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Microstrip direct current superconducting quantum interference device radio frequency amplifier: Noise data

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A series of about twenty superconducting quantum interference devices (SQUIDs) has been operated as microstrip-SQUID amplifiers (MSAs) at frequencies ranging from 100 MHz to 2 GHz to study the dependence of their gain and noise temperature on bias current and flux. The measured values were in good agreement with theory. The observed dependence of MSA gain and noise temperature on bias current and flux resembled the static transfer function of the SQUIDs. The gains are relatively insensitive to changes in bias current and bias flux; the noise temperature is strongly dependent on the bias flux. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.3702825]

The direct current superconducting quantum interference device (dc SQUID) is a leading candidate for a low noise, low power dissipation radio frequency (rf) amplifier[1–10] exhibiting a sensitivity approaching the quantum limit.[7,8] This is more than an order of magnitude better than the sensitivity of the best available semiconductor amplifiers. One (among others) promising configuration for a SQUID rf amplifier is the so-called microstrip SQUID amplifier (MSA), where the input coil is configured as a microstrip resonator with the SQUID washer acting as a groundplane.[3] At the fundamental resonance of the microstrip resonator, there is substantial coupling between the magnetic field of the microstrip mode and the SQUID. A dc SQUID configured this way will act as an amplifier offering high gain and low noise at frequencies up to 5 GHz and beyond.

In a practical application, the SQUID amplifier will have to be optimized for lowest noise and (possibly) highest gain. To this end, two parameters, the bias current in the SQUID I_b and a static bias flux applied to the SQUID Φ_b to bias it at the steepest point of its flux-to-voltage transfer function V_Φ, have to be carefully set. Measurements have shown that the operating point for optimum sensitivity (lowest noise) and optimum gain does not coincide, as is the case for most amplifiers. Hence, a sole optimization for highest gain will not likely optimize the amplifier for lowest noise. To study the dependence of their gain and noise on the operating points of the MSAs in more detail, we made measurements on a larger number of MSAs ($\approx$20) operated at different frequencies and with and without external feedback. In this letter, we will report on the results of this study.

We performed measurements on 20 MSAs having two different geometries. MSAs with a center frequency of about 300 MHz had a conventional washer SQUID with overlaying 9-turn coil. The washer had an inner size of $200 \times 200\,\mu m^2$, an outer size of 1×1 mm^2, an estimated $L \approx 350$ pH, and typical values $I_0 \approx 8$–$11\,\mu A$ and $R \approx 16$–$24\ \Omega$; L is the SQUID inductance, I_0 the critical current of the SQUID, and R the shunt resistance. Measured values of V_Φ are typically 100 GHz. MSAs with center frequency of 1.7 GHz had a SQUID washer with inner and outer dimensions of $10 \times 200\,\mu m^2$ and $500 \times 500\,\mu m^2$, an estimated $L \approx 70$ pH, and typical values $I_0 \approx 25$–$30\,\mu A$ and $R \approx 10$–$14\ \Omega$. The coil forming the microstrip resonator had 14 turns. Measured values of V_Φ are typically 500 GHz. The linewidth of the coils was 5 μm in both cases.

We measured the gain of our amplifiers with a scalar network analyzer; it was also used to determine the input impedance of our devices.[11,12] For frequencies below 1.5 GHz, a HP8970A noise figure meter was used to determine the noise temperature of our amplifiers, in combination with a calibrated HP346B noise source. For higher frequencies, we used the network analyzer as a receiver for the noise produced by the MSAs. As the noise source produces thermal noise with a very high temperature $T_{ns} \approx 9300$ K, we used a commercial 30 dB attenuator (its measured attenuation was 31 dB) directly at the input of the MSA to reduce the noise source power to values which do not saturate the MSA. The overall attenuation due to attenuator and coaxial cables was measured to be 31.5 dB. We note that the attenuator has only 31 dB attenuation if the input impedance of the MSA is close to 50 ohms. A larger input impedance of the MSA will reduce the attenuation and lead to a higher noise power at the input of the MSA. If the input impedance (or the input reflection factor) of the MSA is known, the real attenuation factor can easily be calculated. Thus, for a MSA with a 50 ohm input impedance, the noise produced at the input of the SQUID corresponds to a temperature of 4.2 K (the SQUID is cooled in liquid helium), if the room temperature noise source is switched off, and 10.8 K if the noise source is switched on. The change in noise power $Y \approx 2.6$. If the MSA had zero noise temperature, we would expect the noise at the output of the MSA to change by a factor of 2.6 if the noise source was switched on and off. For a finite noise temperature T_n, the noise ratio $Y = (T_n + T_H)/(T_n + T_C) > 1$, where $T_H = 10.8$ K and $T_C = 4.2$ K in our case. Again, if the input impedance of the MSA is different from 50 Ω, the attenuation factor has to be adjusted accordingly. For an infinite input impedance of the MSA, the attenuation of the (nominally) 31 dB attenuator would be only 25 dB, and the noise ratio $Y \approx 6$. Depending on the resonant frequency of the MSA, an input impedance of 200–800 ohms can be expected, if no negative or positive feedback is applied.[11–13]

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Taking into account all errors in the noise source power, attenuation, and input impedance of the MSA, the error in the measured noise temperature of an MSA is on the order of ±30%.

As the noise temperature of the noise figure meter and the network analyzer was substantial (1000 K and 10 000 K, respectively), a cold semiconductor amplifier with $T_N \approx 9$ K and $G \approx 25$ dB and another warm semiconductor amplifier with $T_N \approx 70$ K and $G \approx 26$ dB were used to amplify the output noise of the MSA. To reduce interaction of the cold semiconductor amplifier with the MSA, a 4 dB attenuator was inserted between the MSA and the cold postamplifier, raising its apparent noise temperature from 9 K to 23 K.

A thorough theoretical treatment of the noise of SQUID amplifiers has been given by Clarke and coworkers,[14–18] Tesche,[19,20] and Koch.[21] They could show that when one takes into account all the voltage and current noise sources in the SQUID, the optimum noise temperature of a SQUID amplifier is given by $T_{optN} \approx (S_V S_J)^{1/2}\omega/2k_B V_\Phi \approx 7T\omega/V_\Phi$. Thus, T_{optN} scales as the ratio ω_0/V_Φ. Here, S_V and S_J are the spectral densities of the voltage and current noise in the SQUID, and ω is the frequency. We expect for our MSAs a best noise temperature of 0.6 K at 300 MHz and 0.7 K at 1.7 GHz if the SQUID is cooled to liquid helium temperature. The expected optimum gain is $G_{opt} \approx V_\Phi/\omega \approx 18$ dB in both cases. As both, G and T_n scale as V_Φ, we expect them to be closely correlated.

We measured the gain and noise temperature of about 20 MSAs as a function of bias current in the MSA and applied bias flux. Most measurements were done in a storage dewar at 4.2 K; some MSAs were measured in a pulse-tube cooler at $T \approx 2.6$ K. The MSAs were magnetically shielded by inserting them into a niobium tube, which in turn was inserted into a high-permeability Co-Netic-AA tube to prevent frozen flux in the superconducting shield. The current and flux biases were supplied by batteries that could be floated relative to the system ground; the flux was generated by a copper coil.

Fig. 1 shows a typical result of a measurement of gain and noise temperature as a function of the dc bias current in the SQUID. The critical current of this MSA was $I_0 \approx 11\,\mu$A. As the bias current in the SQUID is increased, the gain of the MSA also increases at first. This is due to an increase in the transfer function of the SQUID V_Φ. For bias currents at I_0 or slightly above, the gain is constant and decreases at higher I_b due to a decrease in V_Φ. Over a range of about 2 μA, a nearly constant, substantial gain is achieved. The noise temperature is closely correlated to the gain: The noise temperature decreases with increasing gain and is nearly constant as long as the gain is constant as well. Only at a bias current of 11.8 μA does the noise temperature increase somewhat, consistent with a (small) decrease in the gain. This is caused by a resonant structure at this bias current in the current-voltage characteristic of the SQUID used. On the average, the gain of this MSA is higher at lower frequencies where T_n is somewhat lower. This is in good agreement with $G \propto 1/\omega$ and $T_n \propto \omega$.

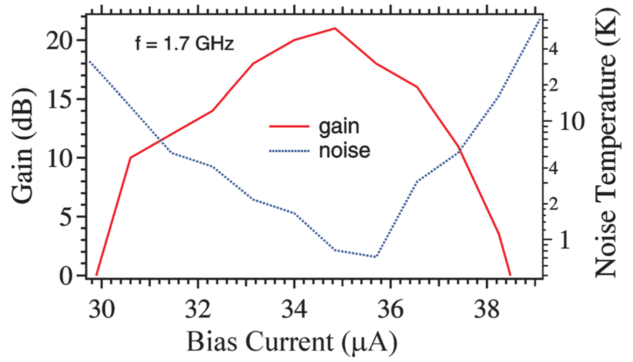

FIG. 2. Gain and noise temperature of a MSA operated at 1.7 GHz as a function of the bias current I_b in the SQUID. The flux bias was optimized for lowest noise temperature for each data point taken.

MSAs working at higher frequencies (1.7 GHz) and having higher critical currents (30 μA) showed a very similar behavior, see Fig. 2. Again, at lower bias currents, the gain increases due to an increase in V_Φ; the noise temperature is lowest around the point of highest gain. As the modulation voltage is much higher for these low-inductance SQUIDs ($L \approx 70$ pH compared to $L \approx 350$ pH for the SQUID shown in Fig. 1), the range over which V_Φ—and thus G and T_n—is constant is rather small. This is due to resonant structures appearing in the current-voltage characteristic of these SQUIDs above 80–100 μV, reducing V_Φ above this voltage.

The dependence of gain and noise on the bias flux is similar to that on bias current, as can be seen from Fig. 3 (for clarity, we express the noise in terms of the noise ratio Y in

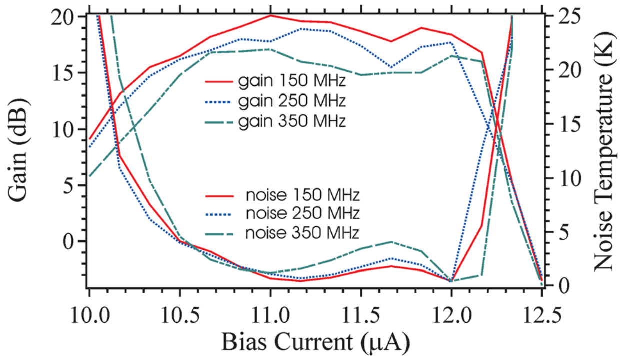

FIG. 1. Gain and noise temperature of a MSA operated at three different frequencies (150 MHz, 250 MHz, and 350 MHz) as a function of the bias current I_b in the SQUID. The flux bias was optimized for lowest noise temperature for each data point taken.

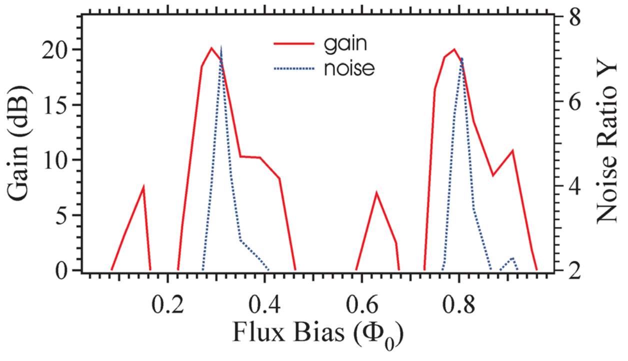

FIG. 3. Gain and noise ratio of the MSA shown in Fig. 1 at 350 MHz as a function of static bias flux Φ_b, measured with a bias current in the SQUID of 12 μA.

Figs. 3 and 4). This SQUID was operated with grounded washer,[3] so there was no negative or positive feedback between input and output of the SQUID. In this case, the gain of the MSA is largest for maximum $|\partial V/\partial \Phi|$ and shows two very similar maxima for a flux change of half a flux quantum. The noise temperature of the MSA follows a similar pattern. It is lowest (noise ratio Y is a maximum) at a point of high gain, which, however, is slightly offset from the bias flux for maximum gain. The dependence of noise on bias flux is very similar for negative and positive $\partial V/\partial \Phi$, although some minor differences are visible in Fig. 3. We see that the dependence of the MSA gain on bias flux is not very strong. If the MSA is biased for maximum gain, small changes in Φ_b will not change the gain noticeably. The dependence of the noise temperature on Φ_b is much stronger: Even a small change in Φ_b will increase the MSA noise. There was not much difference in this behavior at different bias currents or frequencies.

We observed the same behavior of gain and noise on Φ_b in all our 20 devices. The dependence of T_n on I_b was always much less critical than that of T_n on Φ_b. In all cases, the minimum of T_n occurred at a bias flux slightly offset from the point of highest gain. This is only an empirical observation at the moment. It is quite clear that current noise produced by circulating currents in the SQUID (Refs. 17 and 18) is responsible for the increased noise at the bias point for maximum gain. We have, however, no thorough theoretical model, which can explicitly describe this behavior. Nevertheless, this empirical observation can conveniently be used to optimize the MSA for lowest noise as the minimum in noise would always occur at a "higher" bias flux. In this case, one biases the MSA for highest gain and slightly increases the bias flux.

The dependence of gain and noise on Φ_b is strongly dependent on the transfer function of the SQUID, as expected. In Fig. 4, we show the gain and noise of a MSA using a SQUID with a nonsinusoidal transfer function, as depicted in the figure inset. This SQUID had a lower $\partial V/\partial \Phi$ at the flux bias for which normally $\partial V/\partial \Phi$ is a maximum (points a and b in the static transfer function of this particular SQUID shown in the inset). Consequently, the MSA gain is lower and the MSA noise is higher at these flux-bias points. As in the case of the MSA described in Fig. 3, the lowest noise temperature occurs at a slightly higher bias flux than required for maximum gain.

As both G and T_n scale as V_Φ, we expect them to be closely correlated. Our results show that—to a first order approximation—this is indeed the case. Nevertheless, as T_n is more strongly dependent on Φ_b than the gain, there seem to be additional effects that lead to a change in T_n even if V_Φ is nearly constant. For example, if the equivalent noise resistance (the source resistance for which T_n is a minimum) was substantially different[18] from the 50 ohms we used in our experiments, $T_n \propto 1/V_\Phi^2$, which will cause a stronger than linear dependence of T_n on Φ_b.

Most of the MSAs discussed in this paper were operated with a grounded counter electrode,[3] i.e., without any intentional negative or positive feedback. While it is difficult to avoid some kind of negative or positive feedback by the finite inductance of the bond wires used to connect the SQUID washer to ground,[11] we believe that at lower frequencies (300 MHz), this contribution to possible feedback can be made negligible. To this end, we were using seven very short (~1 mm) bond wires in parallel to connect the SQUID washer to ground. Their estimated stray inductance of ~150 pH introduces an impedance of 0.3 Ω (at 300 MHz), which is negligible to the ~50 Ω output impedance. We measured the noise temperature of a few MSAs first without, and then with external negative feedback[11] (gain reduced by 4 dB) but did not observe a noticeable difference in their noise temperature.

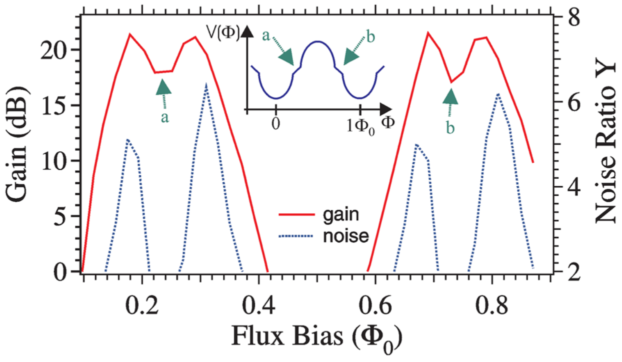

FIG. 4. Gain and noise ratio of a MSA as a function of static bias flux Φ_b, measured at 250 MHz with a bias current in the SQUID of 11 μA. Inset shows sketch of static flux-voltage transfer function of SQUID; for clarity, the nonsinusoidal sections are exaggerated. The arrows show points of reduced $\partial V/\partial \Phi$, leading to a lower gain and higher noise at these bias points.

Finally, Koch *et al.*[22] found an increase in the voltage noise across a single Josephson junction as the bias current in the junction—and thus the dc voltage drop across the junction V_J—increased, as quantum effects become significant if $2\,eV_J > k_BT$. One expects the same to happen in a SQUID amplifier. In our experiments, we did not observe this effect, presumably because the bath temperature was too high; the dc voltage drop across the SQUID was always less than 100 μV for maximum gain, hence $2\,eV_J$ was smaller than or approximately equal to k_BT. Nevertheless, for low bath temperatures (say 40 mK), this mechanism should clearly be visible.

In conclusion, we tested about 20 microstrip SQUID amplifiers. The dependence of gain and noise of the MSAs was strongly correlated with the transfer function (measured at low frequency) of the SQUID used. The values for gain and noise we measured agree well with what is calculated using standard theory. The minimum in the noise and the maximum in the gain do not occur at the same flux bias point; the minimum in the noise occurred at a slightly higher bias flux than required for maximum gain. When we measured the same MSA without, and with external negative feedback, we did not observe a noticeable difference in the noise temperature.

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