of the capital stock  $K_j$  plus the sunk entry costs ( $V_t > K_j + H_j$ ) to make the starting of the production activity profitable. Solving ( $V_t = K_j + H_j$ ) leads to the firm's entry trigger contribution margin  $e_{j,entry}$  in a situation under certainty:

$$e_{j,entry} = i \cdot (K_j + H_j) \quad \text{entry if } e_{j,t} > e_{j,entry} \quad (4.3)$$

Therefore, the contribution margin has to cover at least the interest costs on both, capital stock  $K_j$  and sunk starting/entry costs  $H_j$ .

If the firm was active in the preceding period ( $x_{j,t-1} > 0$ ) it will leave the market and sell the production capital  $K_j$ , if the earnings are getting too low. However, it has to pay sunk exit cost  $F_j (\geq 0)$ , e.g. for writing off firm-specific parts of the capital stock, or for severance payments. Thus, an exit of the firm is optimal if ( $V_t < K_j - F_j$ ), and the corresponding exit trigger  $e_{j,exit}$  for the contribution margin is:

$$e_{j,exit} = i \cdot (K_j - F_j) \quad \text{exit if } e_{j,t} < e_{j,exit} \quad (4.4)$$

We assume ( $F_j \leq K_j$ ), i.e. the exit trigger contribution margin is non-negative. For different situations regarding the utilisation of the production potential, different price reactions are assumed. If the production potential is not fully used, i.e. if ( $x_{j,t} < 1$ ), there is a minimum price floor of 1, thus the turnover  $z_{j,t} (= p_t \cdot x_{j,t})$ , i.e. the value of selling the production on the market, in an underutilisation situation equals the real production quantity:

$$\text{if } x_{j,t} < 1 \text{ (underutilization): } p_t = 1 \Rightarrow z_{j,t} = x_{j,t} \quad (4.5)$$

If the production capacity is completely occupied ( $x_{j,t} = 1$ ), however, a price increase is possible, and now the turnover is equal to the price level:

$$\text{if } x_{j,t} = 1 \text{ (full utilization): } p_t \geq 1 \Rightarrow z_{j,t} = p_t \quad (4.6)$$

Rearranging eq. (4.1) the revenue/turnover  $z_{j,t}$  of firm  $j$  during a period  $t$  is:

$$z_{j,t} = p_t \cdot x_{j,t} = e_{j,t} + k_j \cdot x_{j,t} + c_j \cdot K_j \quad (4.7)$$

We assume, that a starting/entry decision, with the necessity to additionally cover the sunk entry costs, is typically based on a situation where a fully utilised capacity ( $x_{j,t} = 1$ ) of the newly invested capital is expected. Using the turnover/revenue  $z_{j,t} = p_t \cdot x_{j,t}$  representation, rearranging eq. (4.3), the entry trigger  $\alpha_j$  related to the potential turnover, if the production would be sold, can be expressed as:

$$\alpha_j = k_j \cdot x_{j,t} + c_j \cdot K_j + i \cdot (K_j + H_j) \quad \text{entry if } p_t \cdot x_{j,t} = z_{j,t} > \alpha_j \quad (4.8)$$

$\Rightarrow$  for  $x_{j,t} = 1$ :

$$\alpha_j = k_j + c_j \cdot K_j + i \cdot (K_j + H_j) \quad \text{entry if } p_t = z_{j,t} > \alpha_j > 1$$

Exit decisions are typically relevant in an underutilisation situation ( $x_{j,t} < 1$  and  $p_t = 1$ ). The revenue related exit trigger  $\beta_j$  analogously is:

$$\text{for } x_{j,t} < 1: \beta_j = k_j \cdot \beta_j + c_j \cdot K_j + i \cdot (K_j + F_j) \hat{U} \quad (4.9)$$

$$\beta_j = \frac{c_j \cdot K_j + i \cdot (K_j + F_j)}{1 - k_j} \quad \text{exit in } t \text{ if underutilization below } x_{j,t} = z_{j,t} < \beta < 1$$

For a constant interest rate, an (unexpected) change in the (current and expected potential future) turnover and/or capacity utilisation  $z_{j,t} = p_j \cdot x_{j,t}$  results in a supply-pattern of the price-taking firm  $j$  which is a special case of a so called “non-ideal relay”:<sup>6</sup>

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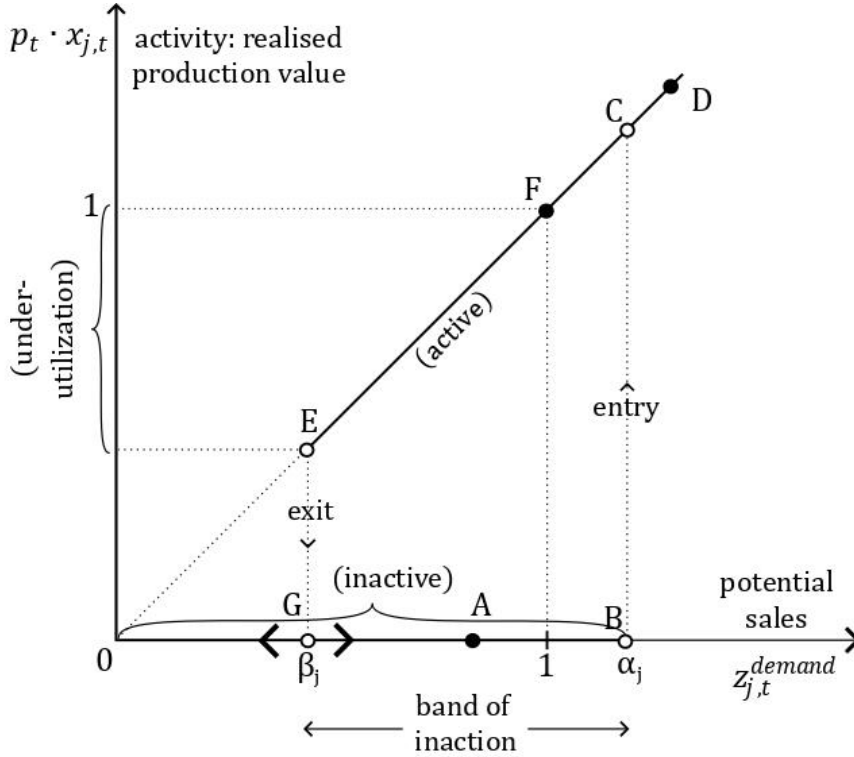
<sup>6</sup> See Krasnosel'skiĭ and Pokrovskii (1989, pp. 14, 234) and Brokate and Sprekels (1996, pp. 23–24) for a general mathematical description of relay-hysteresis. When describing the relay-pattern, the terms ‘input variable’ and ‘output variable’ are meant in a mathematical sense, as inputs and outputs of an operator.

4.1 Sunk cost hysteresis and non-ideal relays

$$z_{j,t} \begin{cases} p_t & \text{if } (x_{j,t-1} = 0 \wedge p_t > \alpha_j > 1) \\ p_t & \text{if } (x_{j,t-1} > \beta_j \wedge x_{j,t} = 1 \wedge p_t \geq 1) \\ x_{j,t} & \text{if } (x_{j,t-1} > \beta_j \wedge x_{j,t} \leq 1 \wedge p_t = 1) \\ 0 & \text{if } (x_{j,t-1} = 0 \wedge p_t \leq \alpha_j) \\ 0 & \text{if } (x_{j,t-1} > \beta_j \wedge x_{j,t} \leq \beta_j < 1) \end{cases} \quad (4.10)$$

*with  $\alpha_j \geq \beta_j$*

Figure 6: A “non-ideal relay” operator of a firm’s realised utilisation (as output) related to potential value of sales (as the input variable)



In Figure 6 the non-ideal relay is depicted with the market situation regarding the potential turnover based on the demand side of the firm’s product  $z_{j,t}^{demand}$ , when goods would be produced and sold on the market in the current situation, as the relay’s input variable on the abscissa, and the actually realised production  $z_{j,t} = p_t \cdot x_{j,t}$  as the relay’s output variable on the ordinate. A non-ideal relay describes a path-dependent multiple-equilibria characteristic. E.g., starting in an inactivity situation at point A (Figure 6) an increase of the demand (as the potential production) exceeding the trigger  $\alpha_j$  induces a starting (‘entry’) of the actual production, i.e. a “jump” from the  $(x_{j,t} = 0)$ -inactivity-line to the  $(x_j > 1)$ -activity-line (from point B to point C). If the demand would

later be higher than the production capacity ( $x_{j,t} = 1$ ), as depicted with point  $C$ , this is a situation based on a rising price level ( $p_t > 1$ ). A later decrease of the demand or potential sales, leads (by passing point  $C$ ) to point  $F$ , where demand and capacity are balanced ( $z_{j,t} = x_{j,t} = p_t = 1$ ). A further decrease in demand/production will result in a stop of production and a market exit, only if the turnover falls below the exit trigger  $\beta_j$  (at point  $E$ ). A switch from one equilibrium-branch to the other only takes place when the triggers are passed. Therefore, the area  $GB$  (or  $EC$ ) between both triggers is often described as a ‘band of inaction’ or ‘hysteresis band’ (Baldwin, 1989, pp. 7–8; Baldwin & Lyons, 1989, p. 11). Depending on the activity status in the past, two different equilibria are possible: The current level of the input variable (demand or potential sales) does not unambiguously determine the current state of the dependent variable (firm’s actual production activity). If a temporary change of the input variable results in a switch between these different equilibrium branches, a permanent effect on the dependent variable remains. The occurrence of this remaining after-effect (remanence) is the constituting characteristic of hysteresis (see section 2.3).

#### 4.1.2 A scenario with stochastic revenue change

The firm’s decision about starting production and entering the market can be understood as an irreversible investment. Thus, in a situation with uncertainty, e.g. due to an expectation of stochastic changes in future revenues, a real option approach applies (Bentolila & Bertola, 1990; Dixit, 1989; Pindyck, 1988, 1991). A firm that is currently inactive has to decide whether to start producing now or not, including the option to enter later. The option to decide on the market entry in the future limits the risk by a “wait-and-see” strategy. By staying passive, a firm can avoid future losses if the stochastic future revenues will be unfavourable, and an instantaneous entry eliminates the option to enter later. Thus, in addition to the sunk costs, an option value of waiting has to be covered in order to make an immediate investment profitable.<sup>7</sup>

The option value effects on the position of the triggers will be demonstrated with a very simplistic example, with similarities to the stochastic model presented in (Belke et al., 2020; Belke & Göcke, 2021): Assume a single non-recurring stochastic change of the revenues, which can be either positive ( $+\varepsilon_a$ ), with the index  $a$  indicating an upward movement, or negative ( $-\varepsilon_d$ ), with the index  $d$  indicating a downward movement (and  $\varepsilon_{a,d} \geq 0$ ). The probability of a positive revenue

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<sup>7</sup> These option value effects may even be amplified if the investment project needs “time to build” Majd and Pindyck (1987).

#### 4.1 Sunk cost hysteresis and non-ideal relays

change  $(+\varepsilon_a)$  is  $(1 - P)$ , and the probability of a negative revenue change  $(-\varepsilon_d)$  is  $P$ . From the next period  $t + 1$  on, the firm will decide under certainty again. In an uncertain situation, there is an option to wait: By staying passive, potential future losses can be avoided if the revenue change will turn out to be negative  $(-\varepsilon_d)$ . However, the firm can still invest and start production in the next period if the future revenue level turns out to be favourable  $(+\varepsilon_a)$ .

A firm that was not producing in the last period will earn no present profits if it continues waiting and being passive for the moment. However, the firm will use its option to enter in  $t + 1$  conditional on a favourable  $(+\varepsilon_a)$ -realisation (via paying the sunk adjustment cost in  $t + 1$ ). In the unfavourable case of a future revenue decrease  $(-\varepsilon_d)$  the firm will remain passive.

If the firm would currently produce, the present period's contribution margin would be  $e_{j,t}$ . In case of a favourable revenue increase, the annuity for the future periods would be  $(e_{j,t} + \varepsilon_a)$ . By waiting during the current period  $t$ , with the option to invest in the next period  $t + 1$  if the  $(+\varepsilon_a)$ -case realizes with probability  $(1 - P)$ , and stay passive in the future in the negative case of  $(-\varepsilon_d)$ , the expected present value of the wait-and-see strategy  $E(W_{j,t}^{entry})$  is:

$$E(W_{j,t}^{entry}) = \frac{1 - P}{1 + i} \cdot \left( \frac{e_{j,t} + \varepsilon_a}{i} - K_j - H_j \right) \quad (4.11)$$

$E(W_{j,t}^{entry})$  has to be compared with the expected present value  $[E(V_{j,t}) - K_j - H_j]$  of an immediate entry (without considering a re-exit):

$$E(V_{j,t}) - K_j - H_j = P \cdot \left( \frac{e_{j,t} + \frac{e_{j,t} - \varepsilon_d}{i}}{1 + i} \right) + (1 - P) \cdot \left( \frac{e_{j,t} + \frac{e_{j,t} + \varepsilon_a}{i}}{1 + i} \right) - K_j - H_j \quad (4.12)$$

The contribution margin  $e_{j,entry}$  triggering a start of production, and the respective revenue trigger  $\alpha_j$ , can be calculated for a situation with indifference between immediate entry and wait-and-see, i.e. if  $E(W_{j,t}^{entry}) = E(V_{j,t}) - K_j - H_j$ :

$$e_{j,entry} = i \cdot (K_j + H_j) + \frac{P \cdot \varepsilon_d}{i + P} \quad \text{in period } t: \text{ entry if } e_{j,t} > e_{j,entry} \quad (4.13)$$

$\Rightarrow$  for  $x_{j,t} = 1$ :

$$\alpha_j = k_j + c_j \cdot K_j + i \cdot (K_j + H_j) + \frac{P \cdot \varepsilon_d}{i + P}, \quad \text{entry in } t \text{ if } p_t = z_{j,t} > \alpha_j > 1$$

A currently active firm, deciding about stopping production now or staying active in  $t$ , with an option to exit later in  $t + 1$  if an unfavourable ( $-\varepsilon_a$ ) revenue change will occur, has to decide analogously. Remaining active and waiting for a period leads to a current margin of  $e_{j,t}$ . In case of a  $+\varepsilon_a$  revenue increase, the firm will continue activity with a future annuity of  $(e_{j,t} + \varepsilon_a)$ . If the negative ( $-\varepsilon_a$ )-change will happen, the firm will use its option to exit/stop in  $t + 1$ . The expected present value of the wait-and-see strategy  $E(W_{j,t}^{exit})$  is:

$$E(W_{j,t}^{exit}) = P \cdot \left( \frac{e_{j,t} - i \cdot K_j - F_j}{1 + i} \right) + (1 - P) \cdot \left( \frac{e_{j,t} + \frac{e_{j,t} + \varepsilon_a}{i}}{1 + i} - K_j \right) \quad (4.14)$$

The present value of waiting with stopping the production  $E(W_{j,t}^{exit})$  has to be compared with the expected present value of an immediate exit (without a re-entry), which is given by the sunk exit costs. The exit/disinvestment-trigger of the revenue  $\beta_j$  results in an indifference between wait-and-see and immediate exit in  $t$ , i.e.  $E(W_{j,t}^{exit}) = (-F_j)$ :

$$e_{j,exit} = i \cdot (K_j - F_j) - \frac{(1 - P) \cdot \varepsilon_a}{i + (1 - P)} \quad \text{exit in period } t \text{ if } e_{j,t} < e_{j,exit} \quad (4.15)$$

$\Rightarrow$  for  $x_{j,t} < 1$ :

$$\beta_j = \frac{c_j \cdot K_j + i \cdot (K_j - F_j) - \frac{(1 - P) \cdot \varepsilon_a}{i + (1 - P)}}{1 - k_j} \quad \text{exit in } t \text{ if } x_{j,t} = z_{j,t} < \beta_j$$

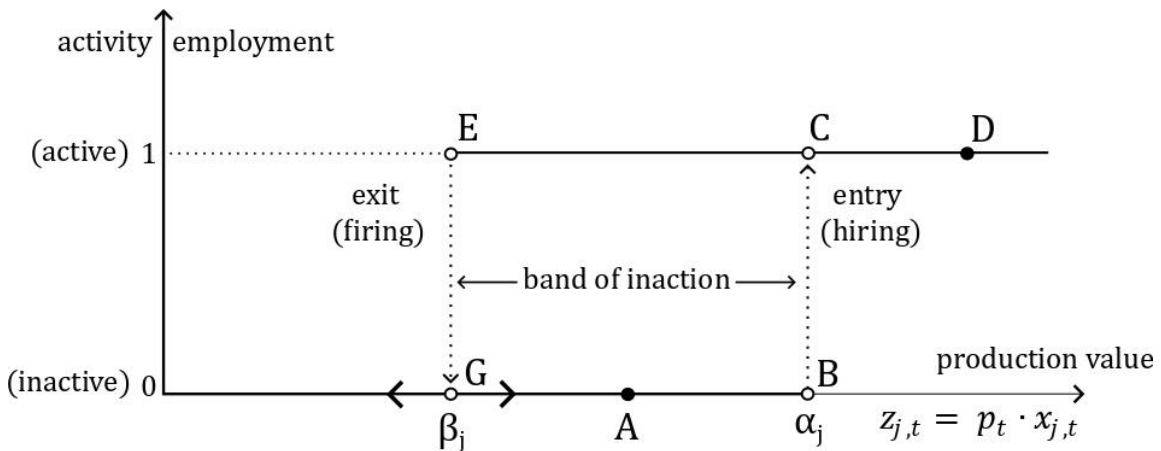
Thus, the investment trigger rate of return  $\alpha_j$  is under uncertainty augmented by a positive option value effect  $[(P \cdot \varepsilon_a)/(i + P)]$ , and on the side of the disinvestment trigger revenue  $\beta_j$  there is a negative effect:  $-(1 - P) \cdot \varepsilon_a/[i + (1 - P)]$ . Thus, the option value effects due to (revenue) uncertainty result in a widening of the ('band-of-inaction')-range between both triggers  $\beta_j$  and  $\alpha_j$ . However, considering the typical pattern of the path-dependence, the non-ideal-relay type dynamics of microeconomic hysteresis have not changed.

4.1 Sunk cost hysteresis and non-ideal relays

4.1.3 A non-ideal relay of employment dynamics

The microeconomic model was, up to now, interpreted with the actual value of production ( $z_{j,t} = p_t \cdot x_{j,t}$ ) as the output variable of the non-ideal relay operator. However, deriving the employment dynamics related to production activity based on the same model is straightforward. Assume that for starting production, one unit of labour for operating the machines (represented by the capital  $K_j$ ) is necessary. Moreover, an employee typically receives a wage not directly related to the utilisation of the production capacity, but as a fixed wage per unit of time (month/year), if (s)he is actually employed and operating the firm’s machines. Thus, the wage of the worker is represented by the cost parameter  $c_j$  related to the stock of machines  $K_j$  (since it is independent of the current utilisation  $x_{j,t}$ , where the cost parameter  $k_j$  is relevant). Thus, a non-ideal relay operator can be derived based on potential sales if actual production activity is directly related to the employment of a worker. Thus, the entry/starting of actual production, triggered by sales value  $\alpha_j \geq \beta_j$ , is directly combined with hiring a worker by firm  $j$ . Analogously, stopping production in an underutilisation situation at revenue trigger  $\beta_j$ , is combined with firing the worker. Actually, the hiring and firing of the worker may result in sunk hiring and firing costs. These sunk costs can be seen as represented by being a part of the sunk cost components  $H_j$  and  $F_j$ .<sup>8</sup> The resulting non-ideal relay, with the employment/activity status as the output variable related to production value as the input variable is depicted in figure 7.

Figure 7: A “non-ideal relay” operator related to a firm’s employment (as the output variable)



Source: Own representation based on (Belke & Göcke, 2001, p. 183).

<sup>8</sup> As stressed in chapter 2.4 labour market hysteresis based on sunk hiring/firing costs was one of the first applications of (sunk costs) hysteresis dynamics in economics.

A transition of the explanation of the non-ideal relay “jumps” when passing triggers resulting in multiple equilibria dependent on the activity status in the past – as the constituting characteristic of microeconomic hysteresis - is straightforward. Just a replacement of the firm’s “actual production activity” by its “employment” (as the output variable of the operator), and of “entry/starting” by “hiring” and “stopping/exit” by “firing” is necessary.

## 4.2 Aggregation and macroeconomic hysteresis

In the following, the Preisach- Mayergoyz-procedure (Mayergoyz, 1986; Preisach, 1935) is applied (originally developed to describe the aggregate magnetization of an entire piece of iron consisting of iron crystals with a heterogeneous orientation). It is an explicit aggregation method for a situation with heterogeneous non-ideal relay agents ( $j = 1, \dots, n; n \gg 0$ ).

As a simplification, we assume the same potential sales situation for every firm in the entire market. In a business cycle boom, the price level for any produced unit is  $p_t$  for every firm, and in a recession, all firms are suffering from the same underutilisation (i.e.  $x_{j,t} = x_t$  is assumed). Thus, the demand side is for simplicity reasons assumed to be homogeneous for all firms. However, even in this simple demand situation, there is heterogeneity regarding the cost structure, as the cost components  $(c_j, k_j, K_j, H_j, F_j)$  are idiosyncratic (different between firms). Therefore, the firms are different regarding the individual entry/exit triggers due to heterogeneity in the firms’ cost structure.<sup>9</sup>

Every potentially active firm  $j$  is characterized by a pair of entry/exit triggers  $(\alpha_j/\beta_j)$ . In an  $\alpha_j/\beta_j$ -diagram (see figure 8), all firms are depicted by individual points in an area above the 45°-line (since  $\alpha_j \geq \beta_j$ ). Firms without any sunk costs ( $H_j = F_j = 0$ ) are represented by points on the 45°-line (with  $\alpha_j = \beta_j$ ). Firms above the 45°-( $\alpha = \beta$ )-line are characterized by a non-ideal relay based on a “band-of-inaction” ( $\alpha_j > \beta_j$ ). To erase any effects of the past, a situation with a zero production level ( $z_{t=0} = 0$ ) is assumed, implying no firm is initially active. A rising potential sales value now results in starting production and hiring workers by the firms with the lowest costs – i.e. the lowest entry triggers  $\alpha_j$ . Firms starting with production activity and hiring workers are included in the hatched triangle  $S_t^+$  (see figure 8 (a)), while the area  $S_t^-$  represents the inactive firms. For a rising

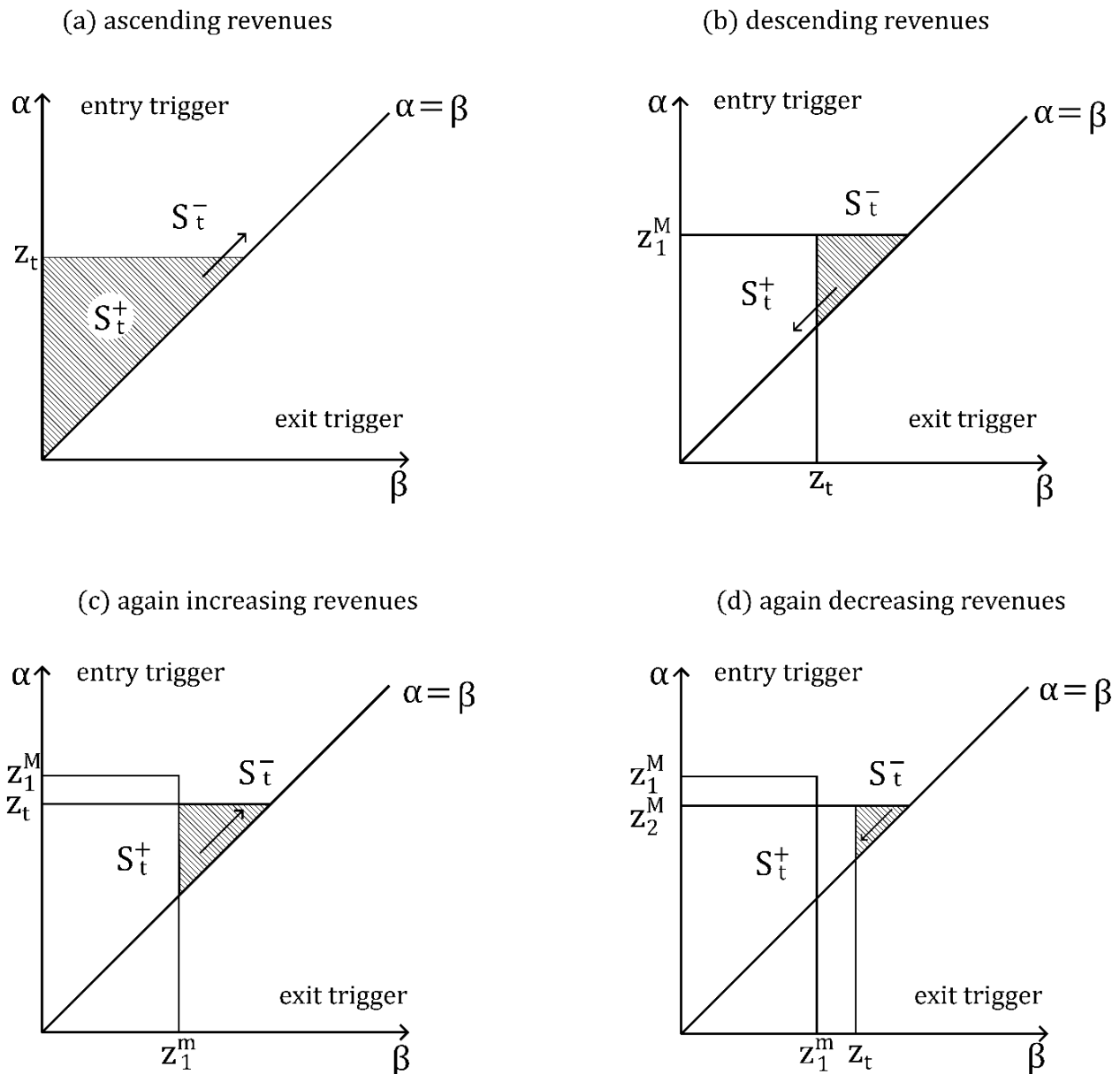
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<sup>9</sup> This procedure was introduced to economics by Amable (Amable et al. (1991)) and Cross (Cross (1993)). See e.g. Cross (1994), Göcke (1994) Piscitelli et al. (2000) Mota and Vasconcelos (2012) Belke and Göcke (2021) for applications of the Preisach-Mayergoyz-model in foreign trade, labour market economics, and for the general case of sunk investments.

#### 4.2 Aggregation and macroeconomic hysteresis

production/employment level, the  $S_t^+$ -expansion is indicated by an upward shift of the horizontal borderline between both areas. If the production of all firms included in the hatched area  $S_t^+$  is summed up (mathematically this is done by an integration operation) the corresponding macroeconomic reaction of the aggregate production quantity is depicted by the path  $AB$  in figure 9. This path of the macroeconomic aggregation shows that a rise in production activity is translated to hiring new employees, therefore, the aggregated output is expressed in employment figures of all active firms combined, while on the abscissa the turnover of all firms is depicted.

Figure 8: Application of the Mayergoyz/Preisach procedure  
 – active firms under a volatile revenue/utilisation

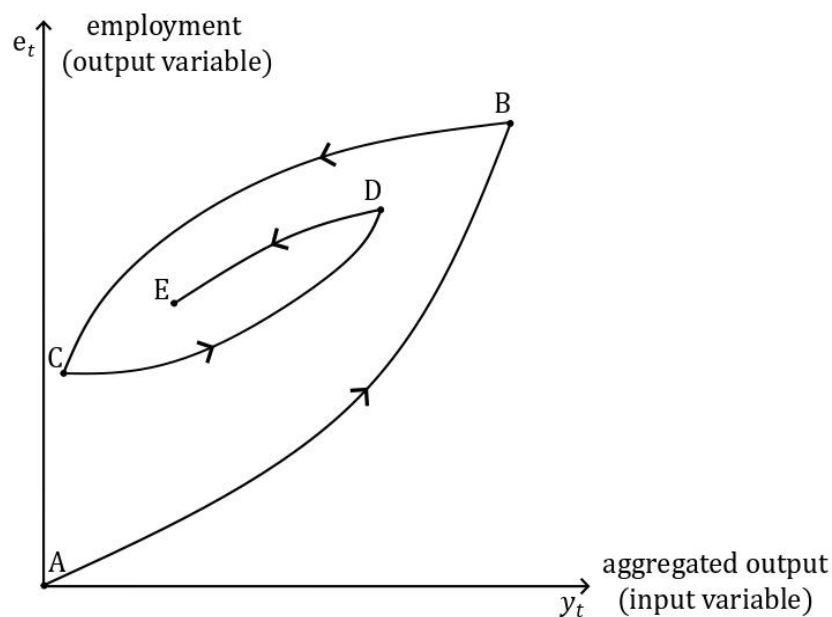


In figure 8 (b) a later decrease in the revenues is traced:  $z_t$  falls after the (local) maximum  $z_1^M$  was reached. Therefore, the area of active firms  $S_t^+$  is now shrinking, since some firms that recently have started production, are stopping and firing again, as revenues are falling below their exit trigger  $\beta_j$ . For a decreasing revenue, the resulting changes of activity (hatched area) are illustrated by a left vertical shift of the  $S_t - S_t^+$ -borderline. The corresponding macro/aggregate quantity reaction is the path  $BC$  in figure 9.

Subsequent to a local minimum  $z_1^m$ , the revenues are rising again, and area  $S_t^+$  is growing again. This is an upward shift of the right-horizontal part of the borderline (see figure 8 (c)), resulting in a macro reaction  $CD$  in figure 9. If the new local maximum is lower than the “old”  $z_1^M$ , the memory of the “old” maximum remains and the new local maximum becomes the second highest, labelled  $z_2^M$ .

In figure 8 (d) the revenue is decreasing again, resulting in a shift to the left of the lower vertical part (path  $DE$  in figure 9). If  $z_t$  does not fall below  $z_1^m$  the new local minimum  $z_2^m$  is now the second lowest minimum. If subsequent local maxima and minima are not as “extreme” as the preceding extrema, a new corner in the “staircase” of the border between the two parts of the triangle is created. These steps/corners of the borderline result from past local extrema and are the base of the “memory” of the system.

Figure 9: The continuous macroeconomic hysteresis loop for employment

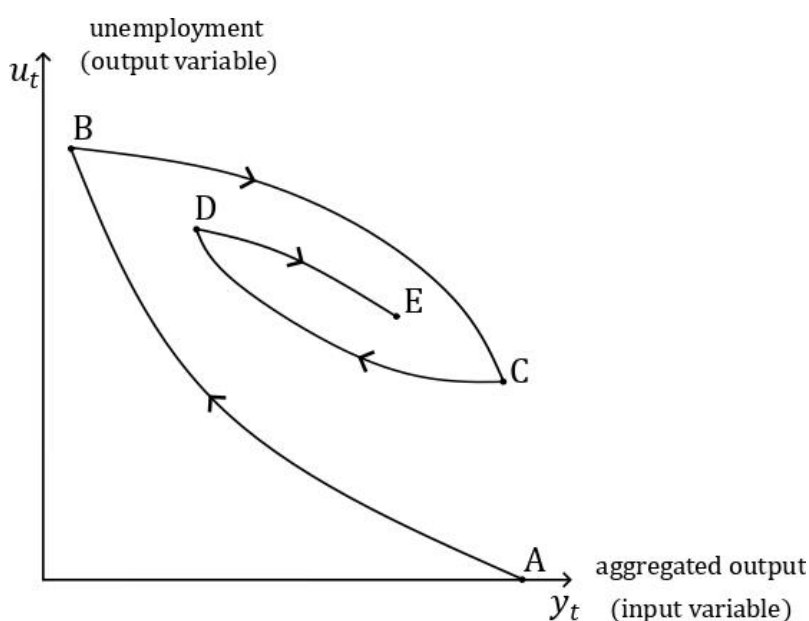


The dynamics were explicitly explained for the derivation of aggregate production quantity over all firms on the market. If instead the corresponding employment of all active firms would be summed

## 4.2 Aggregation and macroeconomic hysteresis

up by the same method (however with different weights for every active firm when doing the integration operation for the number of workers instead of the quantity of production units), the aggregate employment dynamics would be the result. However, the general picture of unemployment dynamics stemming from changes in sales/revenues will have a mirrored (hysteresis) loop pattern/shape as depicted in figure 9. For the sake of completeness, we present it in figure 10.

Figure 10: The continuous macroeconomic hysteresis loop for unemployment



The aggregate system has a memory of past input signal extrema – given by the “staircases” in the borderline of the area  $S_t^+$  of active firms. The character of hysteresis in the aggregate system is stronger, since a transition between different equilibrium branches occurs with *every local extremum* in the path of the input variable; while at the micro/firm level passing of triggers was necessary in order to leave a permanent remanence effect. Macro-hysteresis therefore is called “*strong*” hysteresis (e.g. (1994; Amable et al., 1991; Brokate & Sprekels, 1996, 22–25)).

The *distribution* of the heterogeneous firms in the  $(\alpha_j \geq \beta_j)$ -triangle (and the weight regarding the production quantity/employment of these single points representing a specific cost situation) is important for the results. A continuous distribution of the firms in the  $(\alpha_j \geq \beta_j)$ -triangle implies a *continuous* macro loop. The two-dimensional density of the  $(\alpha_j \geq \beta_j)$ -distribution determines the curvature of the branches of the macro hysteresis loop. If the entry and exit triggers are not too different, since sunk costs are low, and if there is no uncertainty, the upward-leading and the downward-leading branches of the macro loop are close to each other, and the hysteresis effects are

not so important. However, in a situation with high sunk costs and large option value effects on the triggers due to high uncertainty, there is typically a big distance between the entry and exit triggers of the firms. Thus, this situation is characterised by a “blown-up” shape of the macro-hysteresis loop, resulting in severe remanence effects on the macroeconomic level.

## 4.3 Play hysteresis and algorithm

### 4.3.1 Play hysteresis

Belke and Göcke (Belke & Göcke, 2001) worked on a linear approximation of the continuous macroeconomic hysteresis loop (see figure 9). Their model is built upon a sequence consisting of two parts, one called “play”<sup>10</sup> and another one called “spurt”. The former captures the weak economic reactions. While the latter englobes the strong responses, after having passed a play area of weak responses. The interaction between the two parts of the model can be interpreted as a (generalized) case of play hysteresis. By simplifying the hysteresis dynamic to a linear relation, they were able to develop an algorithm capable of computing the play hysteresis for empirical applications.

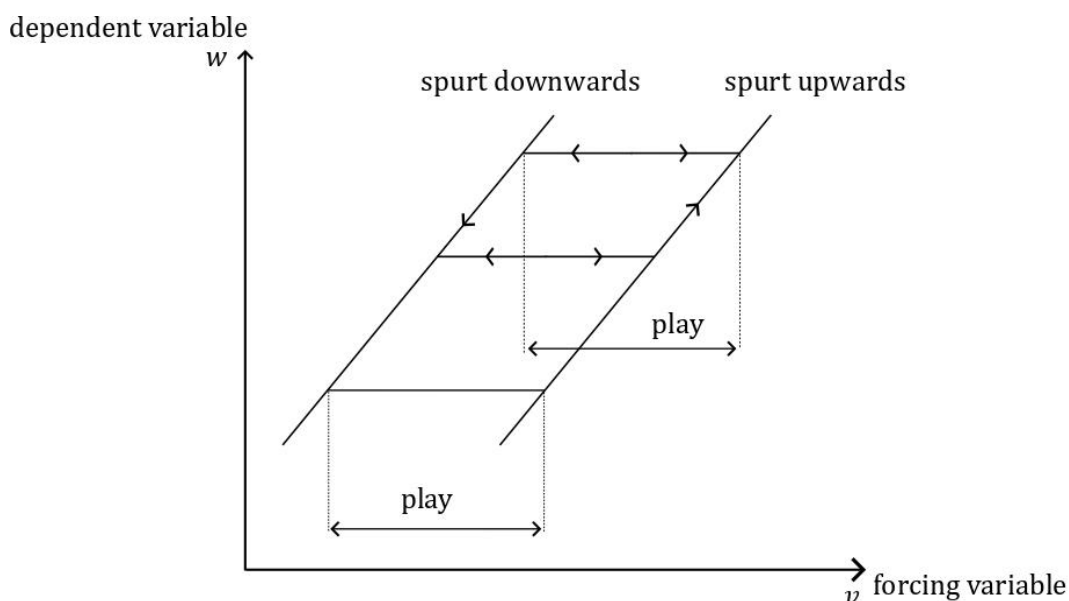
Figure 11 illustrates the mechanics of a supply-side hysteresis model, that could be used for any pair of variables with a positive correlation. The structure of the model consists of two equidistant steep lines, also called spurt lines, and between them, flatter lines with constant lengths, that represent the “play areas”. The spurt lines have a one-way direction of movement, i.e. the spurt line upwards (downwards) can only move up (down), while within the play area, movements in both directions are possible. If applied to a situation where  $v$  is aggregate supply and  $w$  is employment, we would interpret

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<sup>10</sup> Its principle is based on the play operator described in chapter 2.2 using a piston and a cylinder.

### 4.3 Play hysteresis and algorithm

Figure 11: Play hysteresis: Linear spurt lines and constant play width (supply hysteresis)



the relation as: An increase of aggregate supply leads to a rise in employment. Consequently, the slope of the macroeconomic loop is positive. Chapter 6 is built upon this framework. Nevertheless, we also present the demand-side hysteresis, because its structure is mirrored, as seen in figure 12, thus it has a negative slope.<sup>11</sup> This structure is the basis for chapter 5 because it better represents the relationship between GDP and unemployment, as depicted in Okun's law. In both cases, the spurt line represents strong variations of the dependent variables caused by relatively small changes in the forcing variable, while in the play area, changes in the forcing variable do not affect the dependent variable.

To enhance the reader's comprehension of the play dynamics, we give two examples and allude to the macroeconomic labour variable unemployment and aggregate supply (since they are the basis for the traditional Okun's law). Both examples are related to figure 12: The slope of the play areas, e.g.  $GA$  or  $EB$  is not completely flat, but smaller than the slope of the spurt lines.

<sup>11</sup> For further information regarding demand and supply hysteresis see Göcke and Werner (2015).



### 4.3 Play hysteresis and algorithm

unemployment level compared to the initial situation<sup>12</sup>. Another possibility is when, from point  $D$ , GDP decreases from  $y_2$  to  $y_5$ , surpassing point  $E$  and reaching  $G$ . If again GDP starts to rise, the system will move along  $GA$  (the play area) causing no expressive changes in unemployment while in this area. Assuming that GDP returns to the initial value  $y_0$ , unemployment also returns to the initial equilibrium ( $u_0$ ) on the edge of the play area and spurt-up-line, at point  $A$ . Thus, the sequence of points  $A \rightarrow C \rightarrow D \rightarrow G \rightarrow A$  represents a complete/closed hysteresis loop.

So far we have described the whole structure of the model and the two possible patterns that the system can have. Another important characteristic worth attention is the width of the play area. As described in the titles of figure 11 and 12, both have constant play widths. However, in the presentation of the theory on the micro-level of a single firm, we stated in section 4.1.2 that for a firm  $j$  the distance between the entry ( $\alpha_j$ ) and exit ( $\beta_j$ ) triggers, the so called hysteresis band, can be widened due uncertainty, as a consequence of the option value effect. The same is valid for the play areas in the aggregated macro-level (see section 4.2). Note that, uncertainty does not change the shape of the non-ideal-relay nor the structure of the macroeconomic loop. It solely augments the width of the hysteresis band and the play areas. One short example of how changes to the scope of the play area affect the system in fig. 12: Starting from point  $A$ , located on the downwards spurt line (right), a widening of the play area due to uncertainty increase would shift the upwards spurt line (left) horizontally further away from the downwards spurt line. On the contrary, if the current position of the system is point  $G$ , the upwards spurt line would move parallel to the right. If the play width shrinks, the movement of the spurt line has the opposite direction as described above.

Next, we present the algorithm based on the play hysteresis model described in this section.

#### 4.3.2 Play algorithm

The play algorithm was developed by Belke and Göcke (Belke & Göcke, 2001) and implemented within a regression framework. Its first application was the assessment of the impact of the exchange rate on German employment. The following explanation of the algorithm is based on the original paper cited above and considers the general case of a hysteresis system, using variables  $v$  (forcing) and  $w$  (dependent).

A movement of the forcing variable  $v$  ( $\Delta v$ ) can have two outcomes, either it moves within the play area ( $p_a$ ) and thus a weak reaction occurs, or along the spurt lines, causing a strong impact on

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<sup>12</sup> The remanence effects may also be positive, depending on the path of the forcing variable.

the dependent variable  $w$  ( $\Delta w$ ). A shift of  $v$  within the play area is defined as  $\Delta a$  (and cumulated as  $a$ ) and the movement in the spurt area is depicted as  $\Delta s$  (and cumulated as  $s$ ). The initial position of  $v$  is set on the spurt line, moving into the play area, changing its position ( $\Delta v$ ) and then entering the opposite spurt area. This shift is denoted as  $(\Delta v_j^s)$ . The path corresponding to figure 12 is  $B \rightarrow F \rightarrow G$ .  $j$  is the index that represents how many changes between the left and the right spurt line have occurred since the initial position of  $v$ . In general,  $\Delta v_j^s$  can go as far as the opposite spurt line by the amount  $\Delta s_j$  or stop within the play area moving solely  $\Delta a_j$ . In the example given, the initial position was located on the junction between spurt line and play area, therefore the cumulated movement inside the play area  $a_j$  equals the change  $\Delta a_j$  (as seen in figure 12). Bearing in mind these considerations, we have the mathematical definition of  $\Delta v_j^s$  as:

$$\Delta v_j^s = a_j + \Delta s_j, \quad (4.16)$$

and

$$\Delta s_j = \begin{cases} \text{sign}(\Delta v_j^s) \cdot (|\Delta v_j^s| - p_a) & \text{if } (|\Delta v_j^s| - p_a) > 0 \\ 0 & \text{else} \end{cases}$$

Movement of the hysteretic variable  $v$  ( $\Delta v$ ) generates either weak reactions of  $w$  ( $\Delta w$ ) within the play area, characterized by  $\alpha$  as the parameter of the slope of the play line. Or strong reactions along the spurt lines with a steeper slope ( $\alpha + \beta$ ).  $\beta$  is the spurt effect in addition to the steepness of the play area. The overall reaction of  $y$  is described as:

$$\Delta w_j^s = \alpha a_j + (\alpha + \beta) \Delta s_j \quad \text{with } |\alpha| < |\alpha + \beta| \quad (4.17)$$

One of the main characteristics of hysteresis is path dependence, thus we define the cumulated movements induced by all previous movements along the spurt lines ( $i = 0 \dots j - 1$ ) as  $V_{j-1}$ :

$$V_{j-1} = \beta \left[ \sum_{i=0}^{j-1} \Delta s_i \right] = \beta s_{j-1} \quad \text{with } s_{j-1} \equiv \sum_{i=0}^{j-1} \Delta s_i \quad (4.18)$$

The output of the system (position of the dependent variable  $w$ ) is defined as the sum of the historical movements along the spurt lines  $V_{j-1}$  plus the current output realisation  $\Delta w_j^s$ :

### 4.3 Play hysteresis and algorithm

$$w_j = \bar{C} + V_{j-1} + \Delta w_j^s \quad (4.19)$$

Substituting  $V_{j-1}$  and  $\Delta w_j^s$  on the right hand side of equation (4.19) for equations (4.17) and (4.18), we get:

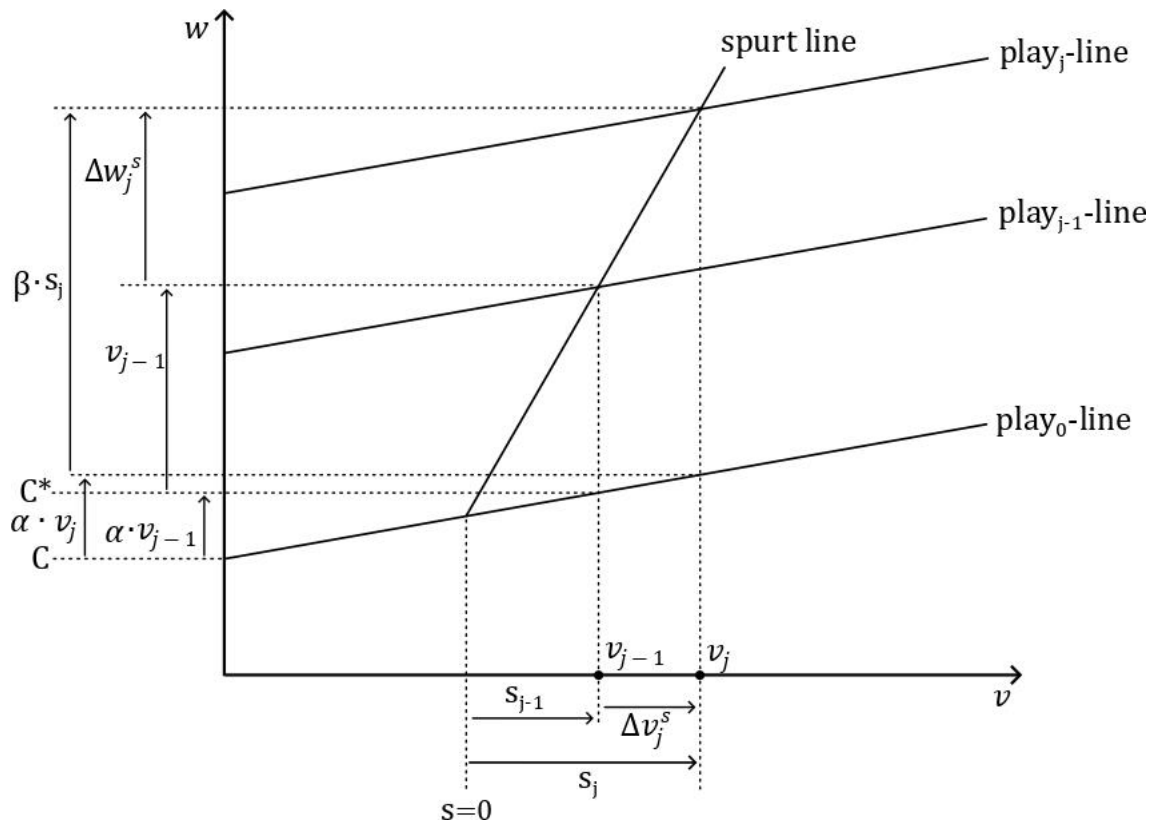
$$C^* + \beta \sum_{i=0}^{j-1} \Delta s_i + \alpha a_j + (\alpha + \beta) \Delta s_j = C^* + \beta \sum_{i=0}^j \Delta s_i + \alpha \Delta v_j^s \quad (4.20)$$

By adding and subtracting the expression  $\alpha \sum_{i=0}^{j-1} \Delta v_i$  on eq. (4.20) results in:

$$\begin{aligned} \Rightarrow w_j &= C^* - \alpha \sum_{i=0}^{j-1} \Delta v_i + \beta \sum_{i=0}^j \Delta s_i + \alpha \left( \sum_{i=0}^{j-1} \Delta v_i + \Delta v_j^s \right) \text{ with } C \equiv C^* - \alpha \sum_{i=0}^{j-1} \Delta v_i \\ &\Rightarrow w_j = C + \alpha v_j + \beta s_j \end{aligned} \quad (4.21)$$

We transformed the output of the system ( $w_j$ ) into a linear equation containing three terms: The constant  $C$ , which now carries the element  $-\alpha \sum_{i=0}^{j-1} \Delta v_i$ , the term  $\alpha v_j$ , which denotes the slope of the play area times the real input variable  $v_j$ . This expression sums up the movements along the play area. The last term is  $\beta s_j$ . It consists of  $\beta$ , the difference in slope between spurt lines and play areas, multiplied by the artificial variable  $s_j$ . This variable is constructed based on the hysteretic variable  $v_j$ . It captures the past and actual effects on  $w_j$  caused by movements of  $v_j$ , but only along the spurt lines (figure 13).

Figure 13: Shift of the play lines



Source: Own representation based on (Belke & Göcke, 2001, p. 189).

Thus, it may be seen as a filter, because it disregards the changes in  $w_j$  caused by movements of  $v_j$  in the play area. To run an OLS model using time series, we can substitute the indicator  $j$ , as denoted so far, for the index  $t$  without any loss of information, and add further non-hysteretic variables ( $\xi_t$ ) multiplied by the respective coefficient  $\phi$ . We then have the final model as follows:

$$w_t = C + \alpha v_t + \beta s_t + \phi \xi_t. \quad (4.22)$$

This linear model allows us to capture the non-linear play hysteresis effects through the artificial spurt variable  $s_t$ , which depends on the size of the play width. We refer to (Belke & Göcke, 2001) for the specifications of the algorithm used to create the spurt and its implementation into an EViews batch program.

Moreover, the program can capture the positive effect of uncertainty on the play width by modelling the play width as a linear function of an uncertainty proxy variable  $\omega_t$ . The estimation's procedure remains the same. The only difference is that the uncertainty component is added to the

### 4.3 Play hysteresis and algorithm

constant play (Belke & Göcke, 2001; 2005). The algorithm now calculates a linear function, which contains a constant and a variable part over time:

$$\gamma = \mu + \delta * \omega_t, \quad (4.23)$$

Where parameter  $\mu$  represents the constant component of the play width,  $\omega_t$  is the uncertainty proxy variable and  $\delta$  is the parameter that represents the slope of the uncertainty variable. The integration of the uncertainty variable in the equation tends to augment the play width, unless it does not play a role and the play width will not show any significant change compared to the scenario without a variable play.

Although the “play” regression model is linear, it is not linear in parameters, due to the model’s structure: It consists of linear segments (play areas and spurt lines), that when adjacent, are linked by knots (e.g. points *A, B, C* in figure 12). The model resembles linear spline functions<sup>13</sup>, because the switch between segments is continuous, and has different slopes. Thus, a change in the input variable may affect the parameters of the model. That is the case if a transition between the segments happens, changing the structure of the model. To identify the position of the knots, the play area ( $p_a$ ) must be determined, which depends on the play width ( $\gamma$ ). Thus, the knots are not a priori known, and solely 3 parameters are necessary to determine the hysteretic dynamic: the slope  $\alpha$ , the slope difference  $\beta$ , and the play width  $\gamma$ . Additionally, the model can account for uncertainty by adding the variable  $\omega$ .

We expect that the following regression model assumptions are valid: the error term is independently, identically, and normally distributed with a constant finite variance over all sections, and the regressors are measured without any error and are not correlated with the error term. Such assumptions imply that the OLS estimators are BLUE (best linear unbiased estimators) for our model. Moreover, the OLS-estimator can be seen as a maximum likelihood estimator, because of the assumption of normal distribution.<sup>14</sup> Generally, in a switching regression model, switches between the branches lead to discontinuities and local maxima when knots are unknown a-priori, and a reliable calculation of the ML-estimator is not possible.<sup>15</sup> Nevertheless, in our case, the adjacent sections are joined and no discontinuities exist. Therefore, the OLS-/ML-estimators should be consistent and asymptotically normally distributed. Unfortunately, due to the small sample properties<sup>16</sup> of equation

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<sup>13</sup> A special case of general switching regression models. For further information on switching linear regressions we refer to sections 7.1 and 7.2 of Poirier (1976), and for spline regression see section 7.3 of the same book.

<sup>14</sup> See e.g. Judge et al. (1984, p. 799)(1984, pp. 802–803).

<sup>15</sup> Properties of ML-(OLS-) estimators for models with unknown but continuous switches can be found in Hinkley (1969); Hudson (1966).

<sup>16</sup> See Hinkley (1969, 1971); Poirier (1976, p. 129) for small sample properties in ML- (OLS-) estimations in a (spline) model.

4.22, local maxima in the likelihood function of the play regression may occur, because the parameter estimates are not even approximately normally distributed, thus the estimator is not BLUE anymore. Moreover, assumptions about the error term may not be valid, e.g. the variables may not be stationary resulting in non-finite variances. Another subject of concern is the very nature of play dynamics, which is a mixture of short- and long-term dynamics, obstructing the use of non-linear cointegration methods. Because of the properties presented and their implications, there is no satisfactory solution for the presented problems, thus we use an OLS estimation. The objective here is to present to the reader issues related to the model. Working in a framework model capable of tackling these issues is definitely a path to be followed for research purposes in the future. In the next chapter, we describe the data used in our first estimations and put into use the play regression model.

# Chapter 5

## Okun's law and hysteresis: An empirical analysis

Chapter 5 belongs to the main contribution of the work. It deals with empirical models to test for hysteresis in the context of Okun's law for several EU countries. The novelty is using the play algorithm, described in chapter 4, to confirm or deny the existence of hysteresis for these countries and quantify the hysteretic effect.

### 5.1 Motivation

Okun's law original version suggests a linear relationship between growth and unemployment with a straightforward interpretation: GDP fluctuations cause the rise or fall of unemployment. Nowadays the law is widely accepted, and its validity is not disputed. Nonetheless, questions arose about the time consistency of the three-to-one ratio as a rule of thumb (Lee, 2000, p. 331). Output and unemployment dynamics may differ over time due to structural economic changes, and so may the parameters estimated. Therefore, it is necessary to reevaluate the coefficients. Moreover, this rule of thumb cannot be applied to all countries equally, due to structural economic differences, reinforcing the necessity of calculating the coefficients for each country. A third argument to reevaluate this law is that, over the years, many scientists refined Okun's finding, giving several reasons to believe that a non-linear relationship between these two variables is more realistic.<sup>17</sup> From the approaches undertaken to model it, one possibility is using hysteresis, since it can be applied to Okun's relationship.<sup>18</sup> Among those scientists who have followed this line of research, none has used the play hysteresis. Thus we present in this chapter estimations to refine Okun's law and possibly use it as a benchmark for policy makers to calculate the cost of growth fluctuations measured in unemployment.

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<sup>17</sup> See chapter 3.2.

<sup>18</sup> We refer to chapter 4 for the theoretical foundation and section 3.2 for a survey of studies that link hysteresis to Okun's law.

## 5.2 Data and variables

Quarterly data on unemployment and GDP were extracted from the Eurostat database to build the regression models. Missing values, seasonal adjustments, and correction of main breaks are some of the adjustments undertaken by Eurostat. In order to guarantee the comparability between reported data of different countries, the EU enforced an accounting framework, the European System of National and Regional Accounts (ESA 2010), that must be used by the Member States since 2014. Several manuals and guidelines were published and updated over time detailing the methods and requirements imposed by the ESA 2010 handbook.

**Unemployment:** Eurostat calculates unemployment based mainly on the EU-LFS<sup>19</sup>, with some adjunctions of national data sources. We use quarterly data<sup>20</sup> that is neither seasonally nor calendar adjusted. Unemployment is calculated as a percentage of the labour force (or workforce) and not on the total population. Contrary to the workforce, the total population includes economically inactive people, i.e. persons that are neither employed nor unemployed, e.g. school children, students, pensioners, and housewives or -men, when not looking for work. The data is aggregated in classes determined by the lifetime span, e.g. from 15 to 24 years. We picked the age class 20 to 64 years old, because it comprises the years when individuals are most likely working. Wider classes, e.g. 15 to 74 years, do not change the data substantially, therefore, similar results are expected when using both age span classes. All other classes have a short time span or do not include crucial periods of working life.

**GDP:** It is defined as the value of the production at market prices of all the goods and services produced within a country. There are three possible approaches to calculate the gross domestic product: a) GDP Output approach/production approach, b) GDP Expenditure approach, c) GDP Income approach. According to the ESA 2010 the composition of GDP in the production approach is the total gross value added (B.1G) + taxes - subsidies on products (D.21 less D.31). Moreover, B.1GQ, B.1G, D.21, and D.31 being their respective acronyms in the accounts' system. The accounting framework of the European Union allows to calculate GDP in these three approaches mentioned, because data reliability differs from one country to another. E.g. when data on intermediate consumption is partially incomplete, then the expenditure method is better suited for the calculation. Or when household consumption expenditure is incomplete, but production data is available, then the

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<sup>19</sup> European Union Labour Force Survey.

<sup>20</sup> The German unemployment data retrieved from Eurostat was missing the value for the period of 1991Q1. We used the data from the German federal statistical office as a substitute for this one missing value. The results are exactly the same, whether the data point is considered missing or not. Also, the parameter settings for the model are not influenced by the inclusion of the data point.

## 5.2 Data and variables

output approach is more suitable (Eurostat, 2016, pp. 10–11). In any case, the ultimate result should converge to one result. Thus, countries balance the different methods utilised to cope with this requirement, resulting in only one GDP measure. Germany, as an example, uses the production and the expenditure approach, then balances the two results into one. The third method is not feasible because the data on entrepreneurial income for periods of less than one year are not available.

GDP is measured as the gross domestic product at market prices and presented as a chain linked volume index with 2015 as basis measured in millions of Euros. Therefore, it is price-adjusted, since the values are linked by rescaling to the level of the reference year. Thus, an augmentation of the price level will not inflate the GDP growth rate. In this case, we deal with the real GDP. Throughout the work, we refer to real GDP as GDP for simplicity reasons.

**GDP gap:** It is derived from GDP and used in the statistical estimations as the independent variable. We apply a filter to the GDP data to obtain the long-term trend development of the productivity, which is also called **potential output**, **productive capacity** or **potential capacity**. In the next step, we deducted the latter from the real GDP, and the difference between them is the **GDP gap** or **output gap**. It corrects a time series for increases (decreases) in GDP over (under) the potential output due to productivity increases (decreases). It measures the percentage deviation of actual GDP from its potential level and reflects whether the economy is operating above or below its potential capacity. I.e. GDP gap represents a level difference from the trend.

Several filters to calculate the gap between actual and potential GDP were tested and their comparison can be found in section 5.3.2.

**SPURT:** It is an artificial variable that approximates the behaviour of the macroeconomic hysteresis dynamics allowing us to distinguish between strong and weak economic reactions of the variable in question. In the presented work hysteresis is derived from the output gap and represents only strong deviations from the theorized productive capacity. One can say that SPURT and GDP gap have the same nature. However, the former results from filtering out small fluctuations (within the play areas) from the latter.

**Control variables:** The variable shift has the value 0 until the period of the structural break, changing its value to 1 until the end of the data set. We included a shift for the financial crisis of 2008/9 for every estimation. Seasonal dummies for the first 3 quarters (D1 to D3) are also included, as well as a linear trend variable.

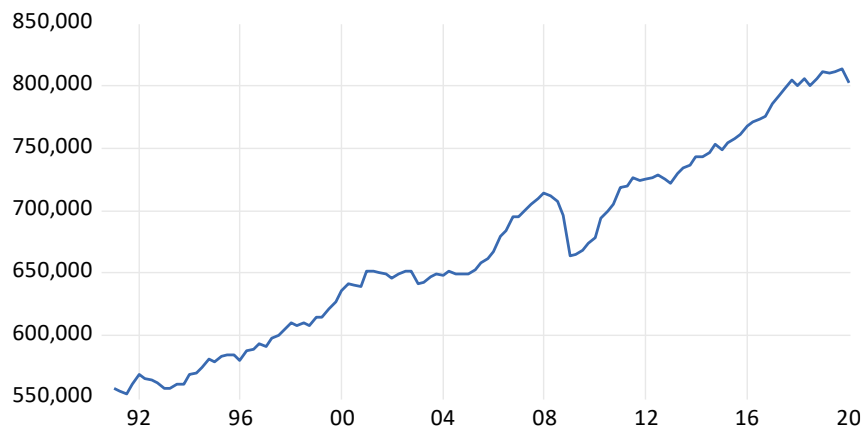
## 5.3 Germany

We conduct a full analysis for Germany. The choice of the country relies on the economic importance within the EU and worldwide. This will be the first detailed analysis of hysteresis in this work. All succeeding countries' evaluations will follow the same structure.

### 5.3.1 Main findings

The real GDP seen in figure 14 was first logarithmized. A logarithmic scale enables the observation of relative changes, so that equal percent changes have the same magnitude independently of the absolute values. Then the series was filtered with the Hodrick-Prescott filter (HP filter). In section 5.3.2 other filters are tested. This filter is a detrending method widely used in economics to separate the trend and the cycle components of a time series. The latter can also be utilised directly as the variable GDP gap, which refers to the GDP being above or below the potential output.

Figure 14: Real GDP Germany



Source: Own representation based on (Eurostat, 2024b).

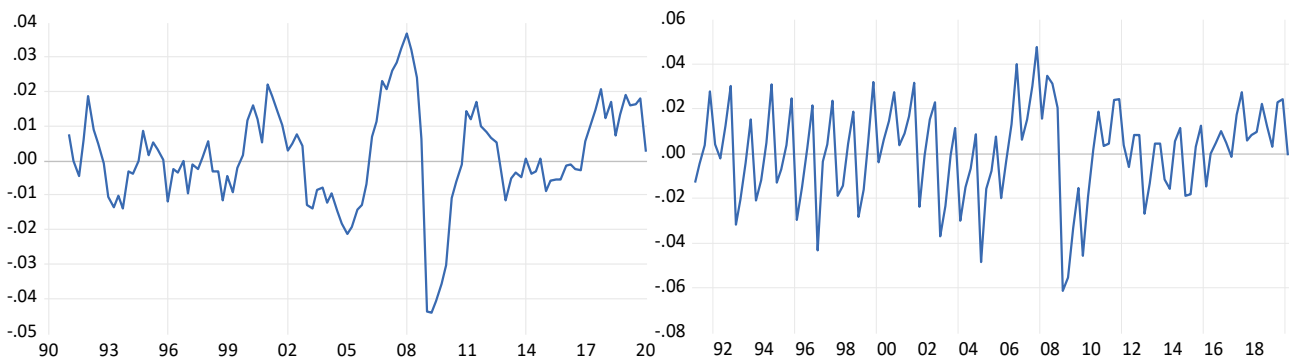
In figure 15 on the left side, the seasonally adjusted GDP gap for Germany is presented, and the seasonally unadjusted GDP gap is on the right side of the figure.

### 5.3 Germany

Every point above the 0% line represents in both cases a capacity overutilisation and below it, an underutilisation. Moreover, GDP gap enters with a lag of one quarter to avoid the problem of reverse causation<sup>21</sup>.

One sees that the use of seasonal adjustments smoothens the peaks and bottoms in the graphic. Higher peaks, e.g. in 1992Q1, 2001Q1, and 2008Q1, represent a utilisation over the potential output, and time periods like 2005Q1 and 2009Q2 are characterized by underutilisation. We utilise seasonally adjusted data, otherwise, we would not be able to determine peaks and bottoms, which is the condition to construct a reasonable artificial variable (SPURT). Moreover, fluctuations of the output gap are overstated by the seasonal effects due to periodic hiring and firing of employees, leading to an increase in the play bandwidth.

Figure 15: German Output Gap - adjusted (left) and unadjusted (right)



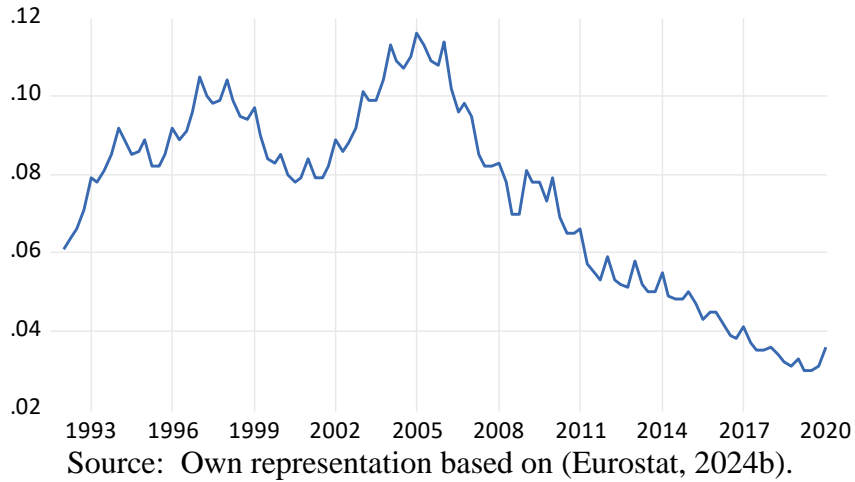
Source: Own calculations with data from (Eurostat, 2024b).

Figure 16 shows the unadjusted unemployment data. The data is seasonally unadjusted since there is no economic reason to transform the data. From 1992 until the first quarter of 2006 high unemployment persisted, peaking in the beginning of 2005 almost 12%. Since then, unemployment has been in a free fall, despite the financial crisis of 2008. It had an extreme negative impact on the German economy, but a relatively low effect on unemployment. The three seasonal dummies D1-D3 capture the effect of cyclical work.

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<sup>21</sup> We choose to use the lagged output gap due to endogeneity effects of the dependent variable (unemployment rate) and regressor (GDP gap).

Figure 16: Seasonally and calendar unadjusted unemployment (Germany)



The benchmark of the analysis will be a simple OLS regression without play effect. In this case the SPURT variable converges to the GDP gap. We regress unemployment in Germany on the German output gap, a linear trend, and dummy variables for the first 3 quarters (D1-D3). The summary of the estimation is the following:

Table 1: Standard OLS regression without play for German data

Dependent variable: Unemployment in Germany

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.104782	0.003877	27.02967	0.0000
GDP_GAP_DE(-1)	-0.319209	0.099072	-3.221982	0.0017
TREND	-0.000539	4.46E-05	-12.07366	0.0000
D1	0.005185	0.004101	1.264472	0.2088
D2	-0.000931	0.004069	-0.228732	0.8195
D3	-0.001633	0.004101	-0.398293	0.6912
R-squared	0.616221	Mean dependent var		0.073850
Adjusted R-squared	0.598288	S.D. dependent var		0.024206
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.042129

Source: Own calculations.

For Germany the GDP gap has a negative sign, being in accordance with the theoretical expectation. The coefficient is relatively low and highly statistically significant. The trend variable is statistically significant but its coefficient approaches 0 due to the filtering process of the real GDP

### 5.3 Germany

data. The three seasonal variables are not statistically significant. The relationship stated by Okun, that GDP and unemployment are negatively correlated, holds for this simple OLS, since an increase of one percent in GDP gap would have a negative impact on unemployment of roughly 0.32 percentage points on average.

The second step of the analysis is to add the play effect, for which the optimal play bandwidth ( $\gamma$ ) must be found. The play algorithm estimates the spurt coefficients by a grid search calculating a multitude of regressions, each of them with a different play width. The settings chosen allow a maximal bandwidth of 10 percentage points of GDP gap, starting with no variable SPURT ( $\gamma = 0$ ) and successively increasing the play width in steps of 0.001 percentage points per regression. Moreover, for the creation of SPURT, we use GDP gap lagged in one quarter to be consistent with the lagged GDP gap used as independent variable in the model.

In order to put the algorithm “on the right track” it is essential for the algorithm to start with a “pronounced” extremum, and thus to use the information that there is a current situation on the spurt line for starting. Therefore, the starting point for the grid search was set in 1992Q2 as a maximum, where GDP gap is above the potential output, reaching the value 0.019 at this point. I.e. that GDP was 1.9% above its potential. We can assure that the starting point is a maximum, because this value is substantially above the average potential output (marked by the 0 line in figure 15 (left)). The difference between the GDP gap in 1992Q2 and the next extremum is crucial for the grid search, because it sets the maximal bandwidth tolerated to trigger SPURT. In the current analysis, after the maximum in 1992Q2 follows a minimum, in 1993Q3, which gives us theoretically a maximal bandwidth of 0.033 percentage points. A maximal play bandwidth from around 1 to 3% is plausible for developed nations like Germany, France, and the Netherlands to trigger a hysteretic behaviour in the context of capacity utilisation.

While for less developed European countries as Greece, Spain, and Portugal 3 to 5% of the play bandwidth is expected. Because these countries tend to have less labour market flexibility, with high unemployment protections. It hinders structural changes in the labour market and thus augments the band on inaction. While more dynamic labour markets, e.g. in the Netherlands, respond faster to shocks and therefore, have a narrower band of inaction.

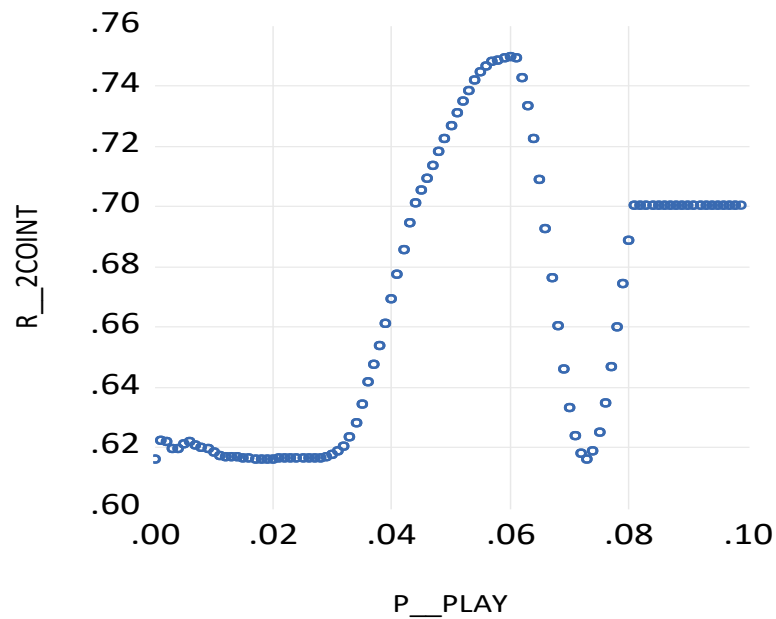
For a higher difference between local extrema, hysteresis is not suitable to analyse the relation. Possibly other significant events could better explain such wide play bandwidths.

Figure 17 shows the measurement of fit,  $R^2$ , on the ordinate axis for each estimation, and in the abscissa the corresponding play bandwidth. The highest  $R^2$  is reached at a play width of  $\gamma = 0.06$ . Such a high value is rather exceptional and typically occurs only in extreme cases (such as during the

financial crisis). The play bandwidth subsequently falls but then stabilises when  $\gamma$  reaches the value 0.08, maintaining a constant  $R^2$  for any higher play width. It also indicates that the variable SPURT converged to a dummy (also due to the financial crisis).

For the grid search in figure 17, we examined a bandwidth of a maximum of 10 percent as an example to demonstrate how the search for the optimal spurt variable can develop. For further analysis, such wide bandwidths are necessary and the boundary of 8 percentage points will be held as standard.

Figure 17: Play width – Germany (no shift)



Next, we add the spurt variable with a bandwidth of  $\gamma = 0.06$  to the OLS, and the following result is retrieved:

### 5.3 Germany

Table 2: Standard OLS regression with play for German data

Dependent variable: Unemployment unadjusted Germany

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.100327	0.003200	31.35210	0.0000
SPURT	2.418590	0.321498	7.522879	0.0000
GDP_GAP_DE(-1)	-0.655900	0.091991	-7.130040	0.0000
TREND	-0.000503	3.65E-05	-13.75708	0.0000
D1	0.004286	0.003329	1.287538	0.2007
D2	-0.001106	0.003301	-0.335008	0.7383
D3	-0.001430	0.003327	-0.430004	0.6681
R-squared	0.749802	Mean dependent var		0.073850
Adjusted R-squared	0.735640	S.D. dependent var		0.024206
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.149224

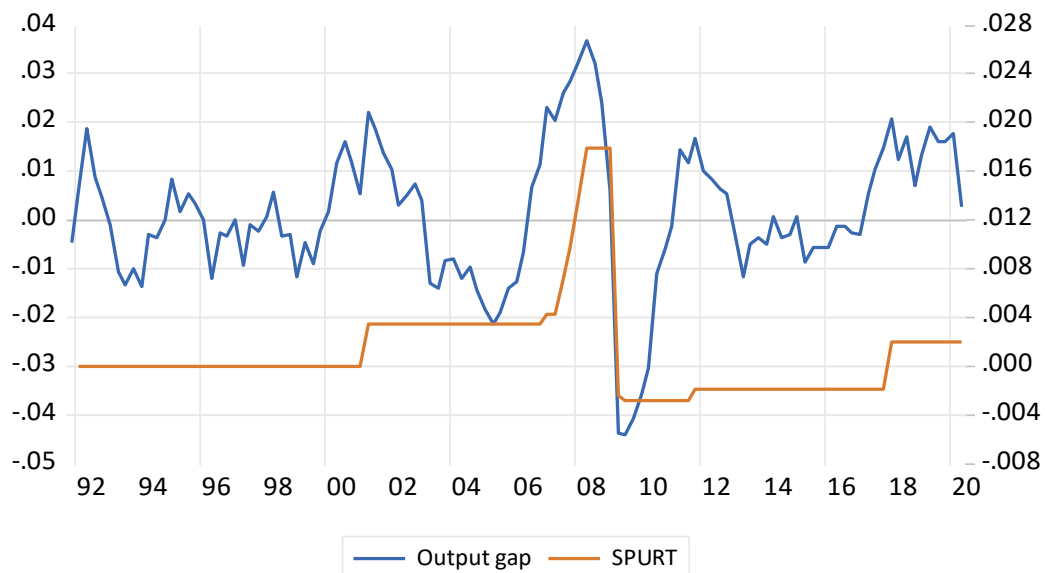
Source: Own calculations.

Contrary to the theoretical expectations, SPURT is positive and its value is remarkably high. Since only the GDP gap was logarithmized and the unit of unemployment is in percentage, one can interpret the coefficients as semi-elastic. For the spurt variable the following statement can be done from the statistical summary: A 1% increase in GDP gap over the play bandwidth of 0.06 percentage points results in an average increase of 2.4 % points in unemployment, which is a coefficient far higher than the benchmark resulted from table 1 (-0.319) and an implausible positive sign. When accounting for the effect of GDP gap, which has a negative coefficient of 0.65, the overall effect is still positive and the sum of the coefficients is 1.75. Thus, the coefficient value is plausible for hysteresis effect, but its sign is not. The trend and seasonal variables did not change compared with the simple OLS. The results of the estimation confirm that the model is not working properly.

The last part of the analysis is a visual inspection of the spurt variable on figure 18. It indicates a likeness between a dummy variable and SPURT: the latter jumps gradually from 0 to approximately 0.34 % of the GDP gap in 2001Q1. And again, rises rapidly to 1.8% after the first quarter of 2008 until 2008Q4, when it falls to -0.28% and remains from this point on roughly constant until 2017Q4, when a last positive jump can be seen to the value of 0.2%. This last value was kept until the end of the estimation. The reason for such behaviour of SPURT relays in the fact that the algorithm recognizes a higher  $R^2$  for a wider play width, since it better fits the financial crisis in the estimation. In the next step, the variable “shift” is included in the estimation as a proxy for the financial crisis. It takes the value 0 in all periods prior to the crisis and 1 during and after the financial crisis. Shift was also lagged

in one quarter. It captures the change in the relationship between the variables in the model caused by the financial crisis on the dependent variable. By doing so, SPURT should not depict a dummy-like variable anymore, separating the hysteretic from the financial crisis effect. For the sake of completeness, we also estimated this very same regression including a dummy variable with value 1 for the crisis period, otherwise 0. The period tested ranged from 2008Q1 to 2011Q3. The resulting play width graphic has a similar shape to the graphic in figure 17 and the resulting spurt path is similar to the path in figure 18 (for the comparison see Appendix A.1), indicating that the dummy does not capture the strong effect of the financial crisis. Therefore, the dummy with value 0 for the period before 2009Q1 and 1 for the period after 2009Q1 is more suitable.

Figure 18: GDP Germany (no shift)



Source: Own calculations.

With the inclusion of shift, we get the following estimation:

### 5.3 Germany

Table 3: OLS regression for German data with shift and without play component ( $\gamma = 0$ )

Dependent variable: Unemployment unadjusted Germany  
Sample: 1992Q2 2020Q2  
Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.086205	0.002508	34.37155	0.0000
GDP_GAP_DE(-1)	-0.682618	0.060848	-11.21849	0.0000
SHIFT(-1)	-0.051842	0.003420	-15.15873	0.0000
TREND	0.000143	5.15E-05	2.765690	0.0067
D1	0.004386	0.002315	1.894037	0.0609
D2	-0.000512	0.002297	-0.223028	0.8239
D3	-0.001100	0.002315	-0.474994	0.6358
R-squared	0.878850	Mean dependent var		0.073850
Adjusted R-squared	0.871993	S.D. dependent var		0.024206
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.305449

Source: Own calculations.

GDP gap now has a negative sign, in accordance with the theoretical expectation. It has a substantial coefficient of 0.68 and is highly statistically significant. Assuming a linear influence of GDP over unemployment (i.e. absence of hysteresis effects), the variable GDP gap is interpreted as follows: A one percent rise in GDP gap induces on average a 0.68 percentage point decrease in unemployment in Germany. GDP has an disproportionately small effect on the dependent variable. For all countries assessed we expect similar values for the variable GDP gap: Under 1 percentage point effect of GDP on the independent variable.

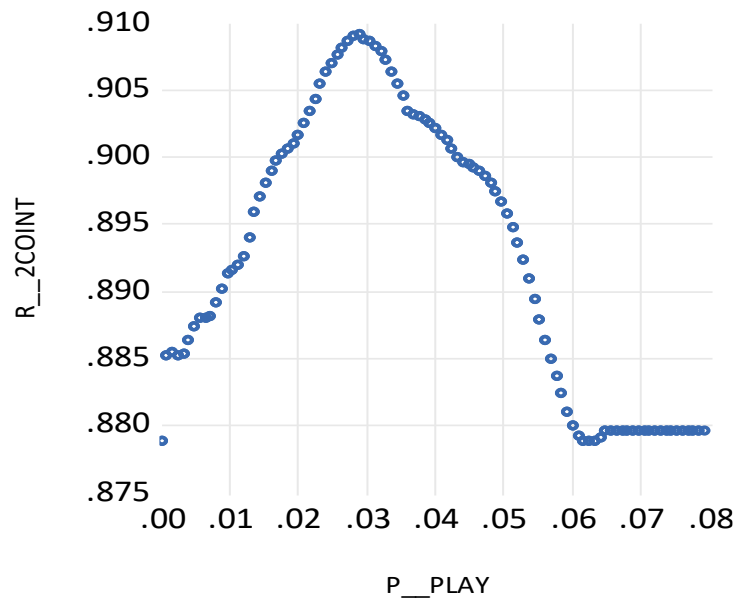
The trend component is also statistically significant although its coefficient almost approaches zero. Of the three seasonal dummies for the quarters 1 to 3 only D1 is statistically significant at the 10% level. Shift is set to have the value 0 for the period before 2009Q1 and assumes the value of 1 for 2009Q1 onwards (2009Q2 after being lagged). Moreover, it is statistically significant as we suspected from the visual inspection and play width graphics. Its negative sign indicates that the financial crisis had a negative influence on unemployment, presumably because of job protection measures undertaken by policy makers and companies. However, the impact of its coefficient is not as expressive as the one from GDP gap.

The next step in the process is to calculate the play widths. fig. 19 gives its graphical representation. The mountain-shaped graph exhibits all 100 estimations made, each of which is a blue dot and has a different play width ( $\gamma$ ). The shape itself already suggests hysteresis effect on the data. On the very left, the first dot represents the simple OLS and its corresponding  $R^2$  is under .880. The

$R^2$  value increases for all estimations as the play width becomes wider until the width reaches 0.028 percentage points of GDP gap (top of the mountain). The play width decreased from 6 percentage points (see fig. 17) to 2.8 (see fig. 19). I.e. in order to trigger the SPURT variable, GDP gap has to exceed or fall short of 0.028 percentage points. The result is economically plausible: An increase (decrease) of 2.8% points of GDP above (below) the normal level of production (0-line) triggers firms to hire (fire) employees. Concluding that only large spreads contribute to determining SPURT.

Taking the play width value of 0.0288 to create the artificial variable SPURT, results in the estimation with the highest  $R^2$ .

Figure 19: Play width – Germany with shift



Source: Own calculations.

In the next step, the results for the OLS equation with the SPURT variable are stated:

### 5.3 Germany

Table 4: OLS regression for German data with shift and with play component ( $\gamma = 0$ )

Dependent variable: Unemployment unadjusted Germany  
Sample: 1992Q2 2020Q2  
Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.081839	0.002304	35.52445	0.0000
SPURT	-0.786603	0.132952	-5.916438	0.0000
GDP_GAP_DE(-1)	-0.334913	0.079101	-4.233974	0.0000
SHIFT(-1)	-0.058559	0.003185	-18.38587	0.0000
TREND	0.000210	4.63E-05	4.546288	0.0000
D1	0.004950	0.002017	2.454381	0.0158
D2	-5.83E-05	0.002000	-0.029126	0.9768
D3	-0.001263	0.002015	-0.626727	0.5322
R-squared	0.909140	Mean dependent var		0.073850
Adjusted R-squared	0.903083	S.D. dependent var		0.024206
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.217657

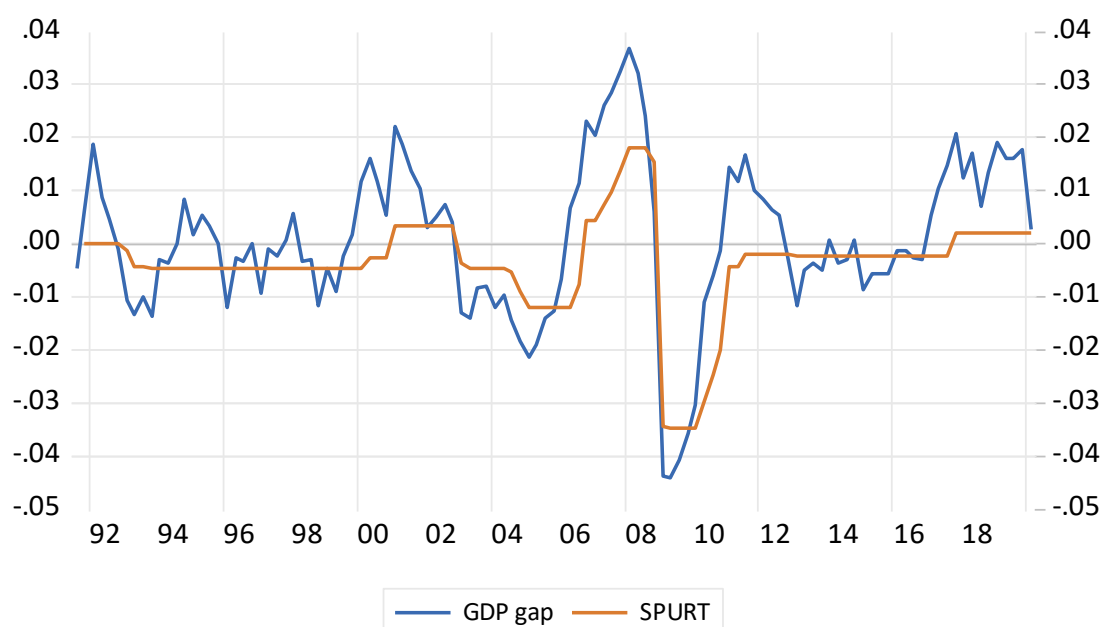
Source: Own calculations.

Table 4 shows a small improvement in the  $R^2$ , an expected negative sign for SPURT's coefficient, which is also statistically significant and has a higher absolute value than GDP gap. The variable GDP gap is now the share of the original GDP gap that was filtered out, and it represents the "weak" reaction inside the play area. It was halved and kept its statistical significance, meaning that a substantial part of the explanatory power was taken by the spurt variable. In its turn, SPURT represents the "strong" reaction, and can be compared to the play-reaction of GDP gap. When summed up, both coefficients add up to the (negative) value 1.12. I.e. that for a 1% increase in GDP gap over the band of inaction, unemployment decreases on average around 1.12 percentage points. I.e. the forcing variable has a less than proportional effect on unemployment while moving within the play area. And the total effect, including the effect of GDP gap on unemployment in the spurt line, has a slightly over-proportional reaction. Lags from 2 to 4 quarters were also tested but did not deliver better results than presented in this chapter (see Appendix A.2).

The seasonal dummies and trend remain roughly as they were in the estimation without the introduction of SPURT, namely coefficients close to 0. Only the statistical significance from D1 changed from a 5% level to a 10% level. Shift is in both estimations statistically highly significant and has a relatively small coefficient. Nevertheless, its statistical significance is an indication that this variable is partially responsible for the hysteresis effect.

In figure 20, one sees that SPURT differs from the shape of a dummy variable. This is because the variable shift partially captured the effect of the financial crisis. A visual inspection of the graphic confirms that SPURT (right scale) is a filtered version of the original GDP gap, where small changes are whipped out: From the starting point SPURT has the value 0 and kept it constant, until GDP gap experienced a significant fall in 1992Q4, thus SPURT was for the first time triggered. It happens because GDP (left scale) fell more than the play width ( $\gamma$ ) of 2.8 percentage points. I.e. in 1992Q1 GDP was 1.9% points above its potential output and in 1993Q4 -1.4% points, representing a span of 3.3% points. Therefore, the threshold of the play variable of 2.8% points was trespassed by 0.5% points and the SPURT variable catches this excess of the fall, dropping from 0 to -0.005. SPURT had another two positive changes subsequently: In 2000Q2 when it rose to -0.003 and in 2001Q1 when SPURT took the value 0.003, when GDP gap reached another peak of 2.2% points. SPURT kept constant until 2002Q4 when GDP fell from its last peak of 2.2% to -1.3% points in 2003Q1 and SPURT absorbed the difference of -0.007, resulting in -0.004 percentage points. The same visual interpretation made until 2003Q1 can be applied to the whole graphic and the variable SPURT behaves as expected.

Figure 20: Spurt and GDP gap – German data



Source: Own calculations.

To back up the interpretation of the play bandwidth and SPURT graphic, and confirm the existence of the play dynamic for any two pairs of variables, in this case, GDP gap and unemployment,

### 5.3 Germany

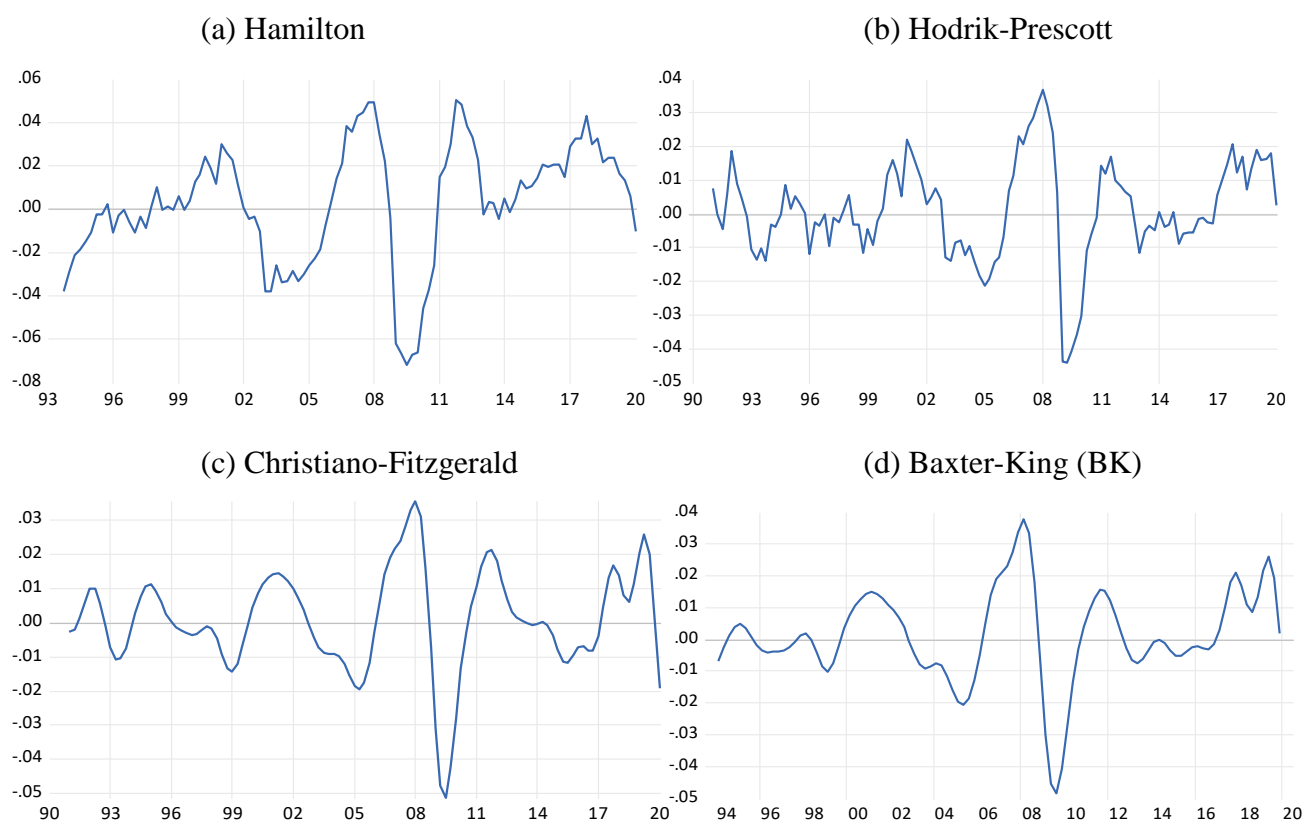
one must assess the estimates with respect to three characteristics. 1) A significantly negative effect of the spurt variable (i.e.  $\beta < 0$ ), since the theoretical expectation is a negative relationship between the two variables. 2) A weaker (or even insignificant) effect of the hysteretic variable when estimated in the same equation as SPURT, since it reflects the reactions in the play area. 3) The absolute spurt effect of output gap, measured by the slope of the spurt-lines ( $\alpha_{play} + \beta$ ) should surpass the output gap effect ( $\alpha_{simple}$ ) of the simple OLS estimation: ( $|\alpha_{play} + \beta| > |\alpha_{simple}|$ ). The first condition is fulfilled since SPURT is negative, as well as the second condition: The value of SPURT is more than double the value of GDP gap, which more than halved from  $|-0.682618|$  to  $|-0.318135|$ , although it kept its statistical significance. The third condition holds:  $|(-0.7866) + (-0.3349)| > |-0.6826|$ , resulting in  $1.12 > 0.68$ , and confirming hysteresis in the data. It suggests that adding SPURT causes the overall effect of GDP on unemployment to rise. Since GDP gap and SPURT have the same nature, their combined effect is only higher than in the simple OLS because the hysteresis plays a role in the relationship between both variables.

To sum up, the full analysis comprises four components: 1) A simple OLS was estimated and serves as a benchmark for a comparison with the estimation with play dynamics. Only a brief statement about the coefficients of the statistical summary was made, because we focus on the play dynamics that follow. 2) The play component was introduced by choosing the optimal bandwidth through the play algorithm. The shape of the grid search was checked for plausibility, and if compliant with our expectations, the bandwidth with the highest  $R^2$  was retrieved and used to generate the artificial spurt variable. 3) This resulting OLS with SPURT is analysed and its coefficients are tested for the three characteristics cited just above. 4) The shape of SPURT must be graphically coherent with the play bandwidth chosen. Steps 3) and 4) do not need to follow this order. Although, throughout the work, we do keep it.

#### 5.3.2 Filters

We calculate the potential output by applying a filter to the original time series, which is the gross domestic product at market prices, in order to separate the long-term trend of real GDP from short-term fluctuations. These fluctuations are also referred as the cyclical component of the time series. After the original series is filtered, the long-term trend is a proxy for the potential production level, which will serve as a reference to calculate the spurt effect. We tested 4 different filters: Hodrik-Prescott (HP), Hamilton (H), Christiano-Fitzgerald (CF), and Baxter-King (BK) on the German data and isolated the cyclical components. A visualisation of the results can be seen in figure 21.

Figure 21: Different cyclical series using filters on German real GDP



Source: Own calculations with data from (Eurostat, 2024b).

The cyclical components exhibit diverging patterns due to the different techniques. HP and Hamilton filters are similar techniques and belong to the category of smoothing filters while the BK and CF belong to the category of band-pass filters.

The CF filter was introduced by Lawrence J. Christiano and Terry J. Fitzgerald (Christiano & Fitzgerald, 2003). And the BK filter by Marianne Baxter and Robert G. King (Baxter & King, 1999). These are frequency filters that allow a specific range of frequencies to pass through, isolating them from the rest of the time series. The “band” or frequencies allowed are defined by two boundaries, a lower and upper bound. The lower bound was set to 6 and the upper 32, representing 1,5 years and 8 years respectively, since each unit represents one quarter of a year.

The frequencies or “bands” are filtered based on a two-sided weighted moving average of the data and there are different ways of computing it. Either it can be estimated utilising the entire series with its weights on the leads and lags differing over time. Or using a fixed number of lead and lag terms. The former is the general approach and is a full sample asymmetric type of filter, while the latter is a fixed length symmetric type and its moving average does not change over time, because the weights do not depend on the observations used. Moreover, for the fixed length symmetric type, the

### 5.3 Germany

number of leads and lags must be the same for the estimation on average, thus the observations at the beginning and end of the series generally cannot be computed.

We computed the CF filtering process in EViews using the following settings: Full sample asymmetry, we assume the series to be an I(1) unit root process (see appendix A.3), and no detrending mode was employed. Since we use the full sample, there is no need to choose the number of leads and lags.

The computation of the BK filter is straightforward, only the leads/lags must be chosen. In our case, it was set to 12. Besides that, no other feature must be chosen because the filter removes up to two unit roots of the data.

The Hodrick-Prescott filter published in 1997 (Hodrick & Prescott, 1997) is commonly used to smooth real GDP. It is a two-sided linear filter, i.e. it takes into consideration future and past observations, to decompose a time series  $X_t$  into a trend and cyclical variable:

$$X_t = g_t + \tau_t \quad \text{for } t = 1, \dots, T. \quad (5.1)$$

$\tau_t$  is the cyclical and  $g_t$  is the trend component. The goal is to estimate the trend component utilising the following objective function:

$$\min_{\{g_t\}_{t=-1}^T} \sum_{t=1}^T (X_t - g_t)^2 + \lambda [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2. \quad (5.2)$$

The first term minimizes the variance of the original series  $y_t$  from the new series  $g_t$ . While the second term applies a penalty for changes in the rate of  $g_t$ , and thus influences the smoothness of the outcome.  $\lambda$  is the smoothing parameter that controls the trade-off between fitting the data and smoothing the trend. If  $\lambda \rightarrow 0$  then the result would be the original series, and if  $\lambda \rightarrow \infty$  the result would be a linear time trend (Hamilton, 2018, p. 831). And the cycle component can be retrieved by subtracting the trend component from the original series.

Despite being well-established in economic literature, the HP filter bears some flaws. Hamilton points out that 1) this technique may generate spurious relations not underlying the data-generating process. 2) The values at the end of the time series to be filtered may be distorted. 3) The choice of  $\lambda$  is based on a common rule of thumb introduced by Hodrick and Prescott in their paper of 1997 and well-established in academic research, rather than a mathematical approach (Hamilton, 2018, pp. 833–835). To tackle these problems, he suggests an alternative without the aforementioned drawbacks. A simple OLS regression of the time series  $X_t$  at date  $t + h$  on a constant, and  $p$  lags of itself as of date  $t$  (Hamilton, 2018). He proposes the use of  $p = 4$  because enhancing the model with more lags or

introducing nonlinear dynamics is not necessary to extract the stationary component. The observed non-stationary time series is described as:

$$X_{t+h} = \phi_0 + \phi_1 X_t + \phi_2 X_{t-1} + \phi_3 X_{t-2} + \phi_4 X_{t-3} + \varepsilon_{t+h}. \quad (5.3)$$

Moreover, the more parameters in the model, the greater is the chance of distorting the results in small samples compared to the long-run. And the cyclical component is obtained from the residuals:

$$\hat{\varepsilon}_{t+h} = X_{t+h} - \hat{\phi}_0 - \hat{\phi}_1 X_t - \hat{\phi}_2 X_{t-1} - \hat{\phi}_3 X_{t-2} - \hat{\phi}_4 X_{t-3}. \quad (5.4)$$

Similarly to the HP filter, the parameter  $h$  must also be set. Hamilton suggests  $h = 8$  as the benchmark for quarterly data (Hamilton, 2018, p. 836).

Hodrick, later on, published a work concluding that for basic time series (i.e. no distinction between trend and cycle components) the Hamilton filter outperforms the HP and BK filter, while the HP filter achieves better results in more complex models (i.e. models with underlying cyclical and trend components) (Hodrick, 2020, p. 2). He did so by running simulations of log US GDP series, then decomposing these series using different filters, namely H, BK, and HP, and finally examining their property (Hodrick, 2020, p. 2). Additionally, in Chapter 11, he compares the USA GDP gap, based on the difference between log GDP and the potential GDP constructed by the Congressional Budget Office, with the GDP gap calculated with HP and H filters. He did not come to a conclusive result of which filter performed better in this case (Hodrick, 2020, pp. 23–24).

We compare the results of the estimations utilising the H, BK, CF, and HP filters for the German data.

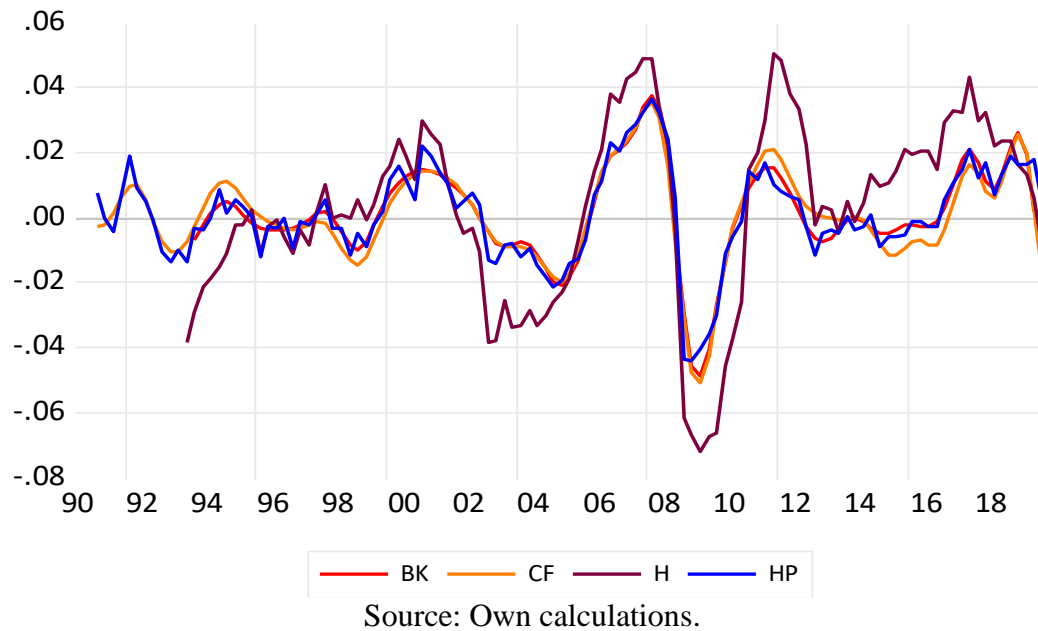
### **Filter comparison:**

The variables were already specified in section 5.2. For the estimations with CK and H, shift takes the value 2009Q3, due to the changes in the time series caused by the filters. All other variables remained unaltered. The beginning of the grid search changed for H and BK because of the reason mentioned above. For the estimation with H, shift was set to 1996Q1, and with BK to 2001Q2.

Figure 22 shows the shortening on the left side of the series when applying the H and BK filters, while the HP and asymmetric CF comprise the whole time series. Mainly the left side was not shortened (only BK lost 1 observation on the right end) because the data sets filtered include more data points than used for the estimations.

### 5.3 Germany

Figure 22: HP, H, BK, and CF filters overlapped



We conduct a compact analysis because the goal is solely to compare the outcomes. We analyse the essential statistics of the four estimations, which are presented below in table 5, without deepening into their interpretation. We follow the three rules of consistency to consider if a hysteretic effect is present in the data set.<sup>22</sup>

According to the table: 1) All four OLS (with play component) have a negative and highly statistically significant coefficient of SPURT (at least within the 10% confidence level). 2) The hysteretic variable output gap loses its explanatory power for the estimations with CF, BK, and HP. Moreover, for the estimation with BK it becomes statistically insignificant, and for the estimations with H filter the value of the coefficient increased and became positive.

3) The inequality  $|\alpha_{play} + \beta| > |\alpha_{simple}|$  clearly holds for the estimations CF and HP. While for H and BK it is also true, but close to not holding. Taking the example of the estimation done with the BK filter, the coefficient of the forcing variable is roughly -0.48, while  $SPURT + GDP\ gap_{play}$  account for -0.59, which is, in value, a small difference close to 0.11. Meaning that SPURT only took the explanatory power of the hysteretic variable but the hysteresis effect, which while existent, is inexpressive and does not contribute to explaining the relationship between GDP gap and unemployment. The estimation utilising the filter H delivers a similar result. For the results using CF and HP filters, the sums of the coefficients  $SPURT + GDP\ gap_{play}$  are 1.22 and 1.12, which are

<sup>22</sup> The three characteristics for the coefficients can be found in section 5.3.1.

significantly higher than the coefficient of the independent variable GDP gap of the regular OLS (0.3187 and 0.6757). For both estimations total effect of the variable GDP grew due to the hysteresis effect: When considering hysteresis, for the estimation CF, the total effect of GDP increased by 0.84, and for HP 0.44. In a matter of comparison, when accounting for hysteresis, the total effect on unemployment, caused by the forcing variable (GDP gap) was for the HP estimation roughly 10 times higher than in the H estimation. I.e. Hysteresis plays a much more significant role in the HP and CK estimations than for H or BK, because the total effect of the forcing variable is higher for the former two. So that, the hysteretic effect for BK and H estimations is inexpressive, and the effect of GDP on unemployment is virtually the same in both OLS (with play effect and without). On the other hand, for the estimation with CF one percent increase in GDP gap decreases on average 1.22 percentage points of unemployment while for the HP the value is 1.12.

Taking a look at the  $R^2$  values shows that all estimations utilising filters had an improvement. Nevertheless, the estimations with HP and CF performed better than H and BK by adding the spurt variable. For the estimation with CF the raise was about 0.02 percentage points, and HP 0.03. Strictly speaking, all estimations fulfil the hysteresis characteristics' requirements, but the BK and H estimations did not deliver satisfactory results, opposed to CF and HP. To underpin these findings, the coefficients of the estimations are scrutinized: For the variable GDP gap of the simple OLS, all coefficients are plausible. They show on average an underproportional response to an increase of 1% of GDP gap.

In the next step, an interpretation of the play bandwidth and SPURT graphic will be done to complete the analysis.

Table 5: Filter statistics

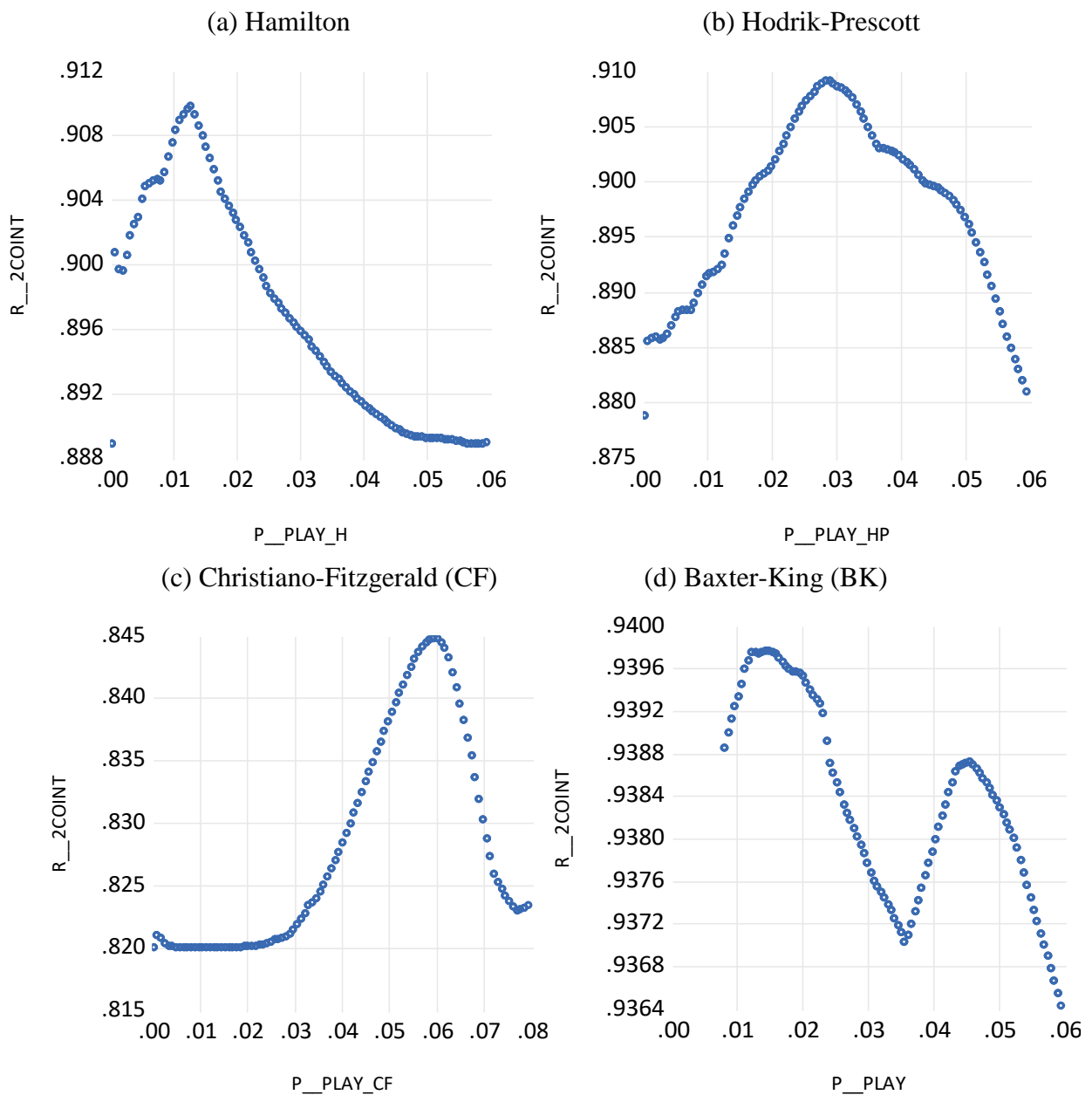
	CF	BK	H	HP
$Output\ gap_{simple}$	-0.38183***	-0.48532***	-0.23313***	-0.682618***
$SPURT$	-0.99607***	-0.38627**	-0.72705***	-0.786603***
$Output\ Gap_{play}$	-0.22558***	-0.21125	0.45152***	-0.334913***
$R^2_{simple}$	0.814323	0.937024	0.890062	0.87885
$R^2_{play}$	0.838269	0.941324	0.911226	0.90914
Play bandwidth	0.057	0.014	0.013	0.028

Source: Own calculations.

### 5.3 Germany

The starting point for the grid search was set differently for every estimation because of the divergent behaviour of the cyclical component around the potential output. For BK it was set to 2001Q2, for H to 1996Q1, and for CF to 1992Q2, while HP was set to 1992Q2. The financial crisis dummies also differ due to the different filter methods (2009Q1 for HP and BK, 2009Q3 for H and CF). They were not included in table 5, together with the seasonal variables, to focus on the coefficients related to hysteresis. The results for the play bandwidth are depicted as follows:

Figure 23: Play width for different filters



Source: Own calculations.

Figure 23 (a) H has the highest  $R^2$  when its bandwidth accounts for 0.013, meaning that an increase or decrease of 0.013 percentage points of GDP gap will trigger SPURT. This result represents an acceptable bandwidth for the German data to filter small variations effectively. Although, in relationship with a high amplitude of the cyclical component, the play width turns out to be proportionally small leading SPURT to closely follow the cyclical component (see figure 24 (a)).

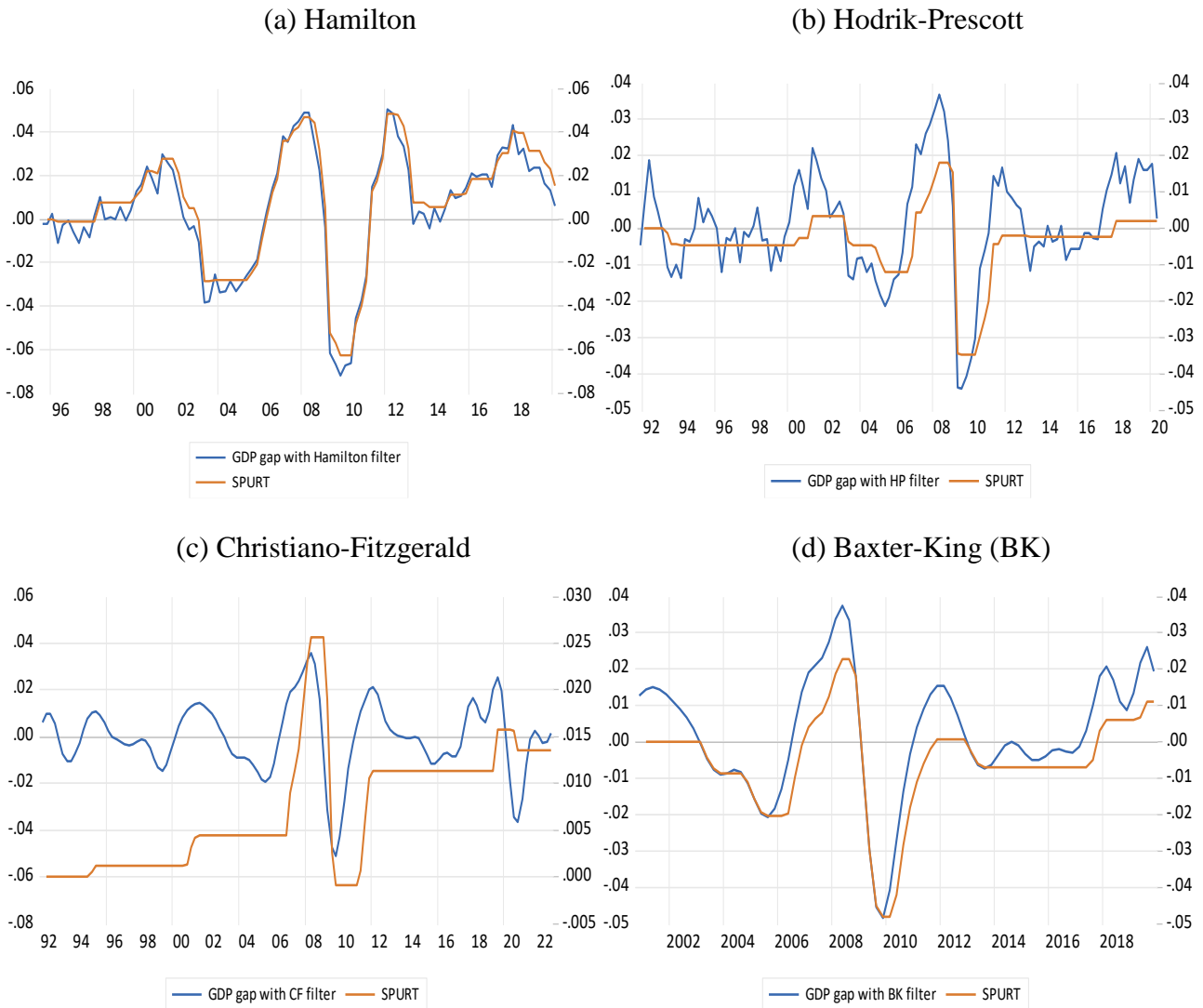
The second grid search analysed is using the CF filter. Figure 23 (c) shows a peak of  $R^2$  when the play bandwidth is 0.057, which means that the GDP gap must surpass 0.057 percentage points to enter the spurt area and have a strong effect on unemployment. The bandwidth is not compatible with real economic data because of its length. Although the rise in  $R^2$  is significant, as well as the equation  $|\alpha_{play} + \beta| > |\alpha_{simple}|$  holds perfectly, the graphic representation of spurt resembles a dummy (see figure 24 (c)) and is mainly triggered during the financial crisis. Therefore, the CF also does not deliver a positive answer to the question whether the relationship between GDP and unemployment can be better modelled using a hysteretic variable.

For the estimation with the BK filter, the bandwidth amounts to 0.014 percentage points and is a plausible value. However, as for the previous filter, the spurt path resembles a dummy (figure 24 (d)). Moreover, a low difference in  $R^2$  and almost the same value of the total effect of the independent variable (for both estimations with and without play) could be detected. These findings indicate a small improvement in the model when the play effect is accounted for.

Contrary to the three cases presented, the HP filter does fulfil all requirements of the play dynamics and its play bandwidth and SPURT variable are in accordance with the theory (as discussed in section 5.3.1). The play width value is 0.028, meaning that a GDP growth of 0.028 percentage points above the potential output is necessary to drastically affect the unemployment rate. Therefore, the HP filter best complies with all the requirements for the existence of hysteresis. For all the following analysis of this work, we employ the HP filter to determine the potential output.

### 5.3 Germany

Figure 24: Spurt and GDP variables for different filters – German data



Source: Own calculations.

### 5.3.3 Uncertainty

To enhance our findings, we add uncertainty to the play variable. Many approaches can be used as a proxy for uncertainty, since there is no objective measure of its influence on macroeconomic variables.

Choi, S., & Loungani distinguish between aggregated and sectoral uncertainty (Choi & Loungani, 2015). For the latter, they use industry-level stock volatility as a proxy for inter-sectoral uncertainty in order to capture the long-term effect of an uncertainty shock on unemployment. They based their approach on Bloom, who uses the aggregated stock market volatility as a proxy for

uncertainty (Bloom, 2009, p. 628). Bachmann and Elstner use firm-survey data (Bachmann et al., 2013), and Baker built an index based on newspaper articles (Baker et al., 2016). Leduc and Liu use consumers' survey data from the US and British industry survey data about decisions on firms' capital expenditures (Sylvain Leduc and Zheng Liu, 2012, p. 4). Jurado and collaborators base their measurement of uncertainty on the unpredictable component of macroeconomic variables (Jurado K. et al., 2015, p. 1178). We use the Economic Policy Uncertainty index (EPU) proposed by Baker.

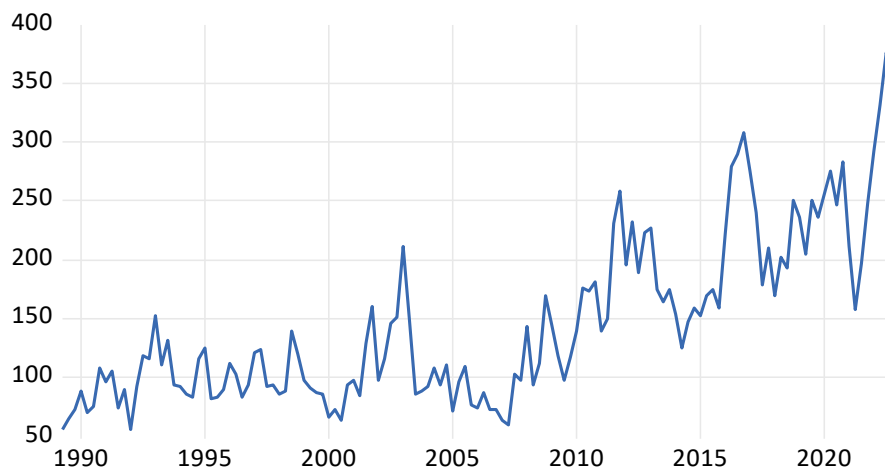
There is a version of the EPU that suits our purpose well. It takes into consideration only European newspapers' news articles regarding policy uncertainty. In total the index is based on 10 newspapers, 2 of each of the following countries: Germany, France, Italy, Spain, and the United Kingdom. The latest version of the EPU focuses on the coverage of newspaper articles and selects them by the criteria that articles must contain terms in three categories, namely "uncertainty, economy, and policy" (Baker et al., 2016).

In section 4.3.2 we defined the play width under uncertainty in eq. 4.23. The EPU index enters the equation as  $\omega_t$ . If uncertainty plays a role, it will extend the play width, otherwise, it will not have any significant change. The results for the regression for Germany are stated next.

### Estimation with uncertainty measures

Uncertainty does not influence the simple OLS estimation (i.e.  $\gamma = 0$ ), since it impacts only the play width and the spurt variable. For the sake of completeness, we present the proxy variable of uncertainty in figure 25 before comparing the results of the play estimation (table 4 in section 5.3.1) with the play estimation containing the uncertainty proxy variable (table 6).

Figure 25: Economic Policy Uncertainty index



Source: Source: Own representation based on (Baker et al., 2024).

### 5.3 Germany

Table 6: Standard OLS regression with variable play including uncertainty

Dependent variable: Unemployment unadjusted Germany

Sample: 1992Q2 2020Q2

Included observations: 113

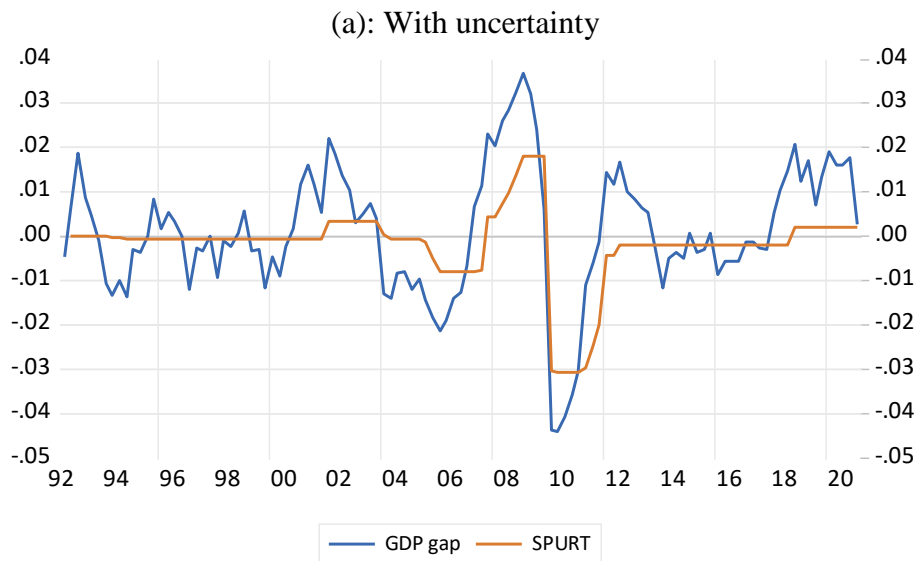
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.068594	0.003202	21.42307	0.0000
SPURT	-1.141061	0.158672	-7.191320	0.0000
GDP_GAP_DE(-1)	0.349735	0.152028	2.300471	0.0234
SHIFT(-1)	-0.053355	0.002820	-18.91698	0.0000
TREND	0.000301	4.78E-05	6.305121	0.0000
D1	0.004477	0.001904	2.350967	0.0206
D2	-0.000225	0.001890	-0.119081	0.9054
D3	-0.001342	0.001904	-0.704664	0.4826
R-squared	0.918829	Mean dependent var		0.073850
Adjusted R-squared	0.913418	S.D. dependent var		0.024206
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.356397

Source: Own calculations.

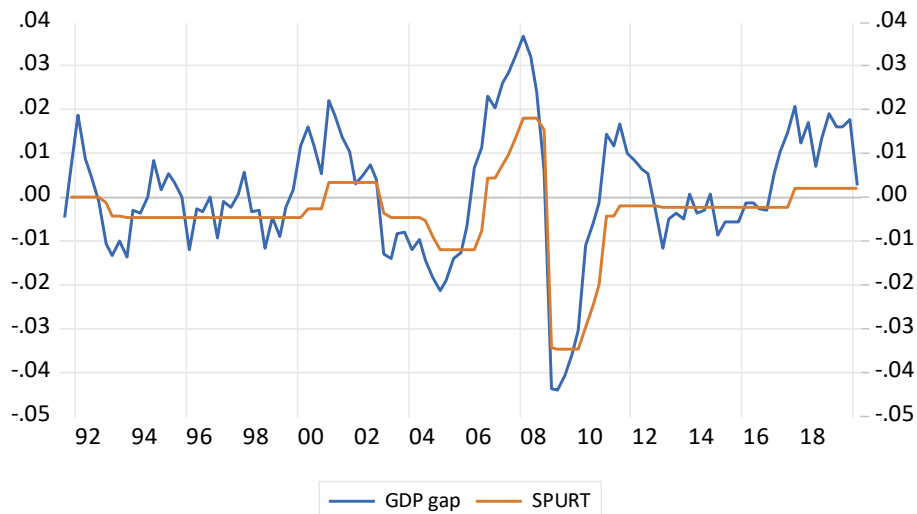
The power of explanation of the  $R^2$  is slightly better (around 1%) in the estimation with a variable play (which accounts for uncertainty) than in the estimation with a constant play (see table 4). However, the first does not significantly outperform the latter, suggesting that uncertainty does not play a major role in the relationship between unemployment and economic growth. I.e. in Germany, the firms' decision to hire and fire employees is not influenced by the uncertainty of the European policy framework. Although the coefficient's value of SPURT increased substantially, from 0.786 to 1.141, the total effect of the forcing variable decreased from 1.121 in the OLS without uncertainty to 0.792 when uncertainty was considered.

The similarity of SPURT's path between the estimation with uncertainty and without it becomes visible in figure 26, and supports the statistical summary that shows just a small change in  $R^2$ .

Figure 26: Spurt path



(b): Without uncertainty



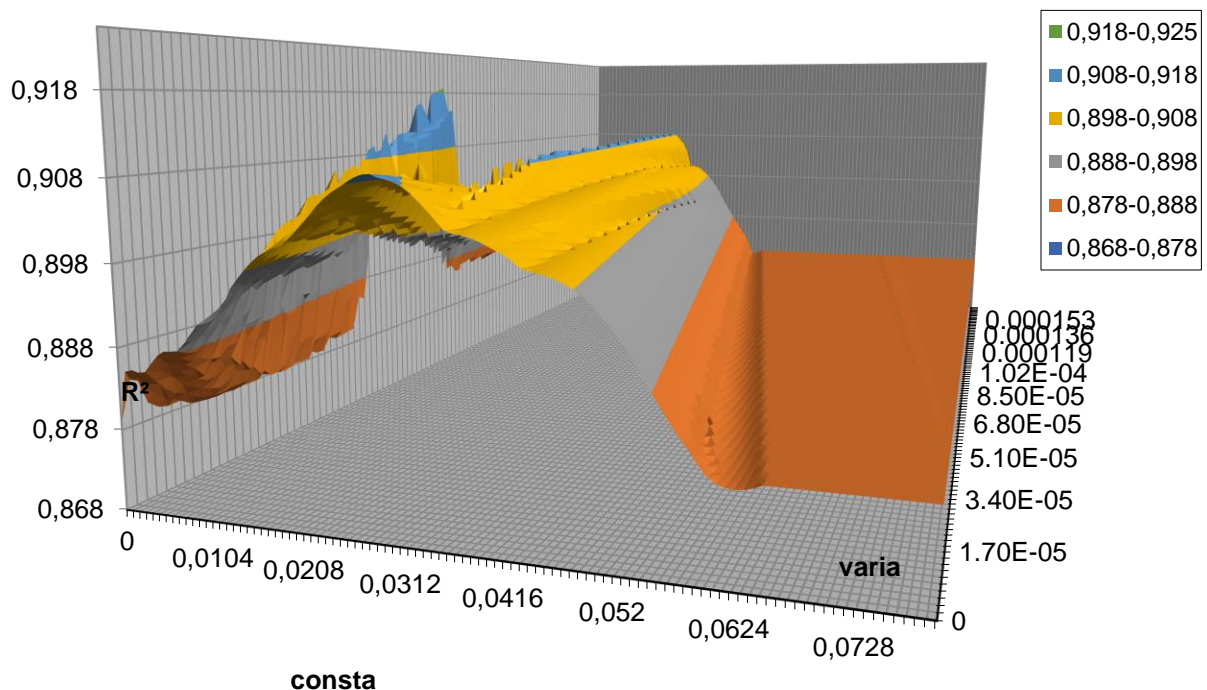
Source: Own calculations.

A visual analysis of the grid search sheds some light on the role of uncertainty in the context presented. Figure 27 shows an extended version of the usual 2-axis –graphic. The inclusion of a second variable for the grid search turns the bidimensional graphic (exhibited in the estimation with constant play) into a 3D graphic. A new layer is added each time the algorithm runs with a new value for  $\delta$ , resulting is the axis “varia”. The axis “consta” represents the constant component ( $\mu$ ) of the play width. For the value  $\delta = 0$ , the 3D representation matches the bivariate graphic, which is the first layer of the 3D graphic and equals the graphic of the constant play grid search.

### 5.3 Germany

In figure 27, we look for an increase in  $R^2$  due to uncertainty. This should appear as a “peak” somewhere in the topography of the layers for  $\delta \neq 0$ . Indeed, the highest peak for  $R^2$  values reaches 0.9188 and is situated on the edge of the graphic, where the constant play equals 0 ( $\mu = 0$ ) and the uncertainty component ( $\delta = 0.0000833$ ). A result on the edge of the graphic is not satisfactory, although possible: Through the linear combination, the variable component of play can substitute the constant component completely. It means that unemployment depends almost solely on the uncertainty proxy, since the constant play is zero. The cause for such a substitution can lie in the widening of the play width, which can occur for any reason. In our case the variable component of play may have a positive trend, because the index used to account for uncertainty only has positive values and augments over time, leading to the substitution. It would mislead the findings, since the play-width augments because of the trend in the underlying data and not on the uncertainty itself. Moreover, with an increment in  $R^2$  of solely 1%, there is no reason to consider uncertainty as an aggregator of value for the estimation, since it increases the complexity of the calculation without contributing to the  $R^2$ . Thus, we rule out the effect of the uncertainty proxy for Germany. Moreover, an in-depth analysis of the cause of uncertainty is beyond the scope of this work.

Figure 27: Grid search with variable play - Germany



Source: Own calculations.

## **5.4 EU-Countries**

In this section, we extended the analysis to 10 other countries of the European Union in order to assess whether hysteresis in the context of Okun's law is a widespread phenomenon across Europe or not. A comparison among countries also gives a better understanding of the magnitude of the hysteretic effect for each country.

The following countries were inspected: Netherlands, Austria, Belgium, Greece, Denmark, Ireland, Spain, France, Italy, and Portugal. The results were mixed, as expected, ranging from no hysteretic effect found in the data to the existence of hysteresis with wide bands of inaction. The latter case happens because some countries enjoy more protective regulations regarding job security. These protections make the labour market more rigid and hinder employees from getting easily fired, hence the play width is expected to be wider.

The data was retrieved from the Eurostat databank and treated in the same way as the previous example of Germany. The lags employed for the variables ranged from 1 to 4 quarters, although mainly one quarter delay delivered the best results. As stated in section 5.3.2 we decided to use the HP filter, based on our previous argumentation and comparison of the 4 filters for the German data. The time series' range of these 10 countries depends on the starting point of the grid search. Because it varies from country to country, so does the series' length, with the Netherlands having the shortest (2001Q1 to 2020Q1) and France the longest (1994Q2 to 2000Q2). We will take a closer look at the results for the Netherlands, Greece, and Italy in order to compare distinct outcomes regarding the presence or absence of hysteresis.

### **Estimation without play**

The procedure used for the German analysis is now repeated, although some steps are not viewed in deep again, since they were already explained. As usual, start with a simple OLS regression for the three countries, i.e. the play effect is non-existent and regress the respective unemployment of each country on the output gap. Then the optimal play width is calculated with the help of the play algorithm and SPURT is created and subsequently added in a second estimation. All estimations contain a shift, trend, and seasonal dummies (D1-D3). The regressor is seasonally adjusted, while the regressand is not. Tables 7 to 9 show the summary of the findings:

## 5.4 EU-Countries

Table 7: Standard OLS estimation - Netherlands

Dependent variable: Unadjusted unemployment Netherlands

Sample: 2001Q1 2020Q2

Included observations: 78

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.028814	0.004171	6.907725	0.0000
GDP_GAP_NE (-1)	-0.652186	0.074769	-8.722628	0.0000
SHIFT(-1)	0.007743	0.003923	1.974065	0.0523
TREND	0.000149	8.47E-05	1.763132	0.0822
D1	0.005230	0.002932	1.783684	0.0787
D2	-0.000668	0.002930	-0.228002	0.8203
D3	-0.001707	0.002967	-0.575335	0.5669
R-squared	0.603133	Mean dependent var		0.043000
Adjusted R-squared	0.569595	S.D. dependent var		0.013931
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.198958

Source: Own calculations.

Table 8: Standard OLS estimation – Greece

Dependent variable: Unadjusted unemployment Greece

Sample: 1998Q2 2020Q2

Included observations: 89

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.072444	0.011740	6.170640	0.0000
GDP_GAP_GR(-1)	-1.063285	0.133700	-7.952780	0.0000
SHIFT(-1)	0.063698	0.013570	4.693931	0.0000
TREND	0.000893	0.000266	3.357035	0.0012
D1	0.005920	0.009002	0.657609	0.5126
D2	-0.008672	0.008905	-0.973831	0.3330
D3	-0.006556	0.009003	-0.728218	0.4686
R-squared	0.810458	Mean dependent var		0.152101
Adjusted R-squared	0.796589	S.D. dependent var		0.066175
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.328897

Source: Own calculations.

Table 9: Standard OLS estimation – Italy

Dependent variable: Unadjusted unemployment Italy  
 Sample: 1999Q1 2020Q1  
 Included observations: 85

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.089529	0.006844	13.08235	0.0000
GDP_GAP_IT(-1)	-0.190680	0.124297	-1.534072	0.1291
SHIFT(-1)	0.032055	0.007367	4.351131	0.0000
TREND	-0.000193	0.000155	-1.246423	0.2163
D1	0.004358	0.004734	0.920596	0.3601
D2	-0.001885	0.004794	-0.393084	0.6953
D3	-0.009403	0.004790	-1.963152	0.0532
R-squared	0.453960	Mean dependent var		0.092235
Adjusted R-squared	0.411957	S.D. dependent var		0.020230
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.128280

Source: Own calculations.

The relationship between GDP gap and unemployment holds for the Netherlands and Greece, where a one percent increase in production leads on average to a decrease of 0.65 percentage points in unemployment in the Netherlands and 1.06 in Greece. While in Italy, the coefficient has the expected sign its coefficient is very low and the correlation of output gap and unemployment is not statistically relevant. The seasonal dummies present similar results. Only D1 is statistically significant for the Dutch data at the 10% level. The financial crisis dummy is statistically relevant for all three countries, and has a small coefficient in all estimations, meaning that the events of 2008/9 contribute in the long term (even if on a small scale) to the rise of unemployment. Another major difference is the  $R^2$  values for the estimations. As expected, the regression with Greek data shows a high  $R^2$  (0.810458), followed by the Netherlands ( $R^2 = 0.603133$ ). While the estimation for Italy ( $R^2 = 0.453960$ ) is low compared to the usual results achieved throughout this work.

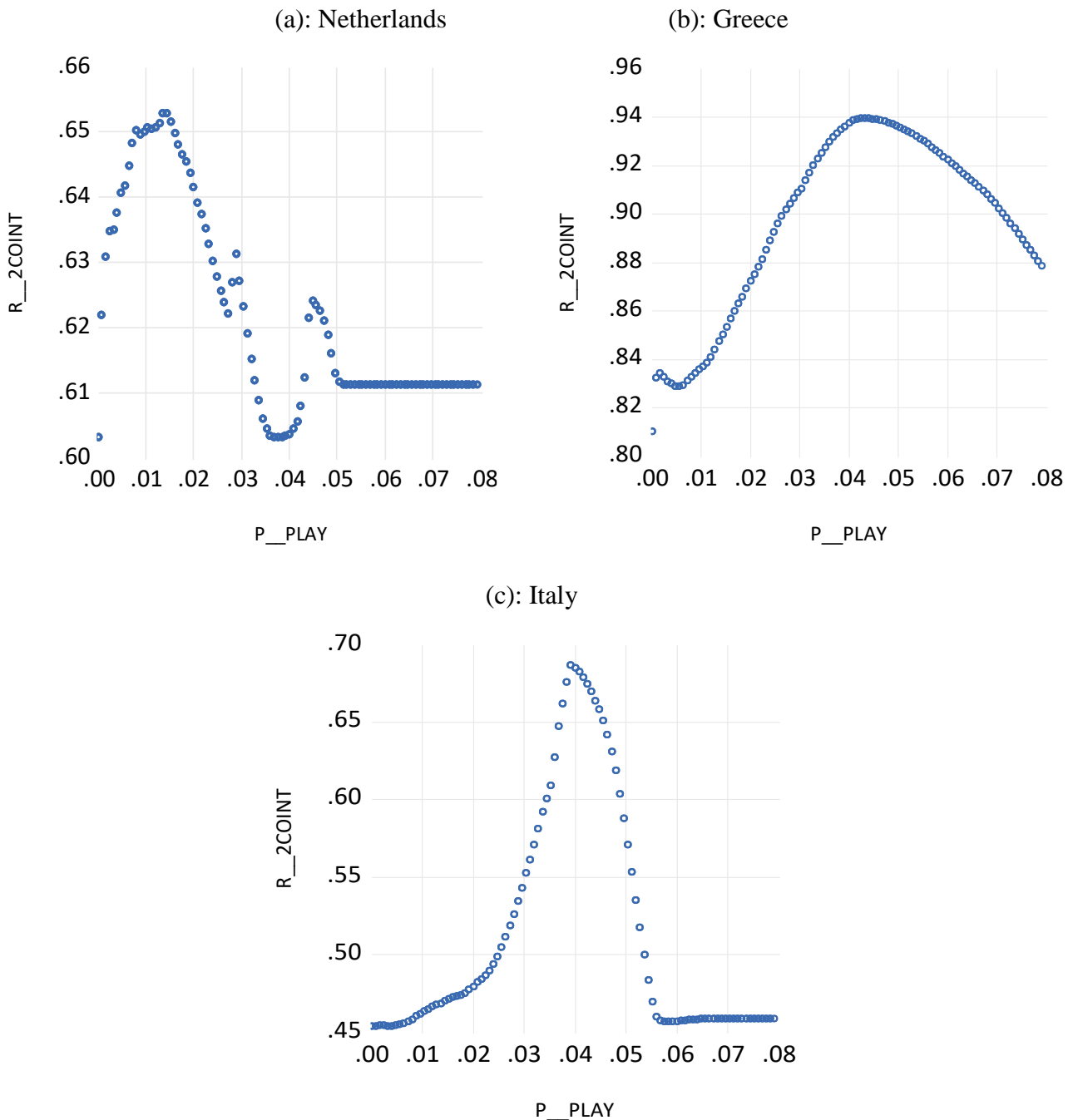
The trend variable is similar for the Netherlands and Greece: They have respectively at the 10% and 1% level of significance positive, although small coefficients. Which means that over time unemployment tends to increase. A positive trend can be interpreted as a long-term rise of unemployment over time as a change in its fundamental dynamics in the context of secular stagnation, when an economy produces under its potential capacity. The Italian result shows a negative trend, although not statistically significant.

5.4 EU-Countries

**Estimation with constant play ( $\gamma_t = \gamma$ )**

In the next step, we calculate several plays using the play algorithm in order to generate, for each of them, the artificial variable SPURT. Then estimate several OLS including SPURT and choose the optimal model based on the play bandwidth that provides the highest  $R^2$ .

Figure 28: Grid search over different values of constant play  $\gamma$



Source: Own calculations.

The very first blue dot, on the bottom left of figure 28 ((a), (b), and (c)) represents the simple OLS without the play component ( $\gamma = 0$ ). Starting from this first point the bandwidth systematically increases at a regular pace of 0.0008, because the maximal bandwidth was set to 0.08 and then divided into 100 tranches for a higher precision of the grid search. The play effect is filtered from the hysteretic variable for each play bandwidth and a SPURT variable is generated as a result. Thus, during this grid, 100 different paths of the SPURT variable based on 100 different play-widths were created. Next, the algorithm estimates, in this case, 100 new OLS models, each including an artificial variable. The shapes in figure 28 for each country show the combination of the corresponding  $R^2$  of each estimation and the belonging bandwidth. The highest  $R^2$  for Greece, the Netherlands, and Italy are achieved when the bandwidths have the values 0.043, 0.014, and 0.039 respectively. I.e. an increase of capacity utilisation of respectively 4.3%, 1.4%, and 3.9% would trigger the SPURT variable.

A high bandwidth for southern European economies is plausible. Italy and Greece have regulations that ensure high employment protection, raising the burden to terminate an employment contract. Thus, the play width for both is as expected.

The scatter plot of play bandwidth and  $R^2$  of the Netherlands also shows a favourable picture, although it differs from the former two. For the Netherlands, the bandwidth of 0.014 slightly over 1 percentage point is plausible, because the Dutch labour market is one of the most flexible within Europe. One of the reasons is the high share of employees with temporary/flexible contracts and self-employed persons as a percentage of total employment. According to the data from the Eurostat database, in the Netherlands, 22.9% of employees had temporary contracts in 2022Q4.<sup>23</sup> For a matter of comparison, in Germany only 11.6% of employees had a temporary contract during the same period, while in Italy 13% and Greece 6.4% (Eurostat, 2022a, 2022b). With flexible contracts, employers can more easily fire and hire new employees, e.g. during economic turmoil or peak seasons. Italy and Greece have been liberalizing their labour markets since the early 1990s and this path was reinforced by the global financial crisis. Based on the three constant plays, for Greece  $\gamma = 0.043$ , Netherlands  $\gamma = 0.014$ , and Italy  $\gamma = 0.039$ , tables 10 to 12 display the estimation results of the spurt/play regressions with an artificial spurt variable.

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<sup>23</sup> We calculated the percentages using tables "Employment by sex, age and citizenship (1 000)" and "Temporary employees by sex, age and economic activity".

## 5.4 EU-Countries

Table 10: Standard OLS estimation with play – Netherlands

Dependent variable: Unadjusted unemployment Netherlands

Sample: 2001Q1 2020Q2

Included observations: 78

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.020680	0.004692	4.407533	0.0000
SPURT	-0.806404	0.254357	-3.170361	0.0023
GDP_GAP_NE (-1)	-0.036265	0.206642	-0.175496	0.8612
SHIFT(-1)	0.011207	0.003852	2.909065	0.0049
TREND	2.14E-05	8.94E-05	0.239442	0.8115
D1	0.005530	0.002763	2.001397	0.0492
D2	-0.000197	0.002763	-0.071193	0.9434
D3	-0.001677	0.002794	-0.600406	0.5502
R-squared	0.652963	Mean dependent var		0.043000
Adjusted R-squared	0.618259	S.D. dependent var		0.013931
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.250735

Source: Own calculations.

Table 11: Standard OLS estimation with play with play – Greece

Dependent variable: Unadjusted unemployment Greece

Sample: 1998Q2 2020Q2

Included observations: 89

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.101648	0.007031	14.45683	0.0000
SPURT	-2.329494	0.177135	-13.15093	0.0000
GDP_GAP_GR(-1)	0.495681	0.140801	3.520446	0.0007
SHIFT(-1)	0.088830	0.007945	11.18121	0.0000
TREND	0.000338	0.000157	2.155310	0.0341
D1	0.004694	0.005116	0.917422	0.3616
D2	-0.006809	0.005062	-1.345059	0.1824
D3	-0.008450	0.005118	-1.650976	0.1026
R-squared	0.939543	Mean dependent var		0.152101
Adjusted R-squared	0.934318	S.D. dependent var		0.066175
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.691498

Source: Own calculations.

Table 12: Standard OLS estimation with play – Italy

Dependent variable: Unemployment unadjusted Italy

Sample: 1999Q1 2020Q1

Included observations: 85

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.103205	0.005517	18.70751	0.0000
SPURT	2.246164	0.296434	7.577272	0.0000
GDP_GAP_IT(-1)	-0.517957	0.104072	-4.976935	0.0000
SHIFT(-1)	0.059716	0.006695	8.919710	0.0000
TREND	-0.000554	0.000127	-4.354136	0.0000
D1	0.003682	0.003607	1.020816	0.3105
D2	-0.001353	0.003653	-0.370433	0.7121
D3	-0.009919	0.003649	-2.718073	0.0081
R-squared	0.687200	Mean dependent var		0.092235
Adjusted R-squared	0.658763	S.D. dependent var		0.020230
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.604811

Source: Own calculations.

Starting with the estimation for the Netherlands: The artificial variable has a high statistically significant coefficient of -0.80. Its value is higher than the output gap's coefficient of the simple OLS (-0.64163). I.e. steep movements of output gap, now represented by SPURT, have a strong negative correlation with unemployment. In fact, the power explanation of GDP gap is now depicted mainly by SPURT, turning the coefficient of GDP gap in table 10 inexpressive. The semi-elasticity of SPURT's coefficient means that a one percent increase in GDP gap, when the play bandwidth of 0.014 is trespassed, decreases unemployment on average by 0.80 percentage points. The sum of both coefficients (-0.8427) is higher in value than the GDP gap coefficient (-0.6521) in table 7. This result indicates that a hysteretic relationship between output gap and unemployment exists, and it is better explained by a hysteretic variable. With a minor change in the p-value of D1, shift and seasonal dummies are virtually the same when compared with the estimation without the play effect. The only difference is the trend variable, which became statistically insignificant in the regression with spurt. The trend effect was partially responsible for the upward movement of the unemployment rate, which is now due to the hysteretic effect. The inclusion of SPURT also raised the  $R^2$  coefficient to 0.933348, i.e. about 5% more of the variance could be explained by the forcing variables.

The Greek regression presents similar results: The coefficient of SPURT (-2.329494) is highly statistically significant, almost three times the coefficient of Netherlands (-0.806404), and roughly

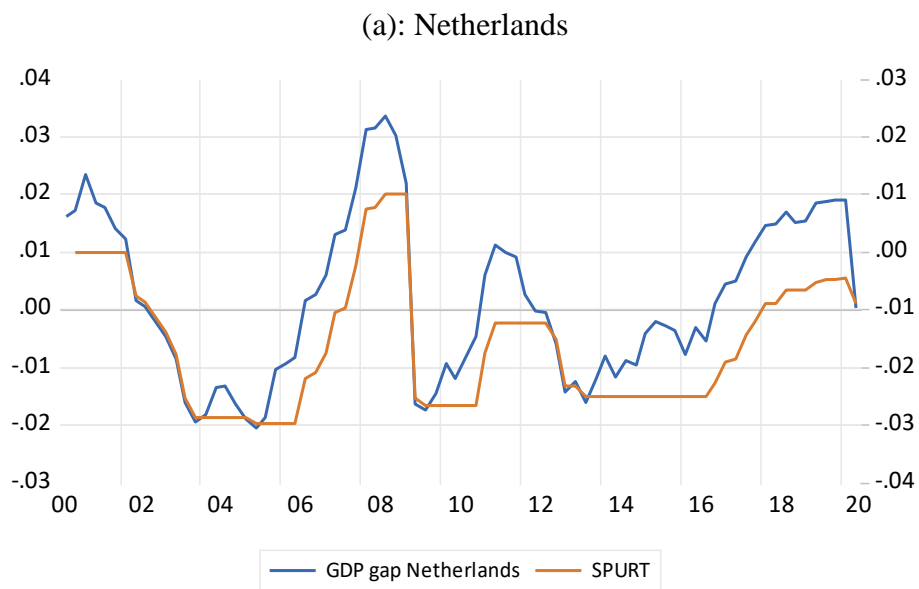
## 5.4 EU-Countries

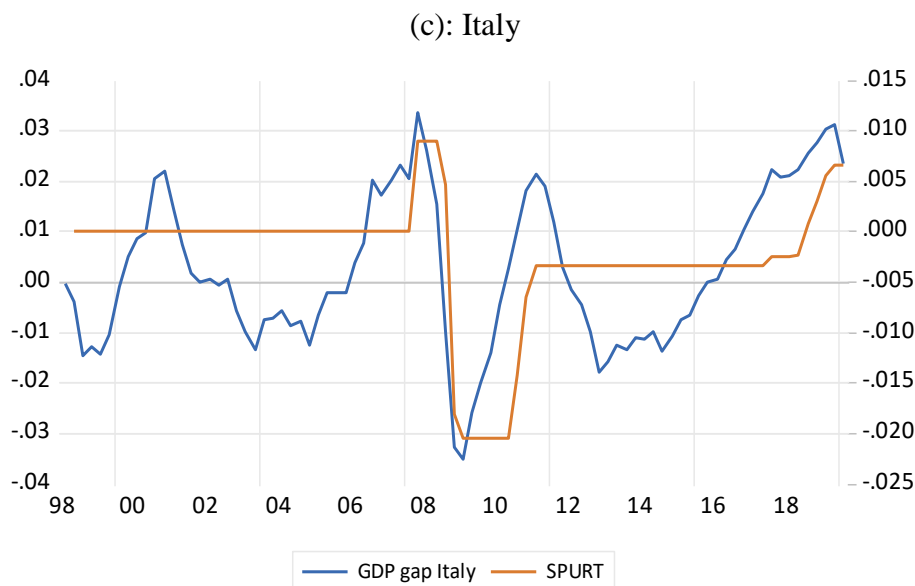
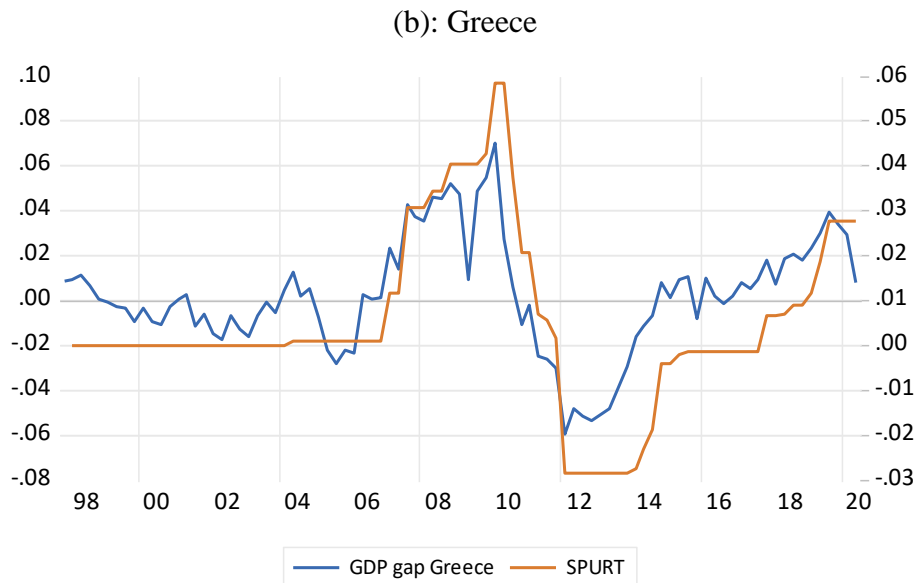
two times the coefficient of GDP gap from the estimation without play (-1.0633). Such a high coefficient is only expected when play width has a high value, as is the case. Adding the SPURT coefficient and the GDP gap coefficient of the same OLS results in the value -1.8337, which is still higher in value than the coefficient of GDP gap of the simple OLS (-1.0632).

The results for Italy with a play component do not deliver the same conclusion. The sign of the SPURT coefficient is positive. In this case, an increase in capacity utilisation would increase unemployment, which is not in accordance with the theory. Moreover, the sum of the coefficients of SPURT (2.246164) and GDP GAP (-0.517957) is positive (1.7288), thus the overall effect remains positive, while a negative effect would be in accordance with the theoretical expectations. As a last remark, this result is higher than the coefficient of GDP gap of the simple OLS (-0.19068), concluding that no hysteresis effect could be found in the data. All other variables remained virtually the same for the Italian estimation.

The last part of the analysis is the visual inspection of the graphic depicting the variables SPURT and output gap for the three countries in figure 29.

Figure 29: Spurt and GDP gap





Source: Own calculations.

In 29 (c) the path of SPURT for the Italian data is delineated. The hysteretic variable spread ranges from -0.03 to 0.034 roughly, while the play bandwidth is 0.039. Consequently, only relatively high changes in output gap will not be filtered out and SPURT has a similar shape to a dummy variable. Therefore, we cannot empirically observe a clear separation between the structural break of the Euro crisis and hysteresis. On the contrary, for figure 29 (b) one can see that for a bandwidth of 0.043 Greece's spurt variable has a smoother path. It is because the output gap's maximum span is about 13 percentage points, so SPURT does not resemble a dummy. Figure 29 (a) shows that the hysteretic

#### 5.4 EU-Countries

variable has a similar range as for Italy (figure 29 (c)). But in the case of the Netherlands, the play bandwidth is considerably smaller (0.014), and therefore a common pattern for the spurt variable is the result.

Regarding all other countries, the main results of all estimations can be seen in table 13. As stated in section 5.3.1, the hysteretic play dynamics exists for any two pairs of variables, here output gap and unemployment, if it fulfils three characteristics: 1)  $\beta < 0$ . 2) A weaker (or even insignificant) effect of the hysteretic variable when estimated in the same equation as SPURT.

3) The inequality ( $|\alpha_{play} + \beta| > |\alpha_{simple}|$ ) holds. Moreover, an interpretation of the play bandwidth and SPURT's graphic must back up the results of the estimation.

Regressions that do not correspond with the first characteristic are marked with grey shading. France and Italy do not cope with the first requirement. The second property is violated by Portugal, since the coefficient's value of GDP gap solely dropped from  $|-1.2247|$  to  $|-0.9417|$ , and the coefficient of the spurt variable is smaller than the coefficient of GDP gap in the estimation with play. The third aspect holds for Austria, Belgium, Denmark Greece, Ireland, Spain, and the Netherlands. The high bandwidth for the Irish data (0.074) rules out the existence of hysteresis in the data set. Therefore, countries that are in line with "typical" play-dynamics are Austria, Belgium, Denmark, Spain, Greece, and the Netherlands.

Table 13: Overview results for European countries

Austria	Belgium	Denmark	Greece	France
$\alpha_{simple} = -0.365105^{***}$ $\alpha = 0.1346$ $\beta = -0.5653^{***}$ $\gamma = 0.007$ $R_{simple}^2 = 0.7768$ $R^2 = 0.821025$	$\alpha_{simple} = -0.5532^{***}$ $\alpha = -0.3688^{***}$ $\beta = -0.5337^{***}$ $\gamma = 0.03$ $R_{simple}^2 = 0.5165$ $R^2 = 0.5621$	$\alpha_{simple} = -0.4349^{***}$ $\alpha = -0.2048^{***}$ $\beta = -0.7374^{***}$ $\gamma = 0.039$ $R_{simple}^2 = 0.6723$ $R^2 = 0.8120$	$\alpha_{simple} = -1.0632^{***}$ $\alpha = 0.4956^{***}$ $\beta = -2.3294^{***}$ $\gamma = 0.038$ $R_{simple}^2 = 0.8104$ $R^2 = 0.9395$	$\alpha_{simple} = -0.1527^*$ $\alpha = -0.4001^*$ $\beta = 0.3243$ $\gamma = 0.01$ $R_{simple}^2 = 0.6556$ $R^2 = 0.6615$
Italy	Spain	Portugal	Ireland	Netherlands
$\alpha_{simple} = -0.1906$ $\alpha = -0.5179^{***}$ $\beta = 2.2461^{***}$ $\gamma = 0.039$ $R_{simple}^2 = 0.4539$ $R^2 = 0.6872$	$\alpha_{simple} = -1.7645^{***}$ $\alpha = -0.8285^{***}$ $\beta = -3.3798^{***}$ $\gamma = 0.0336$ $R_{simple}^2 = 0.806$ $R^2 = 0.8514$	$\alpha_{simple} = -1.2247^{***}$ $\alpha = -0.9417^{***}$ $\beta = -0.6378^*$ $\gamma = 0.037$ $R_{simple}^2 = 0.7322$ $R^2 = 0.7452$	$\alpha_{simple} = -0.1659^{**}$ $\alpha = 0.2464^{**}$ $\beta = -0.6802^{***}$ $\gamma = 0.074$ $R_{simple}^2 = 0.6908$ $R^2 = 0.7790$	$\alpha_{simple} = -0.6521^{***}$ $\alpha = -0.0362$ $\beta = -0.8064^{***}$ $\gamma = 0.014$ $R_{simple}^2 = 0.6031$ $R^2 = 0.6529$

Source: Own calculations.

# Chapter 6

## Modified Okun’s law and hysteresis: An empirical analysis

The variables employment and gross value added (GVA) are used to assess for hysteretic behaviour in European countries, instead of the usual variables unemployment and real GDP. We expect a more prominent effect of hysteresis in the modified version of Okun’s law.

### 6.1 Data and variables

The quarterly data were retrieved from the Eurostat database and treated in the same way as the data presented in section 5.2.

**Employment:** It covers all persons engaged in productive activity that falls within the domestic territory (Eurostat, 2013, p. 307). This is the “domestic concept” of Eurostat’s national accounts. Moreover, workers are defined as employees (working by agreement for another resident unit and receiving remuneration) or self-employed (owners of unincorporated enterprises) (Eurostat, 2024a).

The motivation to use employment and examine beyond Okun’s law traditional relationship between output gap and unemployment relies on the definition of the latter: It is calculated based on the relationship between employment and workforce:

$$unemployment = 1 - \frac{employed\ people}{workforce}. \quad (6.1)$$

According to Eurostat the term workforce “includes both employed (employees and self-employed) and unemployed people, but not the economically inactive, such as pre-school children, school children, students and pensioners.” (Eurostat, 2024c).

An equal rise of both variables on the right equation’s side would keep the unemployment unaltered. For example, a rise in output gap would activate part of the “hidden reserve” augmenting the workforce and employed persons simultaneously. Assuming both effects have the same intensity, unemployment would remain unaltered and the effect of the independent variable would not appear,

despite its existence. Another reason for an augmentation of both variables on the right-hand side of the equation is population growth. Over the years, a larger population tends to produce a higher output, thus a higher GDP /GVA growth is caused by a higher number of employees working, while unemployment can remain unchanged, when the numerator “employed people” and denominator “workforce” grow at the same pace. By using employment as a hysteretic variable, we want to eliminate some compensation effects that may neutralize the rise of the forcing variable (GDP or GVA).

We use employment based on total employment (employees + self-employed people) domestic concept (employment production units within the country boundaries) and it is covered by quarterly, not seasonally or calendar adjusted data. Employment is measured in the units “1000 persons” and “1000 hours worked”.

**Gross value added:** It is defined as Output (P.1) at basic prices less intermediate consumption at purchaser prices (P.2). P.1 and P.2 are the respective acronyms in the accounts' system. The advantage of GVA over GDP at market place used in chapter 5 is that the latter incorporates taxes and subsidies, while GVA does not. Since we are interested only in the capacity of firms, there is no need to add taxes and discount subsidies in the calculation because they do not directly influence the output. GVA figures are of quarterly frequency, chain linked volumes measured in millions of Euros with a basis year of 2015. The data is seasonally and calendar adjusted, and the total gross value added of a country is the sum of the GVA of each of its industries for a given time period.

**GVA gap:** It is derived from GVA and used in the statistical estimations. The procedure is the same as in section 5.2: An HP filter is applied to the data to obtain the long-term trend development of the potential output.

As a last remark, the spurt and seasonal variables remain the same as in section 5.2.

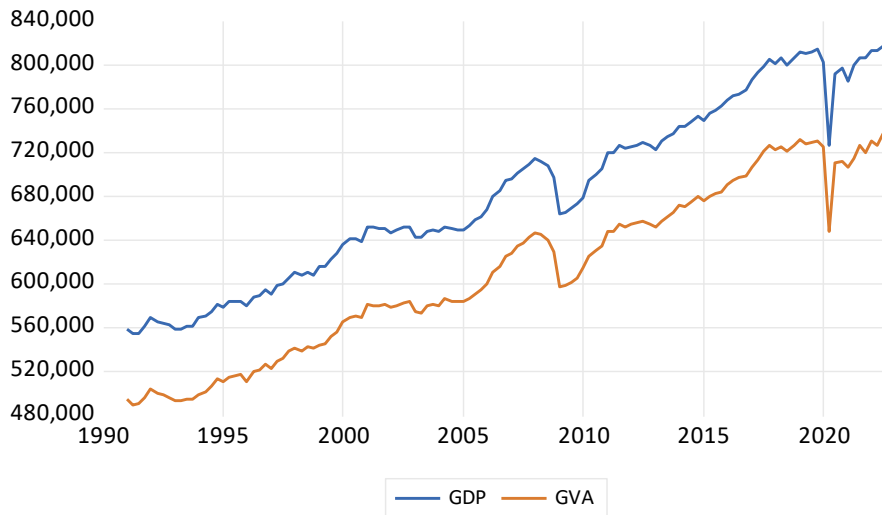
## **6.2 Main findings Germany**

The dependent variable is now GVA, presented in figure 30 alongside GDP, for the sake of completeness. In order to observe relative changes, GVA gap and employment must be logarithmized. Figure 31 shows that the deviations of GDP and GVA from their long-run trend follow a similar path. Note that the scales of the vertical axis are the same for GDP as for GVA figures. Therefore, the use of one or another variable will not substantially change the outcome. Nevertheless, it is more accurate to use the GVA gap as a proxy for potential capacity utilisation than the GDP gap at market price, due

## 6.2 Main findings Germany

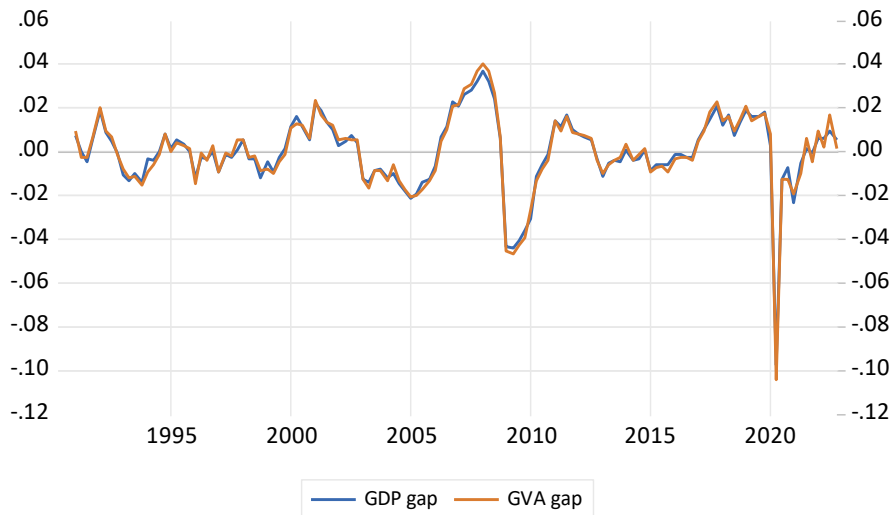
to the inclusion of net indirect taxes, such as Value Added Tax (VAT), that can distort the real value of produced goods and services of a country within a denominated frame period.

Figure 30: GVA and GDP - Germany



Source: Own representation based on (Eurostat, 2024b).

Figure 31: GVA gap and GDP gap - Germany



Source: Own calculations with data from (Eurostat, 2024b)

We substitute the current independent variable unemployment for employment. Since it can be measured by the total number of persons employed or total hours worked, we contrast both for the German data. For both models we estimated several regressions with different lags for GVA, varying from one to four quarters in order to improve the quality of the estimations and to avoid reverse

causation<sup>24</sup>. The estimations with a lag of one quarter presented the best outcome. Moreover, the HP filter was used to filter the series. We start with the linear estimations.

**Estimations without play:**

Table 14: Standard OLS estimation - Hours

Dependent variable: Employment total hours Germany

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	16.50677	0.004656	3545.303	0.0000
GVA_GAP_DE(-1)	0.883471	0.107643	8.207437	0.0000
SHIFT	0.047238	0.006373	7.411759	0.0000
TREND	-0.000152	9.61E-05	-1.585997	0.1157
D1	0.000826	0.004286	0.192753	0.8475
D2	-0.072035	0.004251	-16.94513	0.0000
D3	0.007832	0.004286	1.827457	0.0704
R-squared	0.865733	Mean dependent var	16.50130	
Adjusted R-squared	0.858133	S.D. dependent var	0.042564	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.683270	

Source: Own calculations.

Table 15: Standard OLS estimation - Persons

Dependent variable: Employment total persons Germany

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.53382	0.003561	2958.263	0.0000
GVA_GAP_DE(-1)	0.503452	0.080249	6.273634	0.0000
SHIFT	0.026211	0.004721	5.551773	0.0000
TREND	0.001251	7.12E-05	17.58189	0.0000
D1	-0.013684	0.003335	-4.103030	0.0001
D2	-0.006372	0.003309	-1.925352	0.0569
D3	-0.002662	0.003335	-0.798090	0.4266
R-squared	0.951571	Mean dependent var	10.61144	
Adjusted R-squared	0.948830	S.D. dependent var	0.055152	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.190815	

Source: Own calculations.

<sup>24</sup> See section 5.3.1.

## 6.2 Main findings Germany

For the sake of clarity, we differentiate the estimations by addressing them as estimation “hours” and “persons” depending on the unit used for employment.

In both regressions the coefficient of the forcing variable is positive and statistically significant as the theory foresees: An increase in gross value added augments employment. In chapter 5 we had the opposite case. The coefficient of GVA gap in “hours” is 0.883471, and is higher than the coefficient in estimation “persons”, which has the value 0.573541. A possible explanation for this result is that the output increase leads firstly to an increase in overtime hours before the engagement of new personnel, therefore its effect is higher compared to the case when employment is measured solely in how many persons were hired for a new job. But ultimately, if the output increases substantially, new employees are hired since overtime hours are not sufficient to cover the demand.

In both cases the dummy for the global financial crisis, represented by shift, has a positive yet small impact on employment in the long-run. It is also consistent with the estimation of shift in chapter 5, since the signs are the opposite. Shift was set at 2009Q2 for the estimation “hours” and at 2009Q3 for “persons”, because the amount of hours worked can be more quickly adjusted amid a turmoil situation in some industries before laying off employees.

The trend variable is not statistically significant for estimation “hours”, while “persons” has a small yet statistically highly significant coefficient. D2 and D3 are statistically significant, at least at the 10% significance level, for the estimation “hours”, while D1 and D3 are for “persons”. In the next step, the play width will be generated using the play algorithm.

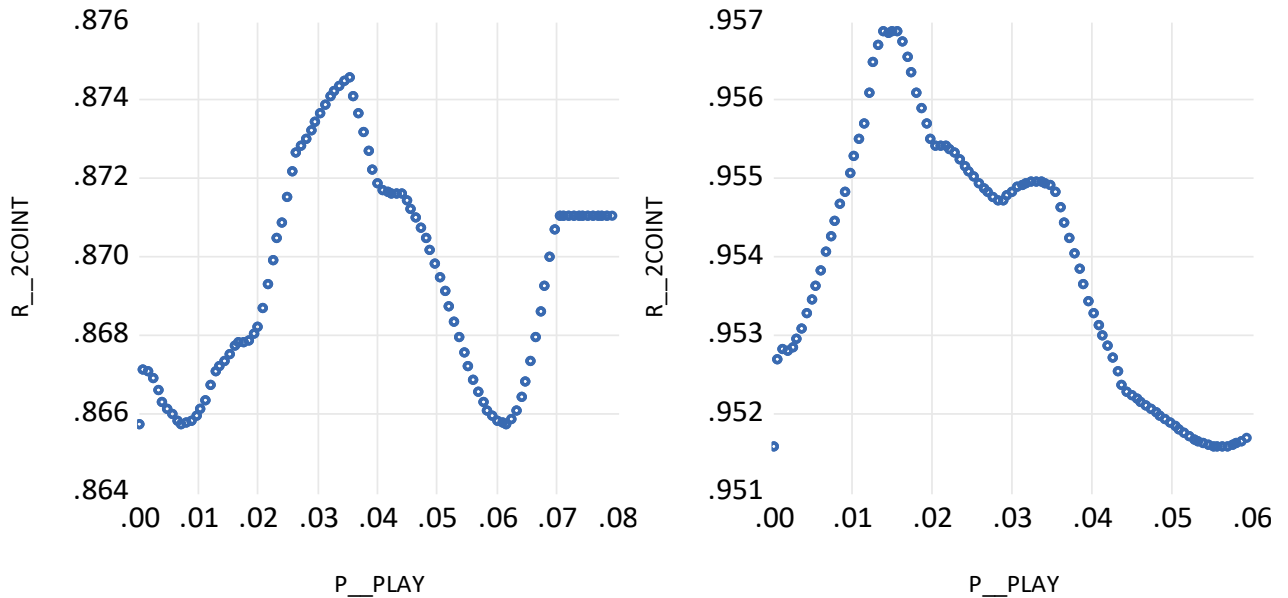
### **Estimation with play:**

The standard length for the grid search of 0.08 points was shortened for the estimation “persons” due to a spike by 0.07, which is not plausible and could not be interpreted in the hysteresis context. Thus, we limited the grid search for “persons” to a play width of a maximum of 0.06 points.

The left graphic of figure 32 depicts a mountain shape, typically encountered when hysteresis effect is present. The ending of the graphic shows blue dots with  $R^2$  values lower than the peak, another piece of information suggesting the existence of hysteresis instead of an extraordinary event represented by a dummy. The highest  $R^2$  is reached for a bandwidth of 0.035. It means that until the play width does not reach the peak, the adjustment is made by overtime hours or through part-time and short-time workers who increase their work hours. Overtime intensifies the hysteresis effect because it allows some degree of flexibility, widening the play width. Just when the full length of the band of inaction is passed, spurt is triggered, i.e. a new employee is hired/fired, because the work

hours' capacity is depleted. Therefore, measuring employment in hours generates a stronger hysteresis effect.

Figure 32: Grid search for estimations “hours” (left) and “persons” (right )



Source: Own calculations.

On the other hand, using “employment” eliminates distortions caused by the hidden reserves and population growth leading to a shorter band of inaction, and thus lesser hysteresis effects. Comparing this result with the play width of unemployment in section 5.3.1, where Germany had a play width of 2.8 % points, the play width of “hours” is 0.007 points higher. Suggesting that accounting for overtime widens the band of inaction more than it is shortened by eliminating the effects of population growth and hidden reserves. As a result, the band of inaction becomes overall wider.

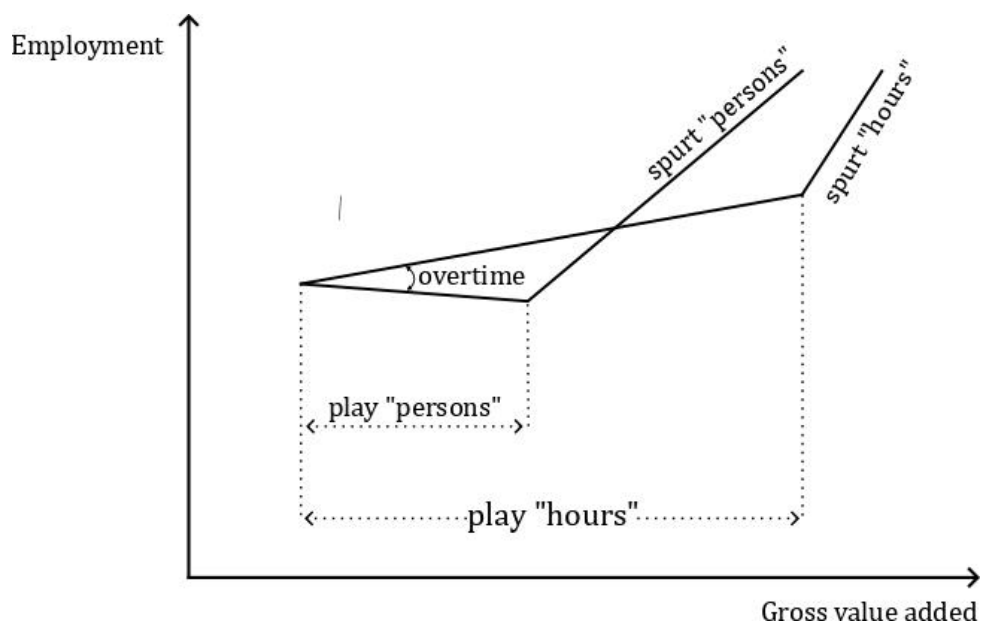
Figure 32 right graphic depicts a grid search for the estimation “persons”. It has the same shape as the estimation “hours”. The play width is 0.016 clearly under the previous result for play width (0.035). The reason may be due to the elimination of the two effects, as explained above, and the absence of overtime in the play area. For the estimation “persons” if GVA gap surpasses (falls short) 0.016 points or more, SPURT is triggered.

Figure 33 is the graphical representation of the system, we take into account the slopes for both estimations (retrieved from tables 16 and 17) in order to underpin our previous arguments. The slope of the play variable ( $\alpha$ ) for the estimation “person” has a small absolute coefficient  $|-0.159310|$ ,

## 6.2 Main findings Germany

compared to 0.599 of the estimation “hours”, due to the overtime that was accounted for in the latter. Moreover, the spurt effect ( $\beta$ ), in addition to the steepness of the play area, is quite high for the equation “persons” (0.834466), suggesting the hiring/firing effect is mostly captured by SPURT. This is not the case for the estimation “hours”, where the coefficients  $\alpha$  and  $\beta$  have similar values, and  $\alpha$  only partially loses its explanatory power to SPURT.

Figure 33: Play hysteresis for “persons” and “hours”



Using the play width with the highest  $R^2$ , an OLS for each case is estimated:

Table 16: Standard OLS with play - Hours

Dependent variable: Employment total hours Germany  
 Sample: 1992Q2 2020Q2  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	16.50946	0.004629	3566.497	0.0000
SPURT	0.779371	0.286615	2.719228	0.0077
GVA_GAP_DE_H(-1)	0.599023	0.147886	4.050576	0.0001
SHIFT	0.057772	0.007302	7.912078	0.0000
TREND	-0.000238	9.84E-05	-2.413793	0.0175
D1	0.000314	0.004166	0.075420	0.9400
D2	-0.072530	0.004132	-17.55160	0.0000
D3	0.007784	0.004162	1.870123	0.0643
R-squared	0.874567	Mean dependent var	16.50130	
Adjusted R-squared	0.866204	S.D. dependent var	0.042564	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.698216	

Source: Own calculations.

Starting with the estimation using worked hours: The conditions that the forcing variable and SPURT must comply with to indicate a hysteretic effect are not completely satisfied. The coefficient of SPURT is positive, statistically significant, and it only partially takes the explanatory power of GVA gap. Therefore, the first condition is given. However, despite being positive, GVA gap's coefficient stays almost as high as before, and statistically relevant, contrary to the second condition. Their sum amounts to  $0.779371 + 0.599023 = 1.3783$  which is greater than the GVA gap coefficient of the estimation without SPURT (0.883471). Hence, only two out of three conditions regarding the forcing and the spurt variables for the existence of hysteresis are, strictly speaking, present. Shift is positive and significant while the trend variable has a negative sign, which is consistent with the sign of the trend variable in the estimation for Germany in section 5.3.1.

## 6.2 Main findings Germany

Table 17: Standard OLS with play - Persons

Dependent variable: Employment persons Germany

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.54496	0.004578	2303.306	0.0000
SPURT	0.834466	0.231856	3.599071	0.0005
GVA_GAP_DE_P(-1)	-0.159310	0.199243	-0.799576	0.4258
SHIFT	0.029396	0.004562	6.443340	0.0000
TREND	0.001210	6.85E-05	17.66554	0.0000
D1	-0.013870	0.003162	-4.386254	0.0000
D2	-0.006319	0.003137	-2.014253	0.0465
D3	-0.002550	0.003162	-0.806393	0.4218
R-squared	0.956890	Mean dependent var	10.61144	
Adjusted R-squared	0.954016	S.D. dependent var	0.055152	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.252152	

Source: Own calculations.

The results for the estimation using the logarithm of employment in “persons” as the dependent variable achieve better results. Starting with the necessary coefficient conditions for hysteresis: SPURT is positive, slightly higher (0.834466) than SPURT’s coefficient of the estimation “hours” (0.779371). And as theoretically expected, it took all the explanatory power of the hysteretic variable GVA gap, which became statistically insignificant and negative. Their sum (0.834466) + (-0.159310) equals 0.6751 and is higher than the coefficient of GVA gap of the estimation without the play dynamics (0.503452). Therefore, all three conditions are met. Shift and trend have a positive sign and are statistically significant, although both have relatively low coefficients. The seasonal effects were controlled by the seasonal dummy variables, from which only D1 and D2 are statistically significant at the 1% and 5% level, respectively.

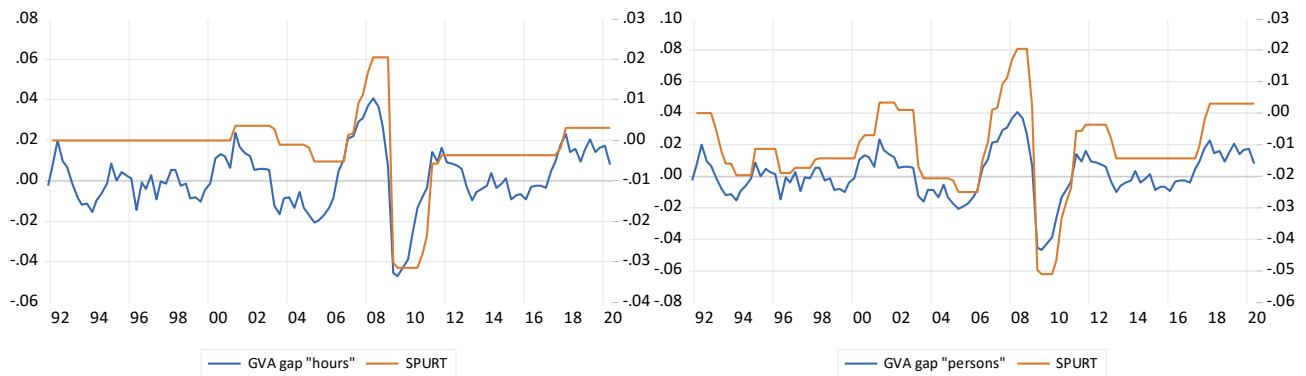
Figure 34 (left) the path of SPURT for regression “hours” is consistent with the pre-sets of the estimation: The grid search starts on a maximum (1992Q2), and, a local maximum follows. SPURT increases from 0 to 0.004 in 2001Q2 and does not suffer any change until 2003Q2, falling to 0.003. For this time frame the GVA gap changed from 0.023 to -0.013, a spread of 0.036, which is consistent with the bandwidth. The rest of the graphic also maintains the consistency between the ups and downs of the forcing variable and the response of SPURT.

The path of the spurt variables in estimation “persons” on figure 34 (right) is also in accordance with our expectations. SPURT starts with the value 0, and the first maximum of GVA gap is set at

1992Q2 with the value 0.02. In 1994Q1 GVA gap then falls in total to -0.015, resulting in a span of 0.035, of which 0.016 points are movements along the play width and 0.019 were integrated in the construction of SPURT. This behaviour of SPURT is consistent with the size of the band of inaction and with the sets chosen to create the model.

As a last remark, the estimation “persons” has more “ups and downs” due to the lower play width of SPURT. Although it kept the typical semblance of a spurt variable, where small changes were filtered out. Therefore, the visual inspection of SPURT suggests that the unit “in 1000 persons” for the dependent variable employment better suits our purpose.

Figure 34: SPURT regression “hours” (left) and SPURT from regression “persons” (right)



Source: Own calculations.

The employment data, when counted in persons, delivers a stronger suggestion of hysteresis effect. To sum up, the estimation “hours” does fail to create a spurt variable that takes the explanatory power of the forcing variable. The estimation “persons” did have a smaller play width, as foreseen, fulfilled the three characteristics and its spurt variable is in accordance with the model.

### 6.3 Main findings EU countries

In this section, we analyse the same 10 other countries of the European Union to assess if hysteresis is more or less present when using the alternative relationship between employment and GVA gap instead of the traditional variables.

After the analysis of the German data we established that the (logarithmized) unit used for employment should be in “1000 persons”. At the this section, table 24 presents an overview of all estimations. Netherlands, Greece, and Italy were chosen for the complete analysis that follows. The

### 6.3 Main findings EU Countries

data was retrieved from the Eurostat databank. All series are at least lagged one quarter<sup>25</sup>, and the range of these 10 countries varies from 2004Q2 to 2020Q2 (Greece) having the shortest time series to 1994Q1 to 2018Q2 (France), which is the longest.

#### Estimation without play

We start with a simple OLS regression for the three countries. So that the play dynamic is not considered and the play width equals 0. We regress employment on the GVA gap and the results are used as a benchmark. All of the estimations contain a shift, a trend, and seasonal dummies (D1-D3) variables. The regressor is seasonally adjusted, while the regressand is not. Tables 18 - 20 show the summary of the findings:

Table 18 Standard OLS (GVA) – Netherlands

Dependent variable: Employment persons Netherlands

Sample: 2001Q1 2020Q2

Included observations: 78

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8.976936	0.006754	1329.054	0.0000
GVA_GAP_NE(-1)	1.004932	0.120523	8.338107	0.0000
SHIFT(-1)	-0.010694	0.006372	-1.678248	0.0977
TREND	0.001710	0.000142	12.03040	0.0000
D1	-0.008644	0.004230	-2.043285	0.0447
D2	0.003542	0.004233	0.836667	0.4056
D3	0.005071	0.004281	1.184651	0.2401
R-squared	0.907311	Mean dependent var	9.080905	
Adjusted R-squared	0.899478	S.D. dependent var	0.041596	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.294122	

Source: Own calculations.

<sup>25</sup> For reasons firstly mentioned in section 5.3.1.

Table 19: Standard OLS (GVA) – Greece

Dependent variable: Employment persons Greece

Sample: 2004Q2 2020Q2

Included observations: 65

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8.407643	0.020844	403.3627	0.0000
GVA_GAP_GR(-1)	0.454830	0.168082	2.706002	0.0089
SHIFT(-1)	-0.074026	0.018896	-3.917563	0.0002
TREND	0.000856	0.000447	1.916025	0.0603
D1	-0.004690	0.008289	-0.565823	0.5737
D2	0.012883	0.008253	1.561087	0.1239
D3	0.013119	0.008326	1.575664	0.1205
R-squared	0.716259	Mean dependent var	8.430760	
Adjusted R-squared	0.686907	S.D. dependent var	0.041874	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.531631	

Source: Own calculations.

Table 20: Standard OLS (GVA) – Italy

Dependent variable: Employment persons Italy

Sample: 2001Q2 2020Q1

Included observations: 76

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.04995	0.009627	1043.933	0.0000
GVA_GAP_IT(-1)	0.259542	0.124017	2.092800	0.0400
SHIFT(-1)	-0.029565	0.007234	-4.086984	0.0001
TREND	0.001083	0.000168	6.440436	0.0000
D1	-0.006803	0.004615	-1.474268	0.1450
D2	0.003133	0.004616	0.678734	0.4996
D3	0.004678	0.004613	1.014187	0.3140
R-squared	0.582744	Mean dependent var	10.11593	
Adjusted R-squared	0.546461	S.D. dependent var	0.021101	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.211091	

Source: Own calculations.

The summary of these three tables has some similarities: All show a positive significant forcing variable, namely GVA gap. For the Netherlands a one percent increase in GVA means that the employment increases on average proportionally (i.e. also 1 %). For Greece, the coefficient is 0.45%, and is comparable to the result of many estimations (see table 24). For Italy, the coefficient's value is 0.25, and is the lowest of all models in the current section. The first two estimations have a highly

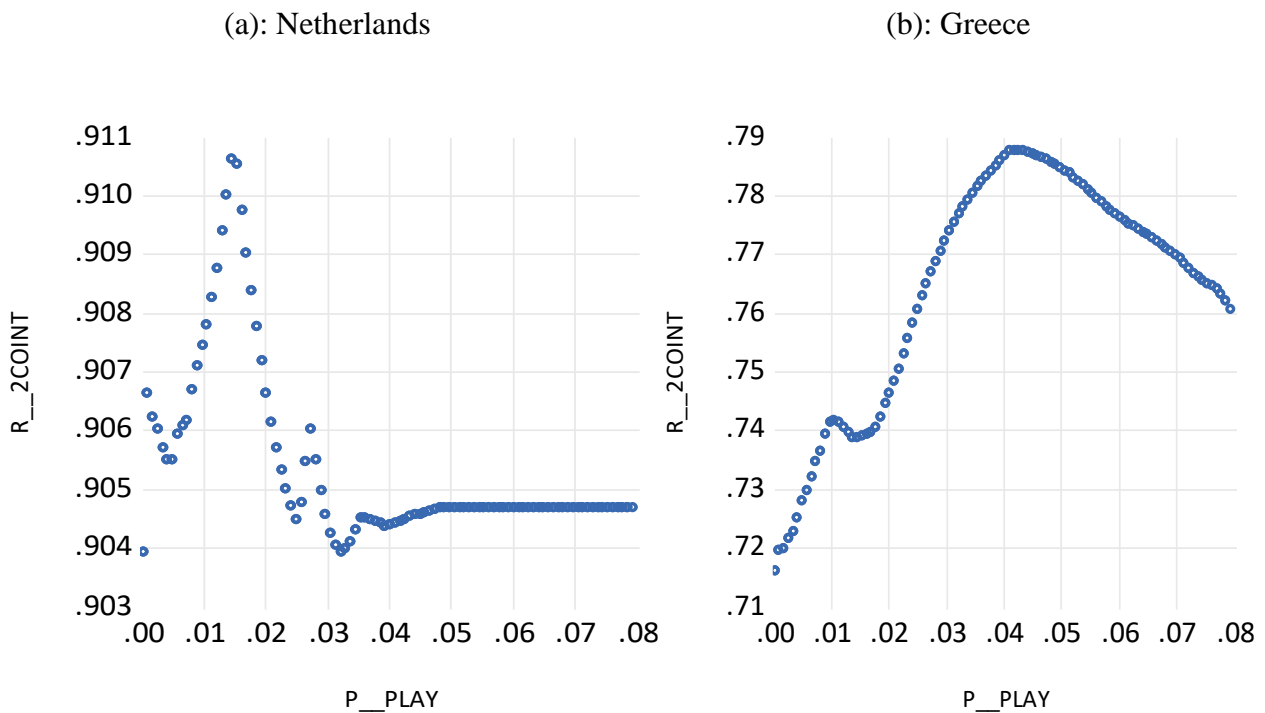
### 6.3 Main findings EU Countries

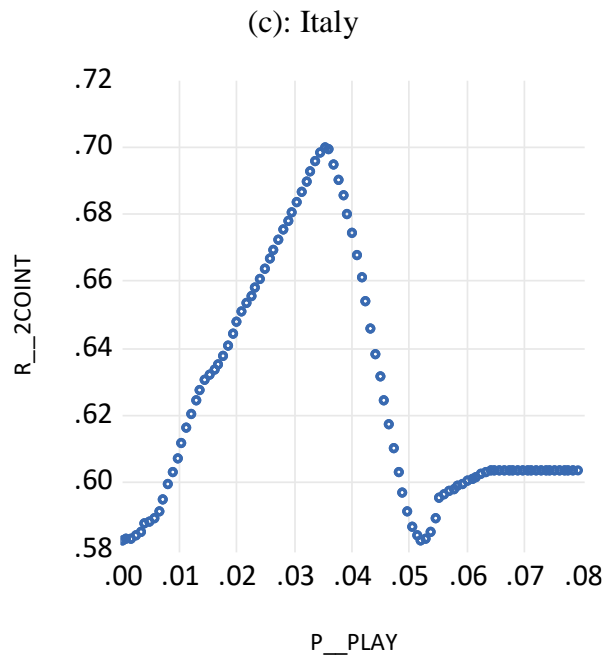
statistically significant GVA gap while the last estimation reaches only the 5% level of statistical significance. Shift has a negative effect on employment for the three estimations. The variable shift in the estimation for Greece represents the Euro debt crisis, since its economic activity started to erode about a year after the financial crisis. All three estimations have a positive and statistically significant trend with a low coefficient. This is compatible with the last models with unemployment as the dependent variable in section 5.3.1. Moreover, D1-D3 are added to control for seasonal fluctuations. The regression with Dutch data has a relatively high  $R^2$  (0.903634), followed by Greece ( $R^2=0.716259$ ). While the estimation for Italy ( $R^2=0.582744$ ) is low compared to the usual results achieved throughout this work.

#### Estimation with constant play ( $\gamma_t = \gamma$ )

We calculate several plays using the play algorithm in order to generate for each of them an artificial variable SPURT. Then estimate several OLS including SPURT and choose the optimal model based on the play bandwidth that proportionated the highest  $R^2$ .

Figure 35: Grid search over different values of constant play  $\gamma$





The grid search for the Netherlands has its peak at  $R^2 = 0.09139$  with a play width of 1.4 percentage points. This is in accordance with the theoretical expectations, since the Netherlands has a flexible labour market, i.e. employers can hire and fire workers relatively easily but also workers can find employment quickly and adapt to changing labour market conditions. Italy has a labour market traditionally less flexible compared to the Netherlands, and achieves the highest  $R^2 (= 0.70)$  when the play width is 3.5 percentage points. Similarly, Greece has a history of strong labour market rigidity and extensive state machinery, thus justifying the 4.1 percentage points of play width, which delivers the estimation with the highest  $R^2$ . The shape of all graphs follows a pattern of a “mountain”, because the  $R^2$  rises with increasing play width until a certain level is achieved followed by a drop, because the play width becomes too wide and no hysteresis effect can be found anymore.

Based on the three constant plays, for Netherlands  $\gamma = 0.014$ , Greece  $\gamma = 0.041$ , and Italy  $\gamma = 0.035$ , tables 21, 22, and 23 display the estimation results of the spurt/play regressions with an artificial spurt variable.

### 6.3 Main findings EU Countries

Table 21: Standard OLS estimation with play (GVA) – Netherlands

Dependent variable: Employment persons Netherlands

Sample: 2001Q1 2020Q2

Included observations: 78

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8.987398	0.007954	1129.976	0.0000
SPURT	0.866201	0.373000	2.322252	0.0231
GVA_GAP_NE(-1)	0.323290	0.315970	1.023165	0.3098
SHIFT(-1)	-0.013465	0.006298	-2.138160	0.0360
TREND	0.001816	0.000145	12.50250	0.0000
D1	-0.008718	0.004105	-2.123732	0.0372
D2	0.003075	0.004113	0.747654	0.4572
D3	0.004844	0.004155	1.165821	0.2476
R-squared	0.913941	Mean dependent var		9.080905
Adjusted R-squared	0.905335	S.D. dependent var		0.041596
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.296518

Source: Own calculations.

Table 22: Standard OLS estimation with play (GVA) – Greece

Dependent variable: Employment persons Greece

Sample: 2004Q2 2020Q2

Included observations: 65

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPURT	1.176170	0.268178	4.385784	0.0001
C	8.432706	0.019058	442.4798	0.0000
GVA_GAP_GR(-1)	-0.067650	0.188907	-0.358114	0.7216
SHIFT(-1)	-0.033884	0.018853	-1.797293	0.0776
TREND	0.000201	0.000417	0.480847	0.6325
D1	-0.002947	0.007241	-0.406972	0.6856
D2	0.014353	0.007206	1.991737	0.0512
D3	0.016835	0.007311	2.302630	0.0250
R-squared	0.787851	Mean dependent var		8.430760
Adjusted R-squared	0.761797	S.D. dependent var		0.041874
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.639107

Source: Own calculations.

Table 23: Standard OLS estimation with play (GVA) – Italy

Dependent variable: Employment persons Italy

Sample: 2001Q2 2020Q1

Included observations: 76

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.03122	0.008989	1115.891	0.0000
SPURT	-1.521688	0.295154	-5.155569	0.0000
GVA_GAP_IT(-1)	0.554597	0.120398	4.606348	0.0000
SHIFT(-1)	-0.050090	0.007350	-6.814748	0.0000
TREND	0.001440	0.000159	9.030845	0.0000
D1	-0.006813	0.003942	-1.728435	0.0884
D2	0.002578	0.003945	0.653547	0.5156
D3	0.004785	0.003940	1.214498	0.2288
R-squared	0.700006	Mean dependent var	10.11593	
Adjusted R-squared	0.669124	S.D. dependent var	0.021101	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.565968	

Source: Own calculations.

Starting with the estimation for the Netherlands: The coefficient of SPURT (0.866201) is positive, lower than the coefficient of GVA gap from the estimation without play (1.004932). GVA gap has partially lost its explanatory power to SPURT and its coefficient now has the value 0.32329 and is statistically insignificant. The sum of both coefficients adds up to 1.1894 suggesting that a hysteresis effect exists, since the overall effect rose from 1.0049 to 1.1894, although the increase of the effect is only about 18%, it is still considered a plausible value for the total effect. The GVA gap coefficient of the simple estimation (1.0049) suggests a proportional increase of dependent and independent variables. Therefore, the findings regarding GVA gap and SPURT suggest a weak, yet existent hysteresis effect. Further variables stayed virtually unaltered when compared with the estimation without play effect. The trend has a positive value, meaning that over time employment tends to go upwards, although the coefficient of 0.001539 is so low, that its effect can be neglected. Similar are the seasonal dummy variables. Only D1 is statistically significant. As the last remark: The inclusion of SPURT did not raise the  $R^2$  coefficient significantly. The  $R^2$  of the simple OLS is 0.907311 while the estimation with SPURT has a coefficient of determination equal to 0.913941. It gives us a hint that the addition of SPURT did not substantially contribute to the explanation of the variation in the model.

Next, we examine the model with Greek data. SPURT is positive and highly statistically significant as theoretically predicted. Its coefficient's value is 1.1761 meaning that a one percent

### 6.3 Main findings EU Countries

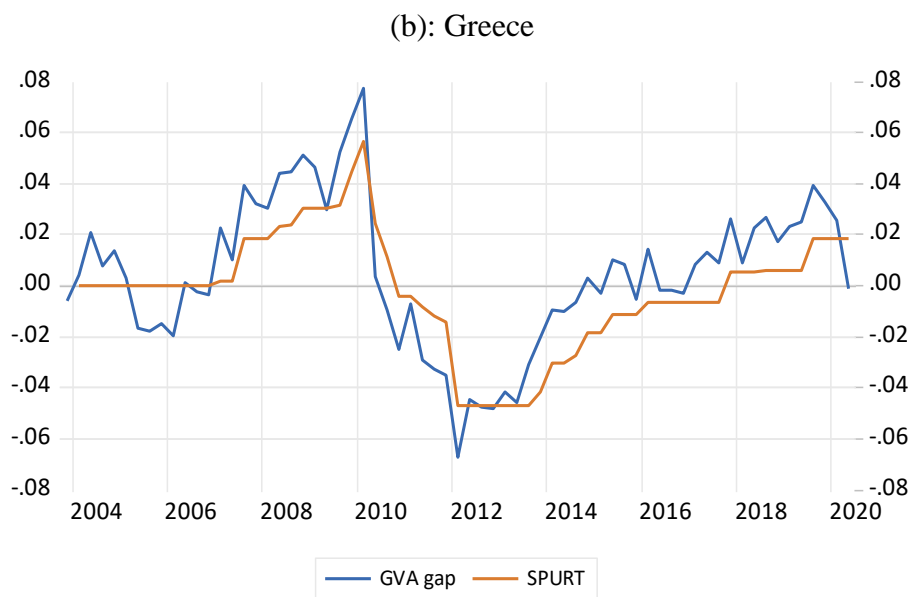
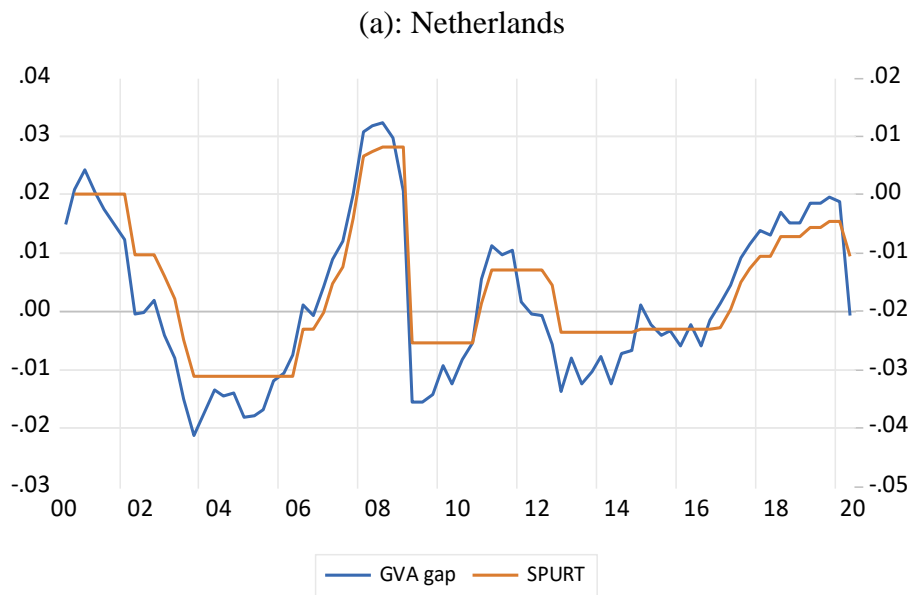
increase in GVA would, on average, increase employment almost in the same proportion. Moreover, SPURT has, in value, over two times the GVA gap of the estimation without play effect (0.4548). Indicating that hysteresis exists and its effect is substantially high. Moreover, the spurt variable is only triggered when the bandwidth of 4.1% points is trespassed. Until this point, every increase of 1% in GVA gap would impact employment on average only 0.067% points negatively. An explanation for the high play width is the huge government apparatus and its many public servants, who cannot be easily fired, as well as labour protection regulations that contribute to the rigidity of the labour market. As expected, the GVA gap became negative and statistically insignificant in the estimation with play dynamics. Its coefficient also shrank from 0.4548 to -0.0676 suggesting that SPURT, representing the play effect, took over all the explanatory power of the variable GVA. The shift variable has the coefficient -0.0338 and is statistically significant at the 10% level. It represents a long-run negative effect of the financial crisis on employment. Probably due to austerity measures forced by European institutions and labour market reforms, which contribute to easing the possibility of firing employees, the financial crisis has a negative effect on employment in the long-run. The variable trend has a very low coefficient (0.0002) and is not statistically significant. D2 and D3 are statistically significant at the 10% and 5% levels, respectively.

Table 23 shows the estimation for the Italian data. At first glance, one can see that the sign of the SPURT's coefficient is negative. In this case, an increase in production would lead to a decrease in employment, which is not in accordance with our expectations. This finding suggests that the model does not fit the theory, although the variable is statistically significant and its absolute value of 1.5216 is plausible. When accounting for the GVA gap of 0.5545 the total effect is still negative, which is another hint for the unfitness of the model. Besides the seasonal dummies, from which only D1 is statistically significant at the 10% level, all the variables are highly statistically significant. Shift has the same sign and similar coefficient as the Greek data. It probably relies on the fact that both countries suffered from the financial crisis in terms of unemployment and negative economic performance.

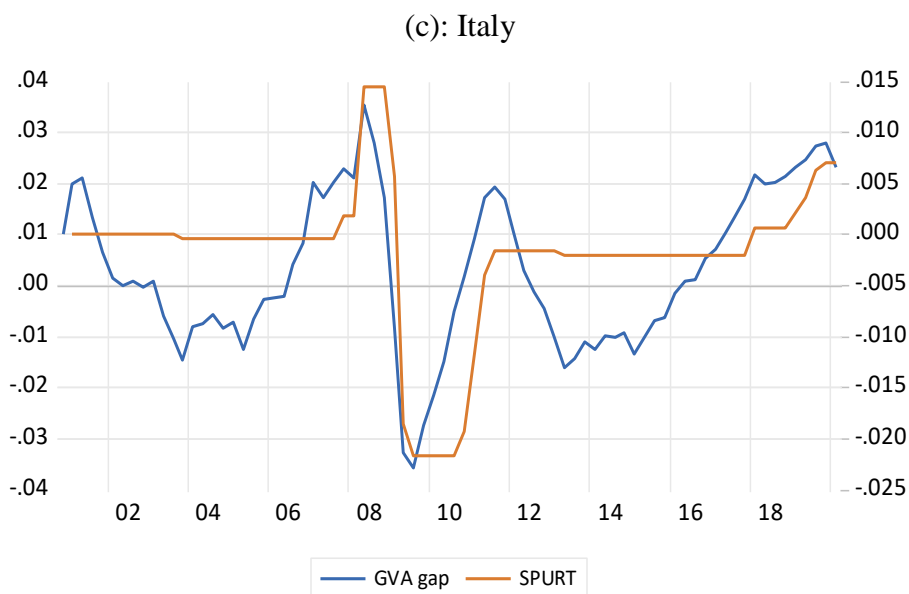
Italian policymakers have attempted to liberalize the rigid labour market for at least 2 decades to boost employment. This is the reason why the trend variable has a positive sign. Nevertheless, the rigidity of the labour market is reflected in the play bandwidth of 3.6% points. Similar to the Greek result, only substantial changes in GVA gap have a substantial influence on the labour market. Contrary to our expectations, the results show that the hysteresis effect could not be confirmed for the Italian data.

The last part of the analysis is the visual inspection of the graphic depicting the variables SPURT and output gap:

Figure 36: Spurt and GVA gap



### 6.3 Main findings EU Countries



Source: Own calculations.

A visual inspection of the path of SPURT for the Netherlands (fig. 36(a)) shows no big variability. Its maximal value is 0.01 and the minimal is -0.030. SPURT does not resemble a dummy around one specific event, e.g. financial crisis, but rather follows the ups and downs of the GVA gap. The graphic for Greece depicts a smoother path due to the higher variability. Hence, one can detect more changes in SPURT over time, even with a high play width as Greece has (e.g. between 2013 and 2016 SPURT's form resembles ascending steps).

For Italy, the contrary is valid. Before and after the financial crisis SPURT is almost not triggered and only between 2008 and 2011 some changes happen. Therefore, it follows to some extent the shape of a dummy variable.

We conclude that Italy delivers the perfect example of when the data set does not confirm hysteresis. At the same time Greece fulfils all the requirements to suggest the presence of hysteresis in the data set. The model for the Netherlands presents strong evidence of hysteresis, although not as robust as for the Greek data. For a complete survey of the results for all countries see table 24.

The 3 characteristics to validate are going to be assessed for all countries. Table 24 depicts SPURT ( $\beta$ ), as well as the play width ( $\gamma$ ), GVA gap for the OLS without play effects ( $\alpha_{simple}$ ) and for play effects ( $\alpha$ ). In table 24, regressions that do not correspond with the first characteristic (positive sign for SPURT) are marked by grey shading. They are the estimations for Austria and Italy. The second property (weaker effect of GVA gap when SPURT is included) is violated again by Italy and Austria. The third aspect, ( $|\alpha_{play} + \beta| > |\alpha_{simple}|$ ), holds for all, but not for the Austrian model. Nevertheless, the play bandwidth of 0.004 for Austria and 0.006 for Ireland and Portugal is too short,

thus not economically plausible. A local maximum was chosen for Denmark because the play width of the global maximum would not properly represent the flexible labour market of this country.

Therefore, countries in line with “typical” play-dynamics are Belgium, Denmark, Greece, France, Spain, Portugal, and the Netherlands. The majority of the countries analysed in this section indicate the existence of hysteresis effect in the data.

### 6.3 Main findings EU Countries

Table 24: Overview results for 10 European countries - alternative Okun's law

Austria	Belgium	Denmark	Greece	France
$\alpha_{simple} = 0.431408^{***}$ $\alpha = 0.510314^{***}$ $\beta = -0.164604^*$ $\gamma = 0.024$ $R_{simple}^2 = 0.9958$ $R^2 = 0.996026$	$\alpha_{simple} = 0.549647^{***}$ $\alpha = 0.390157^{***}$ $\beta = 0.501107^{***}$ $\gamma = 0.032$ $R_{simple}^2 = 0.985544$ $R^2 = 0.987458$	$\alpha_{simple} = 0.4967^{***}$ $\alpha = -0.217051$ $\beta = 0.811463^{**}$ $\gamma = 0.013$ $R_{simple}^2 = 0.757058$ $R^2 = 0.775828$	$\alpha_{simple} = 0.45483^{***}$ $\alpha = -0.06765$ $\beta = 1.17617^{***}$ $\gamma = 0.041$ $R_{simple}^2 = 0.716259$ $R^2 = 0.787851$	$\alpha_{simple} = 0.442026^{**}$ $\alpha = 0.023241$ $\beta = 1.785331^{***}$ $\gamma = 0.026$ $R_{simple}^2 = 0.936532$ $R^2 = 0.936532$
Italy	Spain	Portugal	Ireland	Netherlands
$\alpha_{simple} = 0.2595^{**}$ $\alpha = 0.554597^{***}$ $\beta = -1.521688^{***}$ $\gamma = 0.035$ $R_{simple}^2 = 0.582744$ $R^2 = 0.700006$	$\alpha_{simple} = 1.207967^{***}$ $\alpha = -0.397074$ $\beta = 3.383939^{***}$ $\gamma = 0.04$ $R_{simple}^2 = 0.462913$ $R^2 = 0.577604$	$\alpha_{simple} = 1.2618^{***}$ $\alpha = -1.056308$ $\beta = 2.49307^{**}$ $\gamma = 0.006$ $R_{simple}^2 = 0.640552$ $R^2 = 0.658542$	$\alpha_{simple} = 0.86029^{***}$ $\alpha = -3.86584^*$ $\beta = 4.886313^{**}$ $\gamma = 0.006$ $R_{simple}^2 = 0.737932$ $R^2 = 0.74962$	$\alpha_{simple} = 1.004932^{***}$ $\alpha = 0.32329$ $\beta = 0.866201^{***}$ $\gamma = 0.014$ $R_{simple}^2 = 0.907311$ $R^2 = 0.913941$

Source: Own calculations.

## **6.4 Sectoral analysis for Germany**

In chapter 5 and the last section, we modelled hysteresis for the aggregated data of several economies. In this section, we unravel the German GVA in sectors and perform estimations to identify which industry presents a hysteretic behaviour, explaining the hysteresis effect for the aggregated data.

### **6.4.1 Data and variables**

Sectoral data for Germany was retrieved from the Eurostat databank and is divided according to the statistical classification of economic activities in the European Community NACE Rev. 2 (Eurostat, 2008).

Each industry has its own dynamic: cost structure, regulation, capital intensity, market, production scalability, sector of activity, susceptibility to economic shocks, and thus a different relationship between value added and employment.

The data used is divided into 11 categories. They are: 1) Agriculture, forestry and fishing, 2) Industry<sup>26</sup>, excluding construction, 3) Manufacturing, 4) Construction, 5) Wholesale and retail trade, transport, accommodation and food service activities, 6) Information and communication, 7) Financial and insurance activities, 8) Real estate activities, 9) Professional, scientific and technical activities; administrative and support service activities, 10) Public administration, defence, education, human health and social work activities, 11) Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies. Note that 3) Manufacturing is included in 2) Industry and is its main component. As an example, for the time period 2000Q4 Manufacturing represented roughly 88% of the sector Industry. Hence there are actually only 10 sectors and 11 categories.

Since the variable unemployment is not measurable for each sector of the economy, we utilise the variable employment as independent variable, as described in section 6.1.

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<sup>26</sup> Note that "Industry" with a capital letter refers to the category described in NACE, while industry or economic industry are used as synonyms for economic sector.

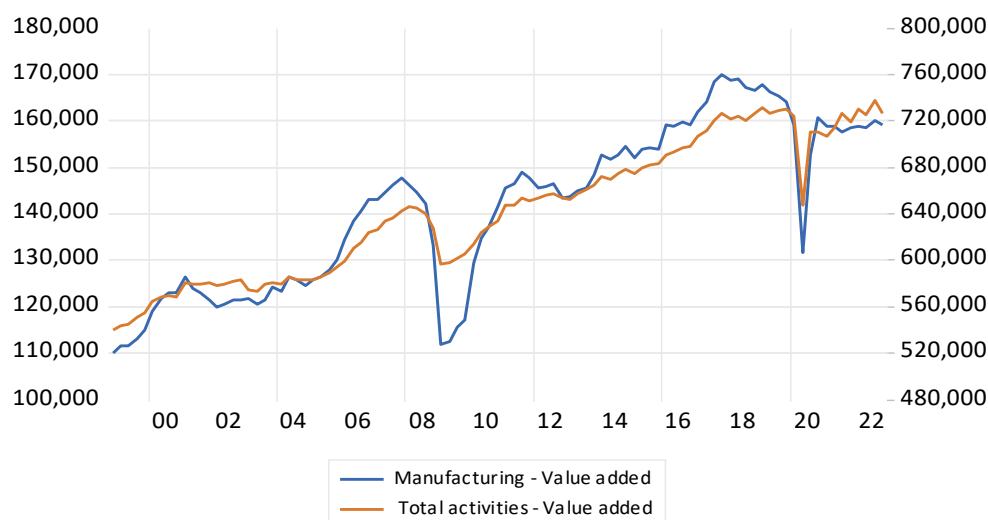
## 6.4 Data and variables

### 6.4.2 Main findings

A comparison among Germany's industries and the aggregated production is conducted to determine if the hysteresis effect is more pronounced in some segments and at the same time, if the aggregation smoothes the hysteretic phenomenon. Similar examination was made by Baldwin and Krugman using the exchange rate as the hysteretic variable (Baldwin & Krugman, 1989, pp. 636–637).

The sectors are not equally represented in the German economy. E.g. in 2000Q4 manufacturing represented around 20% of the aggregated value added, contributing more to the economy than any other sector. A glance at figure 37 suggests that the path of aggregated value added (red line and right scale) is strongly shaped by manufacturing (blue line and left scale).

Figure 37: GVA from total activities and Manufacturing



Source: Own representation based on (Eurostat, 2024b).

For all estimations, the forcing variable GVA gap is lagged by at least one quarter. Some presented better results when the lag was set to two quarters. Consequently, the spurt variable was built with lagged values, and shift was also lagged to maintain consistency. The proxy for the potential capacity was calculated with the HP filter. The time series' range varies from 1992Q1 to 2022Q4 for the sector "Information and communication" as the longest time series, to "Professional, scientific and technical activities; administrative and support service activities" ranging from 2001Q2 to 2020Q2, being the shortest. The full analysis of three economic industries, namely 2) Industry (no construction),

6) Information and communication, and 9) Professional, scientific and technical activities; administrative and support service activities will be presented.

We chose these sectors because of the fundamental differences in their dynamics, which serve as background to corroborate the empirical outcome that follows in this section and demonstrate three cases: Existence of hysteresis, mixed evidence, and no hysteresis.

We expect that hiring employees in manufacturing will be directly linked to the production over the potential capacity, since in the short-term production can only increase through the addition of labour input. Moreover, prices for manufactured goods are generally more stable than commodity prices, and a rise in value added over the potential output is mainly due to an increase in the quantity of produced goods.

On the other hand for sector 6) Information and communication the gross value added may not be the best proxy to measure the sector's production, because of the strategy that companies use to conquer a share of the market in this sector. Due to the high infrastructure costs and network effects, firms invest substantial capital to first expand and gain customers without an operating profit. Therefore, investments in this sector and the generation of employment are not automatically translated into value added, but are firstly intended to gain market share. Hence, employment and value creation are not necessarily correlated. At least, not before the company reaches a certain size, it becomes profitable and the relationship holds. Nevertheless, we do opt to keep value added as the forcing variable for consistency and comparability.

The third sector analysed is 10), which comprises a broad range of services, from legal and accounting activities to management consultancy and veterinary services. Mostly they are services that cannot be scaled and the value added grows proportionally to the labour input. Thus, hysteresis is not a phenomenon that we expect to be detected.

### **Estimation without play**

The standard procedure is used: First, the benchmark estimation is calculated. All of the estimations contain a trend, shift, as well as seasonal dummies (D1-D3). The forcing variable is seasonally adjusted, while the dependent variable is not. Tables 25, 26, and 27 show the summary of the findings:

6.4 Data and variables

Table 25: Standard OLS – 2) Industry (no construction)

Dependent variable: Employment Industry (no construction)

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.324153	0.030381	306.9095	0.0000
GVA_GAP_IND(-1)	0.483110	0.116293	4.154258	0.0001
SHIFT(-1)	0.090869	0.015992	5.682206	0.0000
TREND	-0.002324	0.000241	-9.641870	0.0000
D1	-0.009961	0.010565	-0.942887	0.3479
D2	-0.006191	0.010474	-0.591116	0.5557
D3	-0.001059	0.010561	-0.100282	0.9203
R-squared	0.549130	Mean dependent var	9.020391	
Adjusted R-squared	0.523609	S.D. dependent var	0.057240	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.087419	

Source: Own calculations.

Table 26: Standard OLS – 6) Information and communication

Dependent variable: Employment Information and communication

Sample: 1992Q1 2022Q4

Included observations: 124

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.427276	0.018462	348.1429	0.0000
GVA_GAP_INF(-2)	0.361998	0.079492	4.553868	0.0000
SHIFT(-2)	-0.080479	0.010393	-7.743596	0.0000
TREND	0.004439	0.000145	30.64323	0.0000
D1	-0.006578	0.007379	-0.891368	0.3746
D2	-0.007456	0.007378	-1.010545	0.3143
D3	-0.003998	0.007376	-0.542000	0.5888
R-squared	0.952625	Mean dependent var	7.048759	
Adjusted R-squared	0.950195	S.D. dependent var	0.130099	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.209840	

Source: Own calculations.

Table 27: Standard OLS – 9) Professional, scientific, and administrative activities

Dependent Variable: Employment Professional, scientific, and administrative activities  
 Sample: 2001Q2 2020Q2  
 Included observations: 77

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.563320	0.038339	197.2759	0.0000
GVA_GAP_PRO(-1)	0.291380	0.132200	2.204082	0.0308
SHIFT(-1)	0.050973	0.012409	4.107798	0.0001
TREND	0.005853	0.000272	21.48519	0.0000
D1	-0.023173	0.008901	-2.603299	0.0113
D2	-0.013185	0.008798	-1.498688	0.1385
D3	0.002527	0.008898	0.284045	0.7772
R-squared	0.971439	Mean dependent var	8.541271	
Adjusted R-squared	0.968991	S.D. dependent var	0.155653	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.159754	

Source: Own calculations.

The relationship between GVA gap and employment holds for all three industries, at least at the 5% level (for sector 9). A 1 percent increase in gross value added leads to an average increase of 0.48 percent on employment in sector 2), 0.36 for sector 6), and 0.29 for sector 10).

Sectors 2 and 9 have positive coefficients for shift. It means that employment was positively impacted by the financial crisis, while for sector 6 the opposite is valid. The second case is more plausible, since economic crises tend to have a negative effect on employment.

The categories have similar results concerning the seasonal variables: Mostly they have a negative sign, and are not statistically significant. Only D3 for sector 9 differs from this result and D1 is statistically significant at the 5% level.

The trend component of the estimations also differs. The industrial sector shows a negative and high statistically significant coefficient for the trend variable, meaning that over time the employment tends to diminish in this sector, while the sectors of Information and communication, and Professional, scientific, and administrative activities also have a statistically significant coefficient but with a positive sign. As a result, the variable has a positive effect on employment.

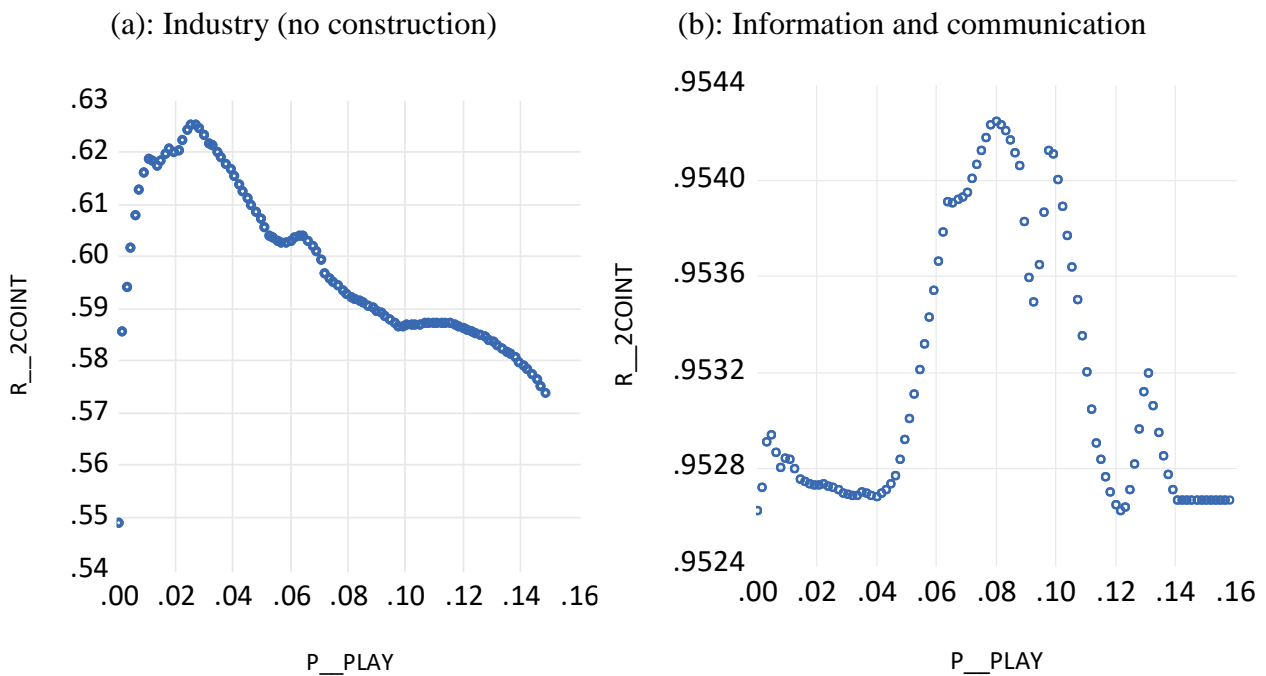
The summary statistics are in conformity with the expectations of Okun's law and diverge mainly only regarding the sign of shift for the financial crisis of 2008/9. Overall, all three estimations cope with our expectations and hold a positive relation between gross value added and employment.

## 6.4 Data and variables

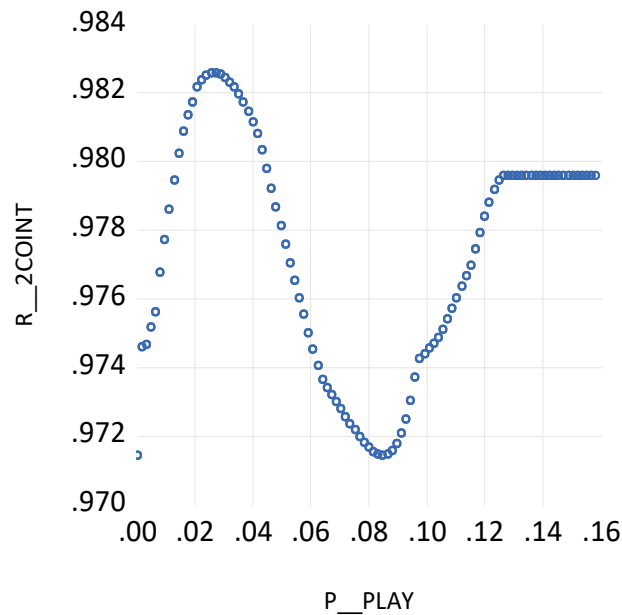
### Estimation with constant play ( $\gamma_t = \gamma$ )

Next, the play algorithm is used to calculate several bandwidths and produces for each of them a spurt variable. Then, estimate several OLS models including SPURT and choose the optimal model based on the play bandwidth that proportionates the highest  $R^2$ . Due to the economic divergences between sectors, the play width was set accordingly to each sector, with a minimum of 3, although most of the estimations use a play width of 16 percentage points.

Figure 38: One-dimensional grid search over different values of constant play  $\gamma$



(c): Professional, scientific, and administrative activities



Source: Own calculations.

In figure 38 we can see the grid search for the three sectors. Starting with the sector “Industry”, the OLS without the play component has a  $R^2$  value of 0.5491, represented by the blue dot, where the play width has the value 0 (bottom left). The addition of SPURT with a play width of 0.027, raises the  $R^2$  significantly to the value of 0.6253. This bandwidth implies that a rise (fall) beyond (under) 2.7 percentage points triggers SPURT. From the peak, the  $R^2$  falls for every further OLS estimated with a higher play width until the end of the graph.

Panel (b) shows another picture: The  $R^2$  value for the estimations with and without SPURT only differ marginally. The total  $R^2$  rise is about 0.0016 percentage points. This indicates that much of the variation is not well explained by the hysteretic effect. Moreover, the shape of the graphic does not follow the regular path for hysteresis, because it has two peaks, the first for the bandwidth 0.08 and the second for 0.098. The existence of these two local maxima, nearly at the same level, turns the criteria used to choose a play width inappropriate, namely choosing the highest  $R^2$ . Note that the bandwidth chosen by the algorithm is quite high: 0.08 percentage points over the long-term value added is needed to trigger SPURT. This reflects the characteristic of the sector, namely that new employees are only hired when production exceeds the potential capacity by far, because high sunk investment costs in infrastructure are needed in order to expand the business and consequently hire more people.

Panel (c) has only one peak for the bandwidth 0.027 and an increase in  $R^2$  of about 0.01, which suits the pattern expected for hysteresis. After the peak, the value of  $R^2$  decreases rapidly until 0.08

## 6.4 Data and variables

when it increases again and stabilizes at around 0.12 percentage points. Its value remains constant until the end of the grid search.

Based on the three constant plays, for Industry (no construction) with  $\gamma = 0.027$ , Financial and insurance,  $\gamma = 0.08$ , and for Professional, scientific, and administrative activities  $\gamma = 0.027$ , tables 28 - 30 display the estimation results of the spurt/play regressions with an artificial spurt-variable.

Table 28: Standard OLS with play - Industry

Dependent variable: Employment persons Industry

Sample: 1992Q2 2020Q2

Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.373751	0.029821	314.3355	0.0000
SPURT	1.562389	0.337949	4.623149	0.0000
GVA_GAP_IND(-1)	-0.871968	0.311858	-2.796042	0.0062
SHIFT(-1)	0.100304	0.014788	6.782942	0.0000
TREND	-0.002458	0.000223	-11.03988	0.0000
D1	-0.009539	0.009676	-0.985851	0.3265
D2	-0.004602	0.009599	-0.479426	0.6326
D3	-1.54E-05	0.009675	-0.001590	0.9987
R-squared	0.625386	Mean dependent var	9.020391	
Adjusted R-squared	0.600411	S.D. dependent var	0.057240	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.162622	

Source: Own calculations.

Table 29: Standard OLS with play - Information and communication

Dependent variable: Employment persons Information and communication  
 Sample: 1992Q1 2022Q4  
 Included observations: 124

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.413062	0.019522	328.5079	0.0000
SPURT	0.400561	0.197485	2.028318	0.0448
GVA_GAP_INF(-2)	0.233851	0.100731	2.321536	0.0220
SHIFT(-2)	-0.083767	0.010385	-8.066392	0.0000
TREND	0.004527	0.000149	30.29928	0.0000
D1	-0.006583	0.007283	-0.903930	0.3679
D2	-0.007173	0.007284	-0.984861	0.3267
D3	-0.003979	0.007280	-0.546553	0.5857
R-squared	0.954247	Mean dependent var	7.048759	
Adjusted R-squared	0.951486	S.D. dependent var	0.130099	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.216063	

Source: Own calculations.

Table 30: Standard OLS with play – Professional, scientific, and administrative activities

Dependent Variable: Employment persons – Professional, scientific, and administrative activities  
 Sample: 2001Q2 2020Q2  
 Included observations: 77

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.523808	0.030742	244.7373	0.0000
SPURT	-2.235078	0.336553	-6.641092	0.0000
GVA_GAP_PRO(-1)	2.067608	0.286969	7.204978	0.0000
SHIFT(-1)	0.038648	0.009937	3.889373	0.0002
TREND	0.005890	0.000214	27.47446	0.0000
D1	-0.023221	0.007003	-3.315964	0.0015
D2	-0.011556	0.006925	-1.668597	0.0997
D3	0.003527	0.007002	0.503803	0.6160
R-squared	0.982576	Mean dependent var	8.541271	
Adjusted R-squared	0.980808	S.D. dependent var	0.155653	
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.664807	

Source: Own calculations.

The summary statistics for the industrial sector in table 28 show a mitigation of the GVA gap coefficient from 0.48 to -0.87 and a positive and highly significant SPURT with a coefficient of 1.56. The GVA gap variable was clearly substituted by SPURT, which is in value higher than GVA gap of

#### 6.4 Data and variables

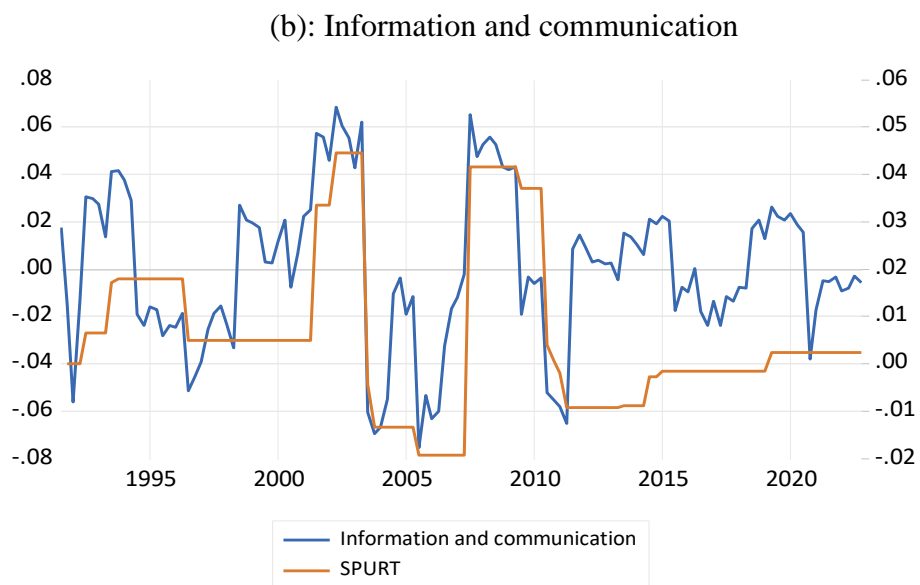
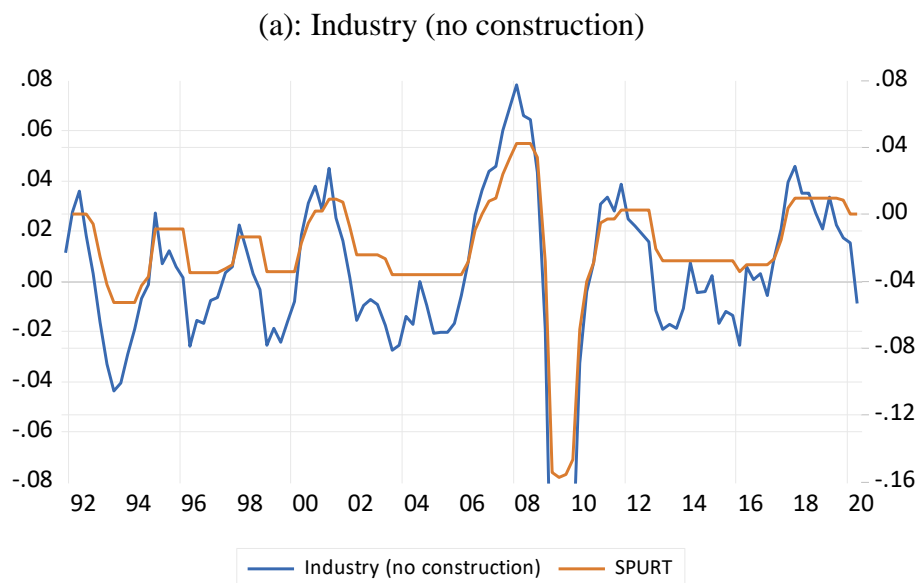
the OLS estimation without play dynamics ( $\gamma = 0$ ). The total effect of the forcing variable on employment is the sum of SPURT (1.56) plus the play effect (-0.87), resulting in 0.69, which is higher than the GVA gap of the simple OLS (0.48). This result reinforces that hysteresis exists. Shift and trend variables stay virtually the same. The coefficients of the seasonal variables also did not undergo any change. All the conditions for suggesting the presence of hysteresis in the industrial sector have been so far confirmed. The inclusion of SPURT has enhanced the model significantly, with an increase of 0.076 percentage points in  $R^2$ , and all other variables are also in accordance with our expectations.

Table 29 depicts the test statistics for the sector Information and communication. GVA gap lost only partially its explanatory power to SPURT and is still statistically significant at the 5% level. Its coefficient lowered from 0.36 in the simple OLS to 0.23 when play dynamics apply. SPURT has a coefficient of 0.40, which is slightly higher than the coefficient of GVA gap of the simple OLS and is likewise statistically significant at the 5% level. Trend, shift, and seasonal components remained unaltered. The statistical summary underpins, to a certain extent, the existence of hysteresis in the data set, only undermined by the findings of the scatter plot. It shows two peaks instead of one, and the resultant play widths (0.08 and 0.098) are quite high, although the sector has high sunk investment costs. Moreover, the  $R^2$  increase is inexpressive. Therefore, the results are not conclusive, since some evidence exists for both claims.

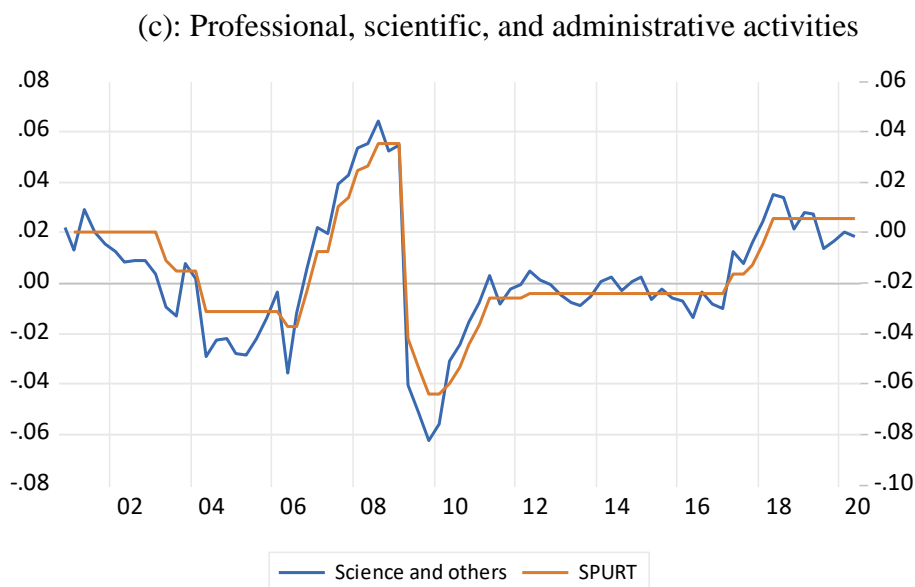
The results for the sector 9) Professional, scientific and technical activities; administrative and support service activities are not as favourable as the previous outcomes. SPURT has a negative sign, indicating that employment falls when production surpasses a certain threshold over the potential capacity. This fact does not comply with the economic theory, which states that an increase in production over its capacity triggers hiring new employees. Moreover, an increase of 1% in value added over the play width causes on average a negative effect of 2.23% points on employment. GVA gap has an opposite effect on employment with a coefficient of 2.06. Their sum accounts to -0.17 as the overall effect. Thus, the impact of SPURT could not be compensated, so that the overall response does not comply with the theoretical expectation. It indicates that no hysteresis was found in this relationship. All other variables remained almost unaltered.

Further, we analyse the path of SPURT and gross valued added-gap in figure 39:

Figure 39: Spurt and GVA variables for different sectors



## 6.4 Data and variables



Source: Own calculations.

Figure 39 (a) contains the path of the variables SPURT and GVA for the Industry sector. The local maximum was set at 1992Q2. As one can see, the spurt variable is 0 at this point (right scale). Two quarters later, in 1999Q4 SPURT is triggered for the first time because the forcing variable GVA gap fell from 0.036 to 0.003. The play width of  $\gamma = 0.027$  was exceeded and SPURT took the difference, amounting to -0.006. And kept falling until 1993Q3 reaching the value of -0.053. Just to be triggered again, but now positively in 1994Q3 until the peak in 1995Q1. Note that the values of the forcing variable were lagged by 2 quarters. Several ups and downs of the variable SPURT followed until the end of 2020Q2 without any disturbances.

The visual inspection was the last analysis made to corroborate the presence of hysteresis in the data set. The same procedure applies for the other two sectors with approving results regarding the path undertaken by the spurt variable.

To sum up, the conclusions of the three models discussed so far: The Industry sector delivered favourable results in every aspect. The statistical summary, the play width, as well as, the spurt path, and the characteristics of the sector suggest the existence of hysteresis. The Information and communication sector delivered mixed evidence. On the one hand, the play width with two peaks and similar  $R^2$  values, and the p-value of GVA gap in the play estimation do not speak for the hysteresis effect in the data set. On the other hand, the model shows some satisfactory evidences: SPURT took partially the explanatory power of the forcing variable and its path is in accordance with the play width. Moreover, the amplitude for the bandwidth for play can be plausible for this specific sector due to high infrastructure costs.

The third sector analysed was 9) Professional, scientific and technical activities; administrative and support service activities: The model did not perform well, as already expected. SPURT has a negative coefficient, and the forcing variable in the simple OLS is not statistically significant. As stated at the beginning of this section, the dynamic of employment in the sector cannot be well described by a hysteresis model.

The main results of the models for all other countries are presented in table 31, and the three requirements regarding statistical summary to validate the hysteresis effect in the model are verified. As elucidated in section 5.3.1 the conditions are 1) A significantly positive effect of the spurt variable (i.e.  $\beta > 0$ ). 2). Grey shaded areas on table 31 indicate that the sector does not comply with the first condition. A weaker (or even insignificant) effect of GVA gap when estimated in the same equation as SPURT. 3) The total effect, which consists of the spurt plus play area effect, should be stronger than the effect of GVA gap in the simple estimation. I.e. the relation ( $|\alpha_{play} + \beta| > |\alpha_{simple}|$ ) must hold. At the same time, we will consider the play width, spurt path, and specific characteristics of each sector in order to assess whether or not the hysteretic effect exists for each of them.

Estimations for sectors 1,5, 8, 9, and 10 have negative SPURT coefficients and thus, fail to fulfil the first requirement. The second and third conditions are fulfilled by all estimations except those which already failed the first test.

For sector 1) Agriculture, Forestry and Fishing not only the model output was not optimal but also the economic background does not suggest the presence of hysteresis: Agricultural and forestry goods, as wheat and wood, are commodities and their prices dependent on the world market price that can vary significantly, e.g. if bad harvests occur, the price for crops will raise and cause a big variance in the value added due to the scarce offer, which do not dependent on the labour used. Therefore, the link between labour input and value added can be distorted in this case.

Sector 5) Wholesale and retail trade, transport, accommodation and food service activities also did not display results in favour of hysteresis. Neither is there a theoretical basis to confirm it: In the tertiary sector the potential output depends rather on the number of employees, thus a rise in value-added is proportional to the quantity persons working. Moreover, the costs of hiring and firing employees are low in this sector. Training new employees is not costly, since most activities require only low-skill levels, thus workers become easily replaceable.

We argue that the economic explanation for sector 8) Real estate activities is the same as for sector 9, which was already extensively explained.

The regression for sector 10) Public administration, defence, education, human health and social work activities also did not deliver any of the three conditions to suggest hysteresis. Moreover, public

## 6.4 Data and variables

administration sector does not strive to create value directly, but rather to create the conditions for a thriving economy, thus its value added is directly linked to the sector's costs. Consequently, the existence of hysteresis is not plausible for this sector for both reasons.

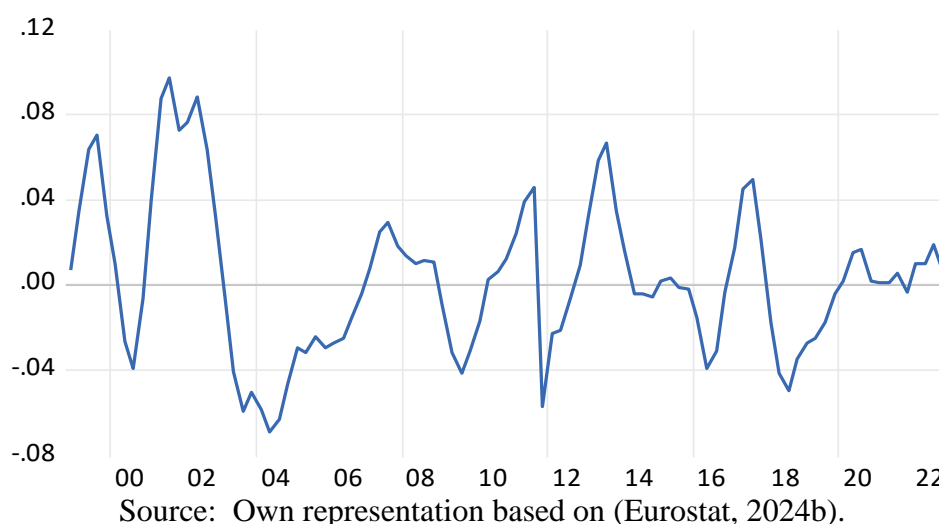
Besides, the excluded sectors 1, 5, 8, and 10, and the three examples extensively discussed of sectors 2, 6, and 9. Four sectors remain to be evaluated: 3, 4, 7, and 11.

Sector 3) Manufacturing is major part of 2) Industry, therefore the results are virtually the same and any further explanation would be redundant.

The estimation for the construction sector 4) exhibits positive results regarding hysteresis. Its statistical summary hints to the existence of hysteresis in the data set, since the coefficients of GVA gap and SPURT correspond to the theoretical expectations, and a 1% increase over the play width causes an overall effect of around 1% increase in employment, while in the benchmark model the coefficient for the hysteretic variable was 0.71. At the same time, the play width of 0.07 means that the value-added through production must surpass 7 percentage points to trigger SPURT. The value is considered quite high, and explained by the high investment costs of the sector. As for manufacturing, construction is a capital-intensive sector. Once the production reaches its potential output, only then are new employees hired, thus the link between value added and employment is given.

The financial and insurance sector (sector 7) does provide promising results regarding the model's coefficients. Nevertheless, its value added is strongly tied up with the securities market. Therefore, an economic boom can drive the prices of financial products up, thus inflating the value added without necessarily hiring more employees, leading to high volatility. Such "spikes" on the value added can be seen in figure 40. These facts suggest that changes in quantity, and not in price, drive the value added of the financial and insurance industry. Therefore, the link between value added and employment in this sector is not always straightforward. A possible solution is to use a quantity index.

Figure 40: GVA - Financial and insurance sector



Sector 11) Arts, entertainment and recreation; other service activities; activities of household and extra-territorial organizations and bodies also fulfils the requirements for the model. It does have a slightly higher play width of 0.046, but it is in accordance with the higher SPURT coefficient of 3.14, since a higher bandwidth normally results in a higher SPURT coefficient. Hence, the overall result indicates the presence of hysteresis in the data.

In overall, sectors 2,3,11 did show signs of hysteretic behaviour underpinned by economical reasoning. While 1,5,8,9,10 were the complete opposite, neither the model nor the economic background suggests the presence of hysteresis effect. Sectors 6,7 delivered good model results but failed partially in giving an economic explanation for the hysteresis phenomenon. Therefore, the main driver of hysteresis in the aggregated data is the Industry/Manufacturing sector due to its relative size to the total German economy, however, the construction sector also contributes to the hysteretic effect.

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Table 31: Overview results for sectors – Germany

**Breakdown in sectors (NACE 2.1)**

1) Agriculture	2) Industry	3) Manufacturing	4) Construction	5) Wholesale and retail	6) Information and communication
$\alpha_{\text{simple}} = -0.06923^{**}$ $\alpha = 0.282751^{***}$ $\beta = -0.44773^{***}$ $\gamma = 0.095$ $R^2_{\text{simple}} = 0.948071$ $R^2 = 0.959007$	$\alpha_{\text{simple}} = 0.48311^{***}$ $\alpha = -0.871968^{***}$ $\beta = 1.562389^{***}$ $\gamma = 0.027$ $R^2_{\text{simple}} = 0.54913$ $R^2 = 0.625386$	$\alpha_{\text{simple}} = 0.252626^{***}$ $\alpha = -0.956569^{***}$ $\beta = 1.286497^{***}$ $\gamma = 0.021$ $R^2_{\text{simple}} = 0.254318$ $R^2 = 0.373289$	$\alpha_{\text{simple}} = 0.71499^{***}$ $\alpha = -2.86773^*$ $\beta = 3.840702^{**}$ $\gamma = 0.07$ $R^2_{\text{simple}} = 0.327222$ $R^2 = 0.361668$	$\alpha_{\text{simple}} = 0.162116^{**}$ $\alpha = 0.399357^{***}$ $\beta = -0.31664^{**}$ $\gamma = 0.028$ $R^2_{\text{simple}} = 0.87504$ $R^2 = 0.880889$	$\alpha_{\text{simple}} = 0.361998^{***}$ $\alpha = 0.233851^{**}$ $\beta = 0.400561^{**}$ $\gamma = 0.08$ $R^2_{\text{simple}} = 0.952625$ $R^2 = 0.954247$
7) Financial & insurance	8) Real estate	9) Professional, scientific, and administrative activities	10) Public administration	11) Arts	
$\alpha_{\text{simple}} = 0.170349^{***}$ $\alpha = 0.021507$ $\beta = 0.410857^{***}$ $\gamma = 0.072$ $R^2_{\text{simple}} = 0.950084$ $R^2 = 0.963227$	$\alpha_{\text{simple}} = 0.121031$ $\alpha = 0.40325^{**}$ $\beta = -0.80398^*$ $\gamma = 0.022$ $R^2_{\text{simple}} = 0.587387$ $R^2 = 0.609561$	$\alpha_{\text{simple}} = 0.29138^{**}$ $\alpha = 2.067608^{***}$ $\beta = -2.235078^{***}$ $\gamma = 0.027$ $R^2_{\text{simple}} = 0.971439$ $R^2 = 0.982576$	$\alpha_{\text{simple}} = 0.866074^{***}$ $\alpha = 1.247612^{***}$ $\beta = -1.26767^{***}$ $\gamma = 0.02$ $R^2_{\text{simple}} = 0.976916$ $R^2 = 0.97911$	$\alpha_{\text{simple}} = 0.101712$ $\alpha = -0.34756$ $\beta = 3.147202$ $\gamma = 0.046$ $R^2_{\text{simple}} = 0.833174$ $R^2 = 0.862757$	

Source: Own calculations.

# Chapter 7

## Conclusion

This work deals with the relationship between economic growth and unemployment, the so called Okun's law, under the framework of hysteresis. The objective is to create a model that better explains the relationship stated by Okun. Due to its non-linear and asymmetric nature, a hysteretic framework is applicable, and because of its time-varying feature, a new assessment of the relationship is needed from time to time. To that end, we created the theoretical fundamentals for the model. This is the first novelty of the work: The microeconomic model consists of a modified version of the "non-ideal relay". In this operator, the firm's realised utilisation (output) is related to the potential value of sales, as the input variable (see section 4.1.1), which was later transformed to an operator related to a firm's employment. It was then aggregated and applied to macroeconomic data. The second novelty is the utilisation of the play algorithm developed by Göcke and Belke (see section 4.3.2) as a tool to assess hysteresis in Okun's relationship. The algorithm is a linearization of the hysteresis loop and mimics its behaviour. It enables the creation of an artificial variable (SPURT) that accounts for hysteresis on the data set and can be implemented in an OLS model. Chapters 5 and 6 constitute the main part of the work and revolve around empirically testing and adapting the Okun's law using the play algorithm implemented in linear regressions for various European countries. An analysis of the German data was conducted, followed by an overview of the results for 10 other EU countries. Of the 11 countries evaluated six (Austria, Belgium, Denmark, Germany, Greece, Netherlands, and Spain) showed evidence of hysteretic behaviour. The result for Italy was not as expected, since hysteresis could not be confirmed in the data. Although southern European countries have more rigid labour market policies despite structural reforms conducted since the 1990s and early 2000s. Ireland is a special case, because the band of inaction reached the value of 0.074, meaning that its GDP has to grow 7.4% over the potential output before SPURT is triggered. This magnitude for a play width cannot be explained by hysteresis and therefore is unreliable. Chapter 6 consists of two sets of empirical estimations: In section 6.2 we tackle some of the problems that the original relationship may hide by changing the variables GDP to

## *Conclusion*

GVA and unemployment to employment, so that the effect of hidden reserves and population growth could be mitigated. Again, estimations for the same 11 countries were made and hysteresis could be observed in seven countries (Belgium, Denmark, Germany, Greece, France, Spain, and the Netherlands). Nonetheless, the estimations for Belgium and France did not show an improvement in  $R^2$ . And the band of inaction of the estimations for Portugal and Ireland are so narrow, that we cannot consider hysteresis in the data. Therefore, roughly half of the countries experience a hysteretic effect in their respective labour markets. Both sets of estimations have a positive result regarding hysteresis for the following countries: Belgium, Denmark, Germany, Greece, Spain, and the Netherlands. Therefore, the existence of hysteresis should be taken into consideration for the following countries and our approach is another tool to quantify the cost of growth fluctuations measured in unemployment. In section 6.3 a sectoral analysis for the German economy was made to find the driving force behind the hysteresis phenomenon in the country. We conclude that the Industry/Manufacturing sector is mainly responsible for hysteresis effects. This finding highlights the importance of sector-specific policies in managing economic cycles to guarantee employment stability and job creation.

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# **Appendices**

A.1

Dependent Variable: Unemployment unadjusted Germany  
 Sample: 1992Q2 2020Q2  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.104722	0.003892	26.90934	0.0000
GDP_GAP_DE(-1)	-0.309866	0.101056	-3.066276	0.0028
SHIFT(-1)	0.002274	0.004416	0.515009	0.6076
TREND_92	-0.000544	4.58E-05	-11.88224	0.0000
D1	0.005274	0.004119	1.280642	0.2031
D2	-0.000902	0.004083	-0.220981	0.8255
D3	-0.001634	0.004115	-0.397173	0.6920

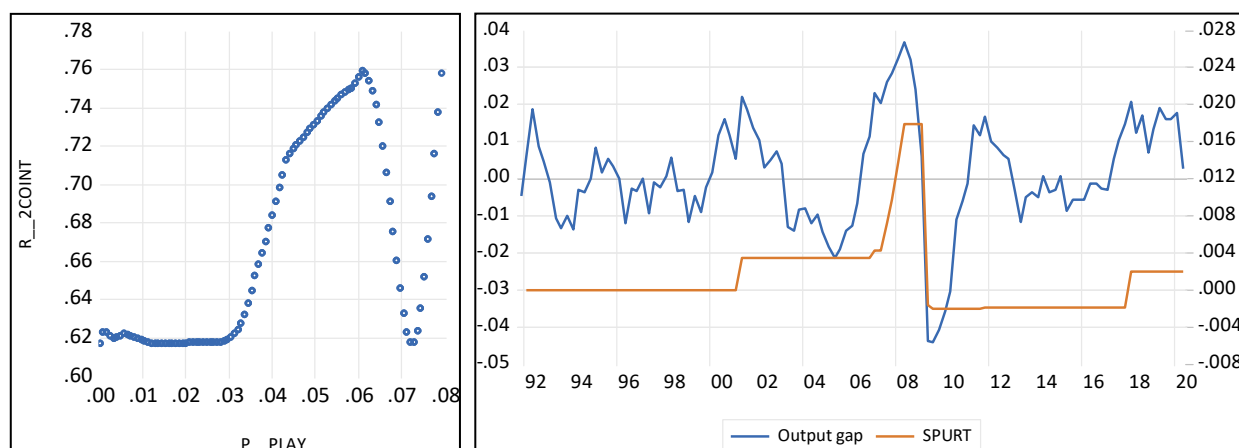
R-squared	0.617179	Mean dependent var	0.073850
Adjusted R-squared	0.595510	S.D. dependent var	0.024206
S.E. of regression	0.015395	Akaike info criterion	-5.449612
Sum squared resid	0.025122	Schwarz criterion	-5.280659
Log likelihood	314.9031	Hannan-Quinn criter.	-5.381053
F-statistic	28.48198	Durbin-Watson stat	0.041139
Prob(F-statistic)	0.000000		

Dependent Variable: Unemployment unadjusted Germany  
 Sample: 1992Q2 2020Q2  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPURT	2.659820	0.338135	7.866142	0.0000
C	0.100136	0.003156	31.72956	0.0000
GDP_GAP_DE(-1)	-0.702826	0.094776	-7.415655	0.0000
SHIFT01(-1)	-0.007685	0.003740	-2.054806	0.0424
TREND_92	-0.000487	3.72E-05	-13.08574	0.0000
D1	0.003905	0.003287	1.188016	0.2375
D2	-0.001254	0.003255	-0.385219	0.7009
D3	-0.001479	0.003280	-0.451090	0.6529

R-squared	0.759126	Mean dependent var	0.073850
Adjusted R-squared	0.743067	S.D. dependent var	0.024206
Prob(F-statistic)	0.000000	Durbin-Watson stat	0.174490



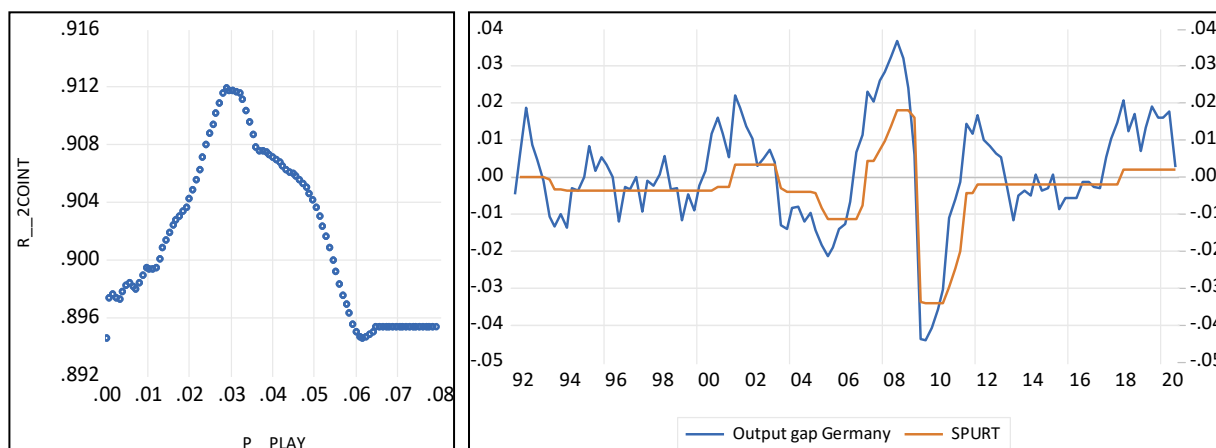
## A.2

Dependent Variable: Unemployment unadjusted Germany  
 Method: Least Squares  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.087347	0.002363	36.96179	0.0000
GDP_GAP_DE(-2)	-0.658947	0.057200	-11.51998	0.0000
SHIFT(-2)	-0.051093	0.003215	-15.89232	0.0000
TREND_92	0.000113	4.85E-05	2.321953	0.0221
D1	0.004906	0.002176	2.254157	0.0262
D2	-0.000207	0.002178	-0.094889	0.9246
D3	-0.002053	0.002159	-0.951039	0.3437
R-squared	0.894614	Mean dependent var		0.073637
Adjusted R-squared	0.888649	S.D. dependent var		0.024398
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.384133

Dependent Variable: Unemployment unadjusted Germany  
 Sample: 1992Q3 2020Q3  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPURT	-0.599137	0.131945	-4.540793	0.0000
C	0.083846	0.002304	36.39574	0.0000
GDP_GAP_DE(-2)	-0.394108	0.078502	-5.020340	0.0000
SHIFT(-2)	-0.056209	0.003161	-17.78272	0.0000
TREND_92	0.000164	4.59E-05	3.574723	0.0005
D1	0.005030	0.001999	2.515760	0.0134
D2	0.000348	0.002004	0.173494	0.8626
D3	-0.001583	0.001986	-0.797223	0.4271
R-squared	0.911912	Mean dependent var		0.073637
Adjusted R-squared	0.906040	S.D. dependent var		0.024398
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.292553



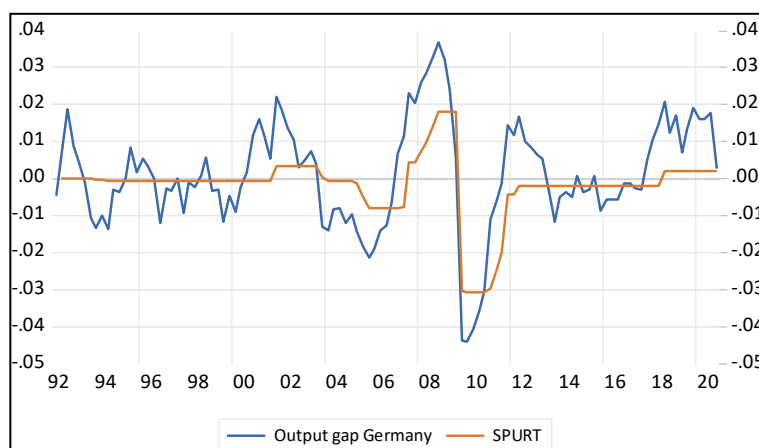
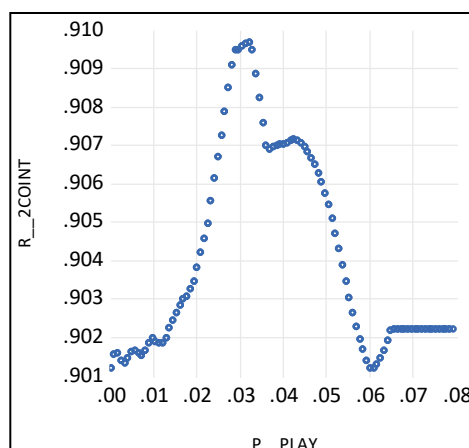
A.2

Dependent Variable: Unemployment unadjusted Germany  
 Sample: 1992Q4 2020Q4  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.087671	0.002348	37.34049	0.0000
GDP_GAP_DE(-3)	-0.611381	0.055880	-10.94088	0.0000
SHIFT(-3)	-0.050162	0.003141	-15.97130	0.0000
TREND_92	7.91E-05	4.73E-05	1.671947	0.0975
D1	0.005859	0.002109	2.778402	0.0065
D2	0.001243	0.002110	0.589275	0.5569
D3	-0.000892	0.002110	-0.422730	0.6733
R-squared	0.901207	Mean dependent var		0.073389
Adjusted R-squared	0.895615	S.D. dependent var		0.024617
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.306607

Dependent Variable: Unemployment unadjusted Germany  
 Sample: 1992Q4 2020Q4  
 Included observations: 113

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPURT	-0.436608	0.138905	-3.143225	0.0022
C	0.086461	0.002288	37.79043	0.0000
GDP_GAP_DE(-3)	-0.446275	0.075102	-5.942222	0.0000
SHIFT(-3)	-0.054650	0.003338	-16.37312	0.0000
TREND_92	0.000115	4.69E-05	2.458218	0.0156
D1	0.005537	0.002028	2.729976	0.0074
D2	0.000975	0.002028	0.480498	0.6319
D3	-0.000827	0.002027	-0.407770	0.6843
R-squared	0.909704	Mean dependent var		0.073389
Adjusted R-squared	0.903684	S.D. dependent var		0.024617
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.246508



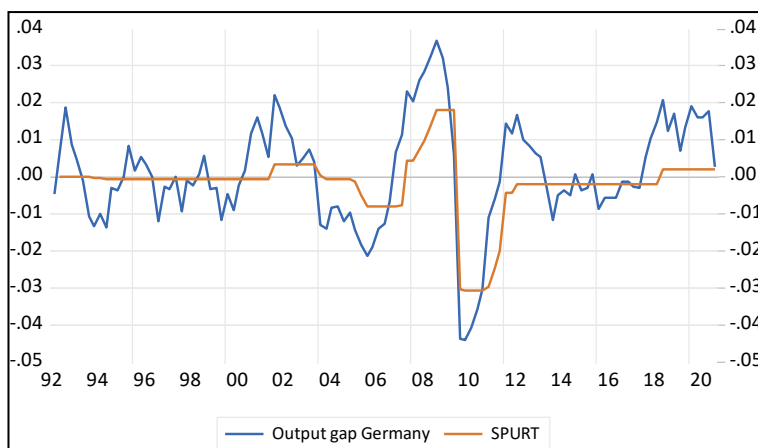
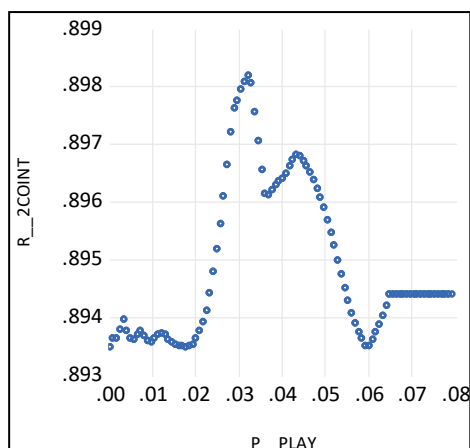
## A.2

Dependent Variable: Unemployment unadjusted Germany  
 Sample (adjusted): 1993Q1 2020Q4  
 Included observations: 112 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.089135	0.002571	34.66578	0.0000
GDP_GAP_DE(-4)	-0.546428	0.058308	-9.371340	0.0000
SHIFT(-4)	-0.048366	0.003281	-14.74179	0.0000
TREND_92	3.86E-05	4.98E-05	0.776395	0.4393
D1	0.005163	0.002222	2.323757	0.0221
D2	0.001175	0.002220	0.529527	0.5976
D3	-0.000498	0.002218	-0.224506	0.8228
R-squared	0.893501	Mean dependent var		0.073411
Adjusted R-squared	0.887415	S.D. dependent var		0.024727
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.307346

Dependent Variable: Unemployment unadjusted Germany  
 Sample (adjusted): 1993Q1 2020Q4  
 Included observations: 112 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPURT	-0.326027	0.148705	-2.192448	0.0306
C	0.088311	0.002554	34.58207	0.0000
GDP_GAP_DE (-4)	-0.422891	0.080348	-5.263214	0.0000
SHIFT(-4)	-0.051686	0.003561	-14.51438	0.0000
TREND_92	6.45E-05	5.03E-05	1.282412	0.2025
D1	0.005059	0.002183	2.317467	0.0224
D2	0.000884	0.002184	0.404561	0.6866
D3	-0.000749	0.002182	-0.343085	0.7322
R-squared	0.898206	Mean dependent var		0.073411
Adjusted R-squared	0.891354	S.D. dependent var		0.024727
Prob(F-statistic)	0.000000	Durbin-Watson stat		0.257659



**A.3**

Null Hypothesis: GDP has a unit root

Exogenous: Constant

Lag Length: 1 (Automatic - based on SIC, maxlag=12)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-0.772039	0.8232
Test critical values:		
1% level	-3.482879	
5% level	-2.884477	
10% level	-2.579080	

\*MacKinnon (1996) one-sided p-values.

## **Eigenständigkeitserklärung**

Ich erkläre hiermit, dass ich die vorgelegte Dissertation selbstständig und nur mit den Hilfen angefertigt, die ich in der Dissertation angegeben habe. Bei den von mir durchgeführten und in der Dissertation erwähnten Untersuchungen habe ich die Grundsätze guter wissenschaftlicher Praxis, wie sie in der Satzung der Justus-Liebig-Universität Gießen zur Sicherung guter wissenschaftlicher Praxis niedergelegt sind, eingehalten.

29.07.2025 \_\_\_\_\_

Datum, Unterschrift