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Non-Controlling Minority Shareholdings and Collusion

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Abstract
This article merges theoretical literature on non-controlling minority shareholdings (NCMS) in a coherent model to study the effects of NCMS on competition and collusion. The model encompasses both the case of a common owner holding shares of rival firms as well as the case of cross ownership among rivals. We find that by softening competition, NCMS weaken the sustainability of collusion under a greater variety of situations than was indicated by earlier literature. Such effects exist, in particular, in the presence of an effective competition authority.

Keywords Collusion · Common ownership · Cross ownership · Minority shareholdings

1 Introduction

Corporations sometimes acquire a minority (less than 50%) equity stake in a rival firm that allows them to share in the profitability of the rival without obtaining control: The buyer acquires a passive interest; see Salop and O'Brien (2000), Tzanaki (2015), and Nain and Wang (2018) for evidence of such acquisitions. Similarly, the majority shareholder of a firm may decide to acquire a non-controlling share in a rival firm (common ownership).

Competition authorities have shown an interest in the economic effects of the acquisition of non-controlling minority shareholdings (NCMS) of both types. For

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example, in July 2014 the EU issued a White Paper that discusses an amendment of the current EU Merger Regulation towards assessing NCMS. Since then, especially common ownership has received increasing attention.

Theoretical literature such as Reynolds and Snapp (1986), Flath (1991), Flath (1992), Malueg (1992), and Gilo et al. (2006) has studied the effects of NCMS on firms' profits and the sustainability of collusion under a variety of assumptions with respect to a number of key parameters such as firms' profit function, their strategic variables (prices versus quantity), and the degree of product differentiation. This diversity of assumptions constitutes an impediment to the comparability of different articles' results.

Our article integrates these earlier models into a more comprehensive one, which fills a gap in the literature because we analyze several combinations of assumptions that have not been studied jointly by prior literature. We also provide analytic proofs for effects that have been established only numerically by prior literature.

The model confirms two insights: First, NCMS of firm i in firm j raise firms' profits in competition. These are the profits that are made when every firm plays its best response to the other firm's choice of its strategic variable. But this increase in competitive profits also increases the incentive to deviate from collusion. This is because the deviator i's profits in the punishment phase will be higher because of the reduced intensity of competition. Second, NCMS decrease firm i's incentive to deviate because firm i (or its majority shareholder) would then receive lower dividends from the cheated firm j. Hence, minority shareholdings increase the deviation incentive if the first effect prevails over the second; as has been shown by Malueg (1992) for convex demand.

We advance the literature in two directions. First, our model is the first to study analytically how a competition authority influences the effect of minority shareholdings on the sustainability of collusion. We find that the collusion-destabilizing effect of NCMS is particularly strong in the presence of an effective competition authority. This effect can be observed even when demand is non-convex.

Second, prior literature has typically concentrated on the question of how firm i's shares in firm j affect i's decision whether to deviate from collusion. In addition, we show that firm j's shareholdings in firm i destabilize collusion by raising firm i's critical discount factor whenever the cross shareholdings raise the firms' profits in competition. This indicates that minority shareholdings destabilize collusion under a wider set of assumption than was suggested by earlier literature.

The article is structured as follows. Section 2 shows how the present study contributes to the existing literature. Section 3 provides the model and studies the effects of NCMS on firms' stage game profits. Section 4 analyzes the effects of NCMS on the sustainability of collusion. Section 5 adds the competition authority. Our results are robust to changes in the assumptions about firms' profit functions, which might either model cross ownership or common ownership, as is demonstrated in Sect. 6. Section 7 concludes the article. Proofs are provided in the "Appendix".

2 Literature Review

Reynolds and Snapp (1986) established that in a static Cournot model with symmetric firms and homogeneous products NCMS make it a best response for the firms to restrict aggregate output and raise aggregate profits.[1] Malueg (1992) later extended this model to an infinitely repeated game and showed that NCMS may have an ambiguous impact on the stability of collusion by reducing the short-run gain from cheating while, at the same time, softening the long-run punishment that is imposed on the deviating firm.

In addition, Flath (1991, 1992) presented a static model to study the effects of NCMS on firms' profits not only in Cournot competition with homogeneous products, but also in Bertrand competition with either differentiated or homogeneous products.[2] Unfortunately, these results cannot be compared to those of Reynolds and Snapp (1986) or Malueg (1992). This is because Flath (1991) assumed that the firms maximize a profit function that is commonly used for cross ownership whereas Reynolds and Snapp (1986) and Malueg (1992) assume that the firms maximize a profit function that is commonly used to model common ownership. We implement our model under both assumptions to enhance comparisons of these earlier contributions.

Gilo et al. (2006) extended the model of Flath (1991) to a dynamic game, which allows them to study the effect of minority shareholdings on collusion. They focus on a Bertrand model with homogeneous products where minority shareholdings cannot raise the profits that the firms make in competition, which however played a prominent role in the analysis of Malueg (1992). We extend their model by assuming also Cournot competition and Bertrand competition with differentiated products where NCMS raise firms' profits in competition. Contrary to Gilo et al. (2006), this points towards a variety of situations where NCMS hinder collusion.

We add a new element to this strand of the literature by assuming that a competition authority behaves along the lines of Aubert et al. (2006): Collusion may be detected with a certain probability and penalized thereafter. We show that in the presence of an effective competition authority NCMS may weaken the sustainability of collusion even under conditions where the literature cited above suggested a stabilizing effect of NCMS on collusion; for example, NCMS are found to destabilize collusion also for non-convex demand, which contributes to Malueg (1992), and even for Bertrand competition with homogeneous products, which extends the results of Gilo et al. (2006).

The present article along with the literature cited above concentrates on acquisition decisions of NCMS that are driven by the rationale of receiving a dividend and raising the acquirer's expected profits. Karle et al. (2011) argued that NCMS

[1] Empirical evidence of the effects of cross ownership and the related issue of common ownership on firms' profits in competition has been provided, for example, by Azar et al. (2016), Trivieri (2007), and Nain and Wang (2018).

[2] A related analysis was conducted by Reitman (1994) in a conjectural variations model that was based on the analysis of joint ventures that was conducted by Kwoka (1992).

may also be driven by efficiency considerations such as the generation of economies of scope in the production process. Ouimet (2013) concentrated on NCMS that are made to provide a financially constrained target firm with additional funds for solving hold-up problems when engaging in joint investment projects. López and Vives (2016) analyzed the effect of cross ownership on firms' R&D-efforts. Our model indicates that even NCMS that were acquired to only raise the profits of the acquirer may have pro-competitive effects by disrupting collusion. To make this point most clearly we abstract from further efficiency considerations.

3 The Model

Section 3.1 presents the setup of the model. Section 3.2 establishes the effects of minority shareholdings on firms' stage game profits.

3.1 Setup

The model assumes two cost-symmetric, risk-neutral firms $i, j \in \{1, 2\}$. The majority owner of firm i may hold a minority stake α_i in firm j while the majority owner of firm j may hold a minority stake α_j in firm i. The stakes may be asymmetric ($\alpha_i \neq \alpha_j$), and they are assumed to have been chosen prior to the game analyzed here, which is in line with Reynolds and Snapp (1986), Malueg (1992), and Gilo et al. (2006).

The firms i and j play an infinitely repeated game with the objective of maximizing the discounted sum of their majority owners' individual profits. The discount factors are denoted $\delta_i, \delta_j \in (0, 1)$. In each period, the firms choose a product market strategy $s_i, s_j \in \{c, k\}$ with c indicating best-response behavior in the product market while k represents the collusive choice of their strategic variable. We analyze the firms' choice of s_i, s_j in Sect. 4.

Our analysis is conducted in the three models that were studied by Flath (1991): Cournot competition; Bertrand competition with differentiated products; and Bertrand competition with homogeneous products. Therefore, the product market profit/operating profit π_i of firm i is a function of the firms' strategic variables – quantities in Cournot competition, and prices in Bertrand competition – and these strategic variables are functions of firms' endogenous choice of s_i, s_j and the exogenous shareholdings α_i, α_j. The profits can be expressed as $\pi_i\big(q_i(q_j, s_i, \alpha_i, \alpha_j), q_j(q_i, s_j, \alpha_j, \alpha_i)\big)$ in Cournot competition and $\pi_i\big(p_i(p_j, s_i, \alpha_i, \alpha_j), p_j(p_i, s_j, \alpha_j, \alpha_i)\big)$ in Bertrand competition.

In Sect. 3.2, we determine the firms' collusive strategies and their non-collusive best responses in the product market. Firms' best responses are initially obtained under the assumption of equation (1). Firm i is assumed to maximize its majority owner's payoff $\tilde{\pi}_i$: Firm i's product market profit π_i, minus the dividend $\alpha_j\pi_i$ that is paid to firm j, plus the dividend $\alpha_i\pi_j$ that is received from firm j (Reynolds and Snapp 1986; Malueg 1992):

$$max\ \tilde{\pi}_i = (1 - \alpha_j)\pi_i + \alpha_i\pi_j. \tag{1}$$

Flath (1991, 1992), Gilo et al. (2006), and Shelegia and Spiegel (2012) modeled cross ownership and used a different profit function where the firms maximize their accounting profits: $\hat{\pi}_i = \pi_i + \alpha_i \hat{\pi}_j$. In Sect. 6, we discuss the exact interpretation of this profit function, and we show that the effects that are presented in this article are qualitatively identical for both profit functions.

Plugging the collusive strategies and competitive best responses into the profit functions for all four combinations of s_i, s_j yields values of the profits that are dependent on the value of the shareholdings α_i and α_j. For reasons of conciseness, we use indices to denote the strategy combinations (s_i, s_j): c for (c, c); k for (k, k); d if i is the deviator in (c, k); and $-d$ if i is the cheated firm in (k, c). Notation is further explicated in Sect. 3.2. In the models that we analyze, the inequalities that are shown in (2) apply, which impose a prisoner's dilemma structure on the game:

$$\pi_{i,d} > \pi_{i,k} > \pi_{i,c} \geq \pi_{i,-d} \text{ and } \pi_{i,d} + \pi_{i,-d} < 2\pi_{i,k} \quad \text{for all } i \in \{1, 2\}. \tag{2}$$

The total payoffs $\tilde{\pi}_{i,c}, \tilde{\pi}_{i,k}, \tilde{\pi}_{i,d}$ of majority owner i can be expressed as in (3)-(5):

$$\tilde{\pi}_{i,c} = (1 - \alpha_j)\pi_{i,c} + \alpha_i \pi_{j,c}; \tag{3}$$

$$\tilde{\pi}_{i,k} = (1 - \alpha_j)\pi_{i,k} + \alpha_i \pi_{j,k}; \tag{4}$$

$$\tilde{\pi}_{i,d} = (1 - \alpha_j)\pi_{i,d} + \alpha_i \pi_{j,-d}. \tag{5}$$

3.2 The Stage Game

This subsection establishes firms' best responses and equilibrium profits in Cournot competition, Bertrand competition with differentiated products, and Bertrand competition with homogeneous products. It demonstrates the effects of α_i, α_j on the firms' stage game profits.

Cournot competition In Cournot competition, the payoff $\tilde{\pi}(q_i(q_j, s_i, \alpha_i, \alpha_j), q_j(q_i, s_j, \alpha_j, \alpha_i))$ of firm i's majority owner, which will be abbreviated as $\tilde{\pi}(q_i, q_j)$, is a function of the outputs q_i and q_j. The term $\tilde{q}_i^R(q_j, \alpha_i, \alpha_j)$ denotes firm i's reaction function after choosing $s_i = c$ if it maximizes payoff function $\tilde{\pi}_i$. The arguments q_j, α_i, and α_j of the best-response function $\tilde{q}_i^R$ are sometimes dropped for reasons of conciseness. The firms are said to compete if both play their best responses – $(s_i, s_j) = (c, c)$ – which generates the payoff $\tilde{\pi}_i\left(\tilde{q}_i^R\left(\tilde{q}_j^R, \alpha_i, \alpha_j\right), \tilde{q}_j^R\left(\tilde{q}_i^R, \alpha_j, \alpha_i\right)\right)$, which we abbreviate as $\tilde{\pi}_{i,c}\left(\alpha_i, \alpha_j\right)$. The discussion below relies on a result that is summarized in Lemma 1:

Lemma 1 $\frac{\partial \pi_{i,c}}{\partial \alpha_i} < 0$ and $\frac{\partial \pi_{i,c}}{\partial \alpha_j} > 0$ if $\frac{\partial q_i^R}{\partial q_j} < 0$.

Proof See "Appendix 1"[3] □

Lemma 1 implies that the product-market profits $\pi_{i,c}$ of firm i in competition *fall* if the majority owner of firm i holds a higher share α_i in firm j. This occurs if quantities are strategic substitutes: $\partial q_i^R/\partial q_j < 0$, with $q_i^R(q_j) = \tilde{q}_i^R(q_j, 0, 0)$ denoting firm i's reaction function when $\alpha_i, \alpha_j = 0$; when maximizing product market profits π_i only. Firm i finds it optimal to reduce both its own output ($\partial \tilde{q}_i^R/\partial \alpha_i < 0$) and its product-market profits $\pi_{i,c}$ in order to raise the total payoff $\tilde{\pi}_{i,c}$ of its majority owner by raising the other firm's profit $\pi_{j,c}$, so that the majority owner of firm i receives a higher dividend from firm j (Reynolds and Snapp 1986). Conversely, a higher value of α_j raises $\pi_{i,c}$: The product-market profits $\pi_{i,c}$ of firm i in competition rise if the majority owner of firm j holds a higher share α_j in firm i.

As in Malueg (1992) and Gilo et al. (2006), the firms are assumed to collude in the product market by setting a 50%-share of the monopoly output: $q_{i,k} = q_{j,k} = Q_k/2$. Because the collusive output is independent of α_i and α_j, the collusive profits are also independent of the value of shareholdings: $\partial \pi_{i,k}/\partial \alpha_i = 0$, $\partial \pi_{i,k}/\partial \alpha_j = 0$.

Deviation profits are defined as $\tilde{\pi}_{i,d}(\alpha_i, \alpha_j) = \tilde{\pi}_i\left(\tilde{q}_i^R(q_{j,k}, \alpha_i, \alpha_j), q_{j,k}\right)$ and $\tilde{\pi}_{j,-d}(\alpha_i, \alpha_j) = \tilde{\pi}_j\left(q_{j,k}, \tilde{q}_i^R(q_{j,k}, \alpha_i, \alpha_j)\right)$: Firm i plays its best response while firm j sets the agreed-upon output. Lemma 2 establishes the effect of α_i on the product market profits in a deviation period:

Lemma 2 $\frac{\partial \pi_{i,d}}{\partial \alpha_i} < 0$ and $\frac{\partial \pi_{j,-d}}{\partial \alpha_i} > 0$.

Proof See "Appendix 1". □

If firm i deviates from collusion, its majority owner receives a lower dividend from firm j as compared to continued collusion ($\alpha_i \pi_{j,-d} < \alpha_i \pi_{j,k}$). The higher is the value of α_i the stronger is this effect, and the lower is the payoff $\tilde{\pi}_{i,d}$ that the majority owner of firm i earns after the payment of dividends. Accordingly, minority shareholdings $\alpha_i > 0$ induce the deviating firm i to set a lower deviation quantity than with $\alpha_i = 0$ and, thus, to earn lower deviation profits in the product market. This leaves higher profits for firm j: $\partial \pi_{j,-d}/\partial \alpha_i > 0$.

Bertrand competition with differentiated products Similar effects are found when we assume Bertrand competition with differentiated products. The payoff $\tilde{\pi}(p_i, p_j)$ of firm i's majority shareholder is a function of the prices p_i and p_j of the two firms. Let $\tilde{p}_i^R(p_j, \alpha_i, \alpha_j)$ denote the best-response function of firm i after choosing $s_i = c$. The firms compete when both play their best responses, making profits $\tilde{\pi}_{i,c}(\alpha_i, \alpha_j) = \tilde{\pi}_i\left(\tilde{p}_i^R\left(\tilde{p}_j^R, \alpha_i, \alpha_j\right), \tilde{p}_j^R\left(\tilde{p}_i^R, \alpha_j, \alpha_i\right)\right)$. Our analysis relies on Lemma 3:

[3] The proof of Lemma 1 relies on the assumption of $\partial q_i^R/\partial q_j < 0$, which applies for the demand for homogeneous products that were assumed by Flath (1991) and Malueg (1992) as well as for several Cournot models with differentiated products. The analysis of the less common case with $\partial q_i^R/\partial q_j \geq 0$ is left for future research.

Lemma 3 $\frac{\partial \pi_{i,c}}{\partial \alpha_j} > 0$ and $\frac{\partial \pi_{i,c}}{\partial \alpha_i} \begin{cases} > 0 \text{ if } \alpha_i < \alpha_j + A \\ \leq 0 \text{ if } \alpha_i \geq \alpha_j + A. \end{cases}$

Proof See "Appendix 1". □

As compared to Cournot competition, firm i's competitive profit rises even for unilateral increases of its share α_i in firm j as long as the shares are not too asymmetric: $\alpha_i < \alpha_j + A$. The exact value of the threshold A depends on the functional form of demand as is shown in the proof of Lemma 3. This effect emerges because in a Bertrand model with differentiated products prices are strategic complements. Shareholdings α_i induce firm i to raise its price, and firm j follows suit. Therefore, even a somewhat asymmetric increase in α_i may cause an increase in both firms' profits. However, for $\alpha_i \geq \alpha_j + A$ the reduction in output that results from the higher prices becomes the dominant force, which leads to $\partial \pi_{i,c}/\partial \alpha_i \leq 0$.

The firms are assumed to collude in the product market by setting the same prices $p_{i,k}$ and $p_{j,k}$ that a jointly profit-maximizing monopolist would set. These prices are independent of α_i and α_j, which implies $\partial \pi_{i,k}/\partial \alpha_i = 0$ and $\partial \pi_{i,k}/\partial \alpha_j = 0$. The deviation payoffs are defined as $\tilde{\pi}_{i,d}(\alpha_i, \alpha_j) = \tilde{\pi}_i\big(\hat{p}_i^R(p_{j,k}, \alpha_i, \alpha_j), p_{j,k}\big)$ and $\tilde{\pi}_{j,-d}(\alpha_i, \alpha_j) = \tilde{\pi}_j\big(p_{j,k}, \tilde{p}_i^R(p_{j,k}, \alpha_i, \alpha_j)\big)$. "Appendix 1" shows that Lemma 2 ($\partial \pi_{i,d}/\partial \alpha_i < 0$ and $\partial \pi_{j,-d}/\partial \alpha_i > 0$) applies also in Bertrand competition with differentiated products.

Bertrand competition with homogeneous products In Bertrand competition with homogeneous products "both firms set prices equal to marginal cost regardless of the state of any partial cross shareholding" (Flath 1991): The firms make zero profits ($\pi_{i,c} = 0$, $\pi_{j,c} = 0$, $\partial \pi_{i,c}/\partial \alpha_i = 0$, and $\partial \pi_{j,c}/\partial \alpha_i = 0$), and minority shareholdings have no effect on firms' profits in competition. Similarly, the collusive and the deviation profits are also independent of the value of minority shareholdings: $\partial \pi_{i,k}/\partial \alpha_i = 0$; $\partial \pi_{j,k}/\partial \alpha_i = 0$; $\partial \pi_{i,d}/\partial \alpha_i = 0$; $\partial \pi_{j,-d}/\partial \alpha_i = 0$. A deviating firm would cut the collusive price marginally and earn $\pi_{i,d} = 2\pi_{i,k}$, while the betrayed firm would earn $\pi_{j,-d} = 0$.

4 The Dynamic Game

Using the framework introduced in Sect. 3, we study the effects of NCMS on the sustainability of collusion. Section 4.1 points out the forces that determine the effect of α_i on the critical discount factor $\tilde{\delta}_i^*$ in a general model. Section 4.2 applies this analysis to specific models of competition. Section 4.3 analyzes how minority shareholdings α_j of firm j's majority shareholder in firm i affect the critical discount factor $\tilde{\delta}_i^*$ of firm i.

4.1 The Critical Discount Factor

Collusion is profitable for the firms if inequality (6) is satisfied. Collusion is sustainable if inequality (7) applies:

$$\pi_{i,k} - \pi_{i,c} > 0 \quad \forall \quad i \in \{1,2\}; \tag{6}$$

$$\frac{\tilde{\pi}_{i,k}}{1-\delta_i} > \tilde{\pi}_{i,d} + \frac{\delta_i}{1-\delta_i}\tilde{\pi}_{i,c} \quad \forall \quad i \in \{1,2\}. \tag{7}$$

In line with Malueg (1992), Aubert et al. (2006), and Gilo et al. (2006), the present value of deviation payoffs–the right-hand side of (7)–assumes a grim trigger strategy (Friedman 1971). With the use of equation (1), $\tilde{\pi}_i = (1-\alpha_j)\pi_i + \alpha_i\pi_j$, the sustainability constraint (7) can be solved for the critical value $\tilde{\delta}_i^*$ of the discount factor as is shown in equation (8). For individual discount factors above this threshold, collusion is a stable outcome:

$$\delta_i > \frac{(1-\alpha_j)(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k})}{(1-\alpha_j)(\pi_{i,d} - \pi_{i,c}) + \alpha_i(\pi_{j,-d} - \pi_{j,c})} \equiv \tilde{\delta}_i^*. \tag{8}$$

Proposition 1 establishes under which condition the critical discount factor rises in the value of minority shareholdings α_i:

Proposition 1 *The inequality* $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$ *applies if inequality* (9) *is satisfied*:

$$\begin{aligned}\frac{(1-\alpha_j)\left[(\pi_{j,k} - \pi_{j,c})\pi_{i,d} - (\pi_{i,k} - \pi_{i,c})\pi_{j,-d} - (\pi_{i,c} - \pi_{j,c})\pi_{i,k}\right]}{(1-\alpha_j)(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k})} \\ < (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}.\end{aligned} \tag{9}$$

Proof See "Appendix 2". □

To interpret condition (9), higher shareholdings α_i have two effects on the sustainability of collusion:

Effect 1: Malueg (1992) argued that higher shareholdings have a destabilizing effect on collusion by increasing firms' profits in competition and, thus, reducing the grim trigger punishment $\pi_{i,c} - \pi_{i,k}$ that follows a deviation. This can be inferred from the right-hand side of (9), which captures the elevating effect of minority shareholdings on firms' profits in competition. One finds $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$ when the right-hand side of inequality (9) is sufficiently high.

Effect 2: NCMS also have a stabilizing effect on collusion. If firm i deviates, the profits of firm j fall from $\pi_{j,k}$ to $\pi_{j,-d}$. This reduces the dividend that firm j pays to the majority owner of firm i. In absolute terms, this reduction of the dividend is stronger if the majority owner of firm i holds a greater share α_i in firm j. This makes firm i more reluctant to deviate because $\alpha_i(\pi_{j,-d} - \pi_{j,k})$ in the denominator of the left-hand side of (9) measures the loss of dividends that are received from firm j if firm i deviates. A higher value of α_i raises the left-hand side of (9) and, thus, contributes to situations with $\partial\tilde{\delta}_i^*/\partial\alpha_i < 0$.

4.2 The Sustainability of Collusion in Different Models of Competition

This subsection explores the effect of minority shareholdings on the stability of collusion by exploring the relative strength of Effect 1 and Effect 2 in different models of competition.

Cournot competition In Cournot competition, an asymmetric increase in the shareholdings α_i cannot destabilize collusion by raising $\tilde{\delta}_i^*$ (Lemma 4), yet a symmetric increase can have this destabilizing effect (Lemma 5).

Lemma 4 *In Cournot competition with* $\partial q_j^R/\partial q_i < 0$, *one finds that* $(1-\alpha_j)\cdot\partial\pi_{i,c}/\partial\alpha_i + \alpha_i\cdot\partial\pi_{j,c}/\partial\alpha_i < 0$.

Proof See "Appendix 2". □

Lemma 1 established that in Cournot competition firm i would want to reduce its output and, thus, soften competition if its majority shareholder acquires shares α_i of its rival j. Because of the strategic substitutability of quantities ($\partial q_j^R/\partial q_i < 0$), firm j would however react by expanding its output, which leads to $\partial\pi_{i,c}/\partial\alpha_i < 0$ and $\partial\pi_{j,c}/\partial\alpha_i > 0$. Because firm i's majority shareholder participates more in the decline of firm i's profits than in the increasing profits of j, $(1-\alpha_j) > \alpha_i$, one finds that $(1-\alpha_j)\cdot\partial\pi_{i,c}/\partial\alpha_i + \alpha_i\cdot\partial\pi_{j,c}/\partial\alpha_i < 0$. Therefore, inequality (9) would be violated, which implies $\partial\tilde{\delta}_i^*/\partial\alpha_i < 0$.

Lemma 5 shows that this is different if the firms choose symmetric shareholdings and also if any increase in the shareholdings is symmetric: $\alpha_i = \alpha_j = \alpha\ \forall\ \alpha_i, \alpha_j$.

Lemma 5 *In Cournot competition with* $\partial q_j^R/\partial q_i < 0$ *and* $\alpha_i = \alpha_j = \alpha\ \forall\ \alpha_i, \alpha_j$, *one finds that* $(1-\alpha)\partial\pi_{i,c}/\partial\alpha + \alpha\cdot\partial\pi_{j,c}/\partial\alpha > 0$.

Proof See "Appendix 2". □

The inequality $(1-\alpha)\partial\pi_{i,c}/\partial\alpha + \alpha\cdot\partial\pi_{j,c}/\partial\alpha > 0$ for symmetric increases of the shareholdings is a prerequisite for the destabilizing effect of NCMS on collusion as was shown by Malueg (1992). If the firms raise their shares in each other symmetrically, both firms' profits $\pi_{i,c}, \pi_{j,c}$ rise. This reduces the loss that is inflicted on a firm by grim trigger punishments and contributes to raising the critical discount factor $\tilde{\delta}_i^*$.

Bertrand competition with differentiated products Collusion-destabilizing effects of NCMS – $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$ – are more likely in Bertrand competition with differentiated products than in Cournot competition. This is because in Bertrand competition with differentiated products even asymmetric increases of α_i may raise the right-hand side of inequality (9) as was shown by Lemma 3. This finding suggests that the pro-competitive effects of NCMS are even more prevalent than was suggested by Malueg's (1992) seminal contribution.

Bertrand competition with homogeneous products NCMS do not have an impact on firms' profits under the assumption of price competition with homogeneous products: $\partial\pi_{i,c}/\partial\alpha_i = 0, \partial\pi_{j,c}/\partial\alpha_j = 0$. Therefore, the right-hand side of inequality (9)

takes a value of zero, which implies $\partial\delta_i^*/\partial\alpha_i < 0 \; \forall \; \alpha_i$. This result is in line with the finding of Gilo et al. (2006), who showed that under the assumption of Bertrand competition with homogeneous products "an increase in firm [i]'s stake in firm [j] never hinders tacit collusion".[4]

4.3 The Effect of α_j on $\tilde{\delta}_i^*$

The previous analysis was concerned with the effects of the stake α_i, which firm i's majority shareholder possesses in firm j, on i's critical discount factor $\tilde{\delta}_i^*$. Proposition 2 goes beyond the prior literature by showing under what condition the stake α_j of firm j's majority stakeholder in firm i makes collusion harder to sustain by increasing firm i's critical discount factor $\tilde{\delta}_i^*$:

Proposition 2 *The inequality* $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$ *applies if inequality* (10) *is satisfied*:

$$\frac{\alpha_i\left[(\pi_{j,k}-\pi_{j,c})\pi_{i,d}-(\pi_{i,k}-\pi_{i,c})\pi_{j,-d}-(\pi_{i,c}-\pi_{j,c})\pi_{i,k}+(\pi_{i,k}-\pi_{j,k})\pi_{i,c}\right]}{(1-\alpha_j)(\pi_{i,d}-\pi_{i,k})+\alpha_i(\pi_{j,-d}-\pi_{j,k})} < (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_j}+\alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_j}. \tag{10}$$

Proof See "Appendix 2". □

The inequality $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$, which results if inequality (10) applies, is a result of firm j's incentive to soften competition after its majority shareholder acquires shares of firm i. This raises the profits that are made by both firms when competing, and it thus softens the grim trigger punishments, which makes it more attractive for firm i to deviate.

Note that inequality (10) in Proposition 2 is easier to satisfy than inequality (9) in Proposition 1. This can be seen particularly well when starting from a situation of symmetry ($\alpha_i = \alpha_j$) before analyzing an asymmetric expansion of α_j. Symmetry causes $\pi_{i,c} - \pi_{j,c} = 0$, $\pi_{i,k} - \pi_{j,k} = 0$, $\partial\pi_{i,c}/\partial\alpha_i = \partial\pi_{j,c}/\partial\alpha_j$ and $\partial\pi_{j,c}/\partial\alpha_i = \partial\pi_{i,c}/\partial\alpha_j$. Together with $\alpha_i < 1 - \alpha_j$, these equalities can be used to show that the right-hand side of (10) is weakly greater than that of (9).

Taken together, this causes $\tilde{\delta}_i^*$ to be increased more easily by an increase in α_j than by an increase in α_i. Because earlier literature focused on $\partial\tilde{\delta}_i^*/\partial\alpha_i$, Proposition 2 implies that a collusion-destabilizing effect of minority shareholdings occurs under a wider set of conditions than was predicted by this earlier literature.

[4] Note that Gilo et al. (2006) assumed that the firms maximize the profit function $\hat{\pi}_i = \pi_i + \alpha_i\hat{\pi}_j$ rather than $\tilde{\pi}_i = (1-\alpha_j)\pi_i + \alpha_i\pi_j$. Although these functions model different aspects of minority shareholdings – common ownership vs. cross ownership – their effect on our results is minor and has no impact on our qualitative conclusions as will be discussed in Sect. 6.

5 Competition Authority

As a further addition to prior literature, we merge our model with that of Aubert et al. (2006) to study the effect of a competition authority on the stability of collusion in the presence of common ownership. This is modeled by assuming that the competition authority detects a collusive agreement with probability ρ in every period and then imposes a fine F on every colluding firm.

Proposition 3 demonstrates the main result of this section: The collusion-destabilizing effect of common ownership is further enhanced by $\rho F > 0$.

Proposition 3 *Given the detection probability ρ and fine F, the critical discount factor takes the functional form shown in (11). Inequality $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$ applies if inequality (12) is satisfied, which is easier to satisfy than the corresponding inequality (9) for $\rho F = 0$:*

$$\tilde{\delta}_i^* \equiv \frac{(1-\alpha_j)(\pi_{i,d}-\pi_{i,k})+\alpha_i(\pi_{j,-d}-\pi_{j,k})}{(1-\alpha_j)(\pi_{i,d}-\rho F-\pi_{i,c})+\alpha_i(\pi_{j,-d}-\rho F-\pi_{j,c})}; \tag{11}$$

$$\begin{aligned}\frac{(1-\alpha_j)\left[(\pi_{j,k}-\rho F-\pi_{j,c})\pi_{i,d}-(\pi_{i,k}-\rho F-\pi_{i,c})\pi_{j,-d}-(\pi_{i,c}-\pi_{j,c})\pi_{i,k}\right]}{(1-\alpha_j)(\pi_{i,d}-\pi_{i,k})+\alpha_i(\pi_{j,-d}-\pi_{j,k})}\\ <(1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_i}+\alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}.\end{aligned} \tag{12}$$

Proof See "Appendix 3".

Proposition 3 suggests that $\rho F > 0$ makes minority shareholdings more likely to destabilize collusion. This is because the competition authority imposes a fine not only on firm i but also on firm j. This causes the expected dividend income of firm i's minority shareholder to be low whether the firm adheres to the collusive agreement or not, which makes a deviation relatively more profitable for firm i. Mathematically, $\rho F > 0$ decreases the left-hand side of (12), which makes it more likely to find $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$.

6 Cross Ownership

Up to this point, we have relied on profit function (1): $\tilde{\pi}_i = (1-\alpha_j)\pi_i + \alpha_i\pi_j$. The payoff $\tilde{\pi}_i$ models the income of a common shareholder who possesses a majority share $1-\alpha_j$ of firm i and a minority share α_i of firm j.

A different profit function was used, for example, by Flath (1991), Gilo et al. (2006), and Shelegia and Spiegel (2012), which is shown in (13):

$$max\ \hat{\pi}_i = \pi_i + \alpha_i\hat{\pi}_j. \tag{13}$$

The profit $\hat{\pi}_i$ models firm i's profit as it appears in its income statement: its operating profit π_i plus its investment income $\alpha_i \hat{\pi}_j$. This profit function is typically used when modeling the effect of cross ownership, with α_i denoting the share of firm i in firm j. An overview about the literature that surrounds the interpretation of profit functions (1) and (13) was given by Schmalz (2018).

In "Appendix 4", we show that only the functional form but not the economic interpretation of our results changes if one instead assumes profit function (13). As above, the critical discount factor is shown to rise in α_i and (even more easily) in α_j if the NCMS raise firms' profits in competition sufficiently strongly. A high value of the expected fine facilitates these situations.

7 Conclusion

This article presents a framework for the analysis of collusion in the presence of common ownership or cross ownership. We combined the established models of Reynolds and Snapp (1986), Flath (1991, 1992), Malueg (1992), and Gilo et al. (2006). These earlier contributions had not always been easy to compare: First, because they used different profit functions. Second, some combinations of assumptions had so far remained unexplored. Third, they sometimes relied on numerical proofs. We fill these gaps in the literature by studying the 'missing' combinations of assumptions and thus improving the comparability of these earlier contributions. The properties of our model are proven analytically. Additionally, we extend this literature by adding a competition authority (Aubert et al. 2006) and analyzing the effect of firm i's shareholdings on the critical discount factor of firm j.

Malueg (1992) pointed out that NCMS raise the profits that firms make when competing, which contributes to weakening the sustainability of collusion. Our study indicates that NCMS weaken the sustainability of collusion under an even greater variety of circumstances than was acknowledged by this earlier literature.

The integrated framework that we proposed in this article establishes a basis for further theoretical extensions. For example, it will be interesting to endogenize firms' decisions to acquire NCMS. The firms might decide about splitting the collusive profits unequally or making side payments, which might be necessary if the firms were asymmetric in costs. Future work should extend our duopoly model to the case of n firms and, additionally, explore the impact of other than grim trigger punishments. Several approaches have been proposed for modeling the effect of competition authorities on collusion among independent firms (see Harrington (2017) for a review), which may now also be applied to the study of collusion in the presence of common ownership or cross ownership.

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Appendix to Section 3

The following proofs assume that the firms produce subject to constant marginal costs taking a value of zero.

Proof Lemma 1 proposes $\partial\pi_{i,c}/\partial\alpha_i < 0$ and $\partial\pi_{i,c}/\partial\alpha_j > 0$ for Cournot competition with $\partial q_i^R/\partial q_j < 0$ and profit function (1). To prove this, reaction function (A.3) is determined by deriving profit function (A.1) for q_i and solving first-order condition (A.2) for q_i, while we define $q_i^R(q_j) = \tilde{q}_i^R(q_j, 0, 0)$:

$$\tilde{\pi}_i = (1-\alpha_j)p(q_i, q_j)q_i + \alpha_i p(q_i, q_j)q_j; \tag{A.1}$$

$$\frac{\partial\tilde{\pi}_i}{\partial q_i} = (1-\alpha_j)\left[\frac{\partial p}{\partial q_i}q_i + p(q_i, q_j)\right] + \alpha_i\frac{\partial p}{\partial q_i}q_j = 0; \tag{A.2}$$

$$\tilde{q}_i^R(q_j, \alpha_i, \alpha_j) = q_i^R(q_j) - \frac{\alpha_i}{1-\alpha_j}\theta q_j. \tag{A.3}$$

The variable θ is a scaling factor with $0 < \theta < 1$. For a specific homogeneous products demand curve $p = (1 - q_i - q_j)^x$ with $x > 0$ (Malueg 1992) one finds $\theta = x/(1+x)$, and one can show that $-1 < \partial q_i^R/\partial q_j < 0$. Given $\partial q_i^R/\partial q_j < 0$, equation (A.3) implies $\partial\tilde{q}_i^R/\partial q_j < 0$ and, thus, $\partial\tilde{q}_j^R/\partial q_i < 0$.

To prove $\partial\pi_{i,c}/\partial\alpha_i < 0$, note that equation (A.3) implies $\tilde{q}_i^R(q_j, \alpha_i, \alpha_j) < q_i^R(q_j)$ for an unchanged value of q_j. Because of $\partial\tilde{q}_j^R/\partial q_i < 0$ firm j will optimally expand its output, which causes firm i to contract its output even further. This implies that $\tilde{q}_i^R(\tilde{q}_j^R, \alpha_i, \alpha_j) < q_i^R(q_j^R)$ and $\tilde{q}_j^R(\tilde{q}_i^R, \alpha_j, \alpha_i) > q_j^R(q_i^R)$, $\partial\tilde{q}_i^R/\partial\alpha_i < 0$ and $\partial\tilde{q}_j^R/\partial\alpha_i > 0$. Firm i's product market profit is reduced by selling a lower than the optimal quantity $\tilde{q}_i^R < q_i^R$ while firm j raises its output, which implies and proves $\partial\pi_{i,c}/\partial\alpha_i < 0$.

Because the output reduction of firm i exceeds the output expansion of firm j the equilibrium price rises: From $-1 < \partial\tilde{q}_j^R/\partial q_i$ it follows that $\partial\tilde{q}_j^R/\partial\alpha_i = (\partial\tilde{q}_j^R/\partial q_i)\cdot(\partial\tilde{q}_i^R/\partial\alpha_i) < -\partial\tilde{q}_i^R/\partial\alpha_i$, such that $\partial(\tilde{q}_i^R + \tilde{q}_j^R)/\partial\alpha_i < 0$ and $\partial p(\tilde{q}_i^R + \tilde{q}_j^R)/\partial\alpha_i > 0$. Combining $\partial p(\tilde{q}_i^R + \tilde{q}_j^R)/\partial\alpha_i > 0$ and $\partial\tilde{q}_j^R/\partial\alpha_i > 0$ proves that $\partial\pi_{j,c}/\partial\alpha_i > 0$ for Cournot competition and profit function (1). □

Proof Lemma 2 proposes for Cournot competition that $\partial\pi_{i,d}/\partial\alpha_i < 0$ and $\partial\pi_{j,-d}/\partial\alpha_i > 0$. Firm j keeps its output $q_{j,-d} = q_{j,k}$ constant by assumption: $\partial q_{j,-d}/\partial\alpha_i = 0$. Reaction function (A.3) with $\partial\tilde{q}_i^R(q_{j,-d}, \alpha_i, \alpha_j)/\partial\alpha_i < 0$ causes $\partial(q_{j,-d} + \tilde{q}_i^R(q_{j,-d}, \alpha_i, \alpha_j))/\partial\alpha_i < 0$ and thus $\partial p_d/\partial\alpha_i > 0$. This proves that $\partial\pi_{j,-d}/\partial\alpha_i = (\partial p_d/\partial\alpha_i) \cdot q_{j,-d} > 0$. Inequality $\partial\tilde{q}_i^R(q_{j,-d}, \alpha_i, \alpha_j)/\partial\alpha_i < 0$ also implies that with $\alpha_i > 0$ firm i sets a lower output as in a case with $\alpha_i, \alpha_j = 0$, i.e., $\tilde{q}_i^R(q_{j,-d}, \alpha_i, \alpha_j) < q_i^R(q_{j,-d})$, which causes $\pi_i(\tilde{q}_i^R, q_{j,-d}) < \pi_i(q_i^R, q_{j,-d})$. This proves $\partial\pi_{i,d}/\partial\alpha_i < 0$. □

Proof Lemma 3 proposes for Bertrand competition with differentiated products that $\partial\pi_{i,c}/\partial\alpha_i > 0$ if $\alpha_i < \alpha_j + A$, $\partial\pi_{i,c}/\partial\alpha_i \leq 0$ if $\alpha_i \geq \alpha_j + A$, and $\partial\pi_{i,c}/\partial\alpha_j > 0$. The term $\tilde{p}_i^R(p_j, \alpha_i, \alpha_j)$ in equation (A.6) denotes the reaction function of firm i, and it is determined by maximizing profit function (A.4) with respect to p_i. Equation (A.5) presents the first-order condition. The term $p_i^R(p_j)$ in (A.6) denotes the reaction function if firm i maximizes its own product market profits π_i only, i.e., $p_i^R(p_j) = \tilde{p}_i^R(p_j, 0, 0)$:

$$\tilde{\pi}_i = (1 - \alpha_j) q_i(p_i, p_j) p_i + \alpha_i q_j(p_j, p_i) p_j; \tag{A.4}$$

$$\frac{\partial\tilde{\pi}_i}{\partial p_i} = (1 - \alpha_j)\left[\frac{\partial q_i}{\partial p_i} p_i + q_i(p_i, p_j)\right] + \alpha_i \frac{\partial q_j}{\partial p_i} p_j = 0; \tag{A.5}$$

$$\tilde{p}_i^R(p_j, \alpha_i, \alpha_j) = p_i^R(p_j) + \frac{\alpha_i}{1 - \alpha_j} B p_j. \tag{A.6}$$

For a specific demand function such as $q_i = 1 - p_i + \beta p_j$ with $0 < \beta < 1$, one can prove that $0 < \partial p_i^R/\partial p_j < 1$, and one finds that $B = \beta/2 > 0$. Equation (A.6) indicates that $\partial\tilde{p}_i^R/\partial\alpha_i > 0$ for a given value of p_j. Moreover, firm i will increase its price if firm j acquires shares of firm i, i.e., $\partial\tilde{p}_i^R/\partial\alpha_j$, which can be written as in (A.7):

$$\frac{\partial\tilde{p}_i^R(\tilde{p}_j^R, \alpha_i, \alpha_j)}{\partial\alpha_j} = \frac{\partial p_i^R}{\partial p_j} \cdot \frac{\partial\tilde{p}_j^R}{\partial\alpha_j} + \frac{\alpha_i B p_j}{(1 - \alpha_j)^2} + \frac{\alpha_i B}{1 - \alpha_j} \cdot \frac{\partial\tilde{p}_j^R}{\partial\alpha_j}. \tag{A.7}$$

Given (A.7), $\partial p_i^R/\partial p_j > 0$, and $\partial\tilde{p}_j^R/\partial\alpha_j > 0$, one finds that $\partial\tilde{p}_i^R/\partial\alpha_j > 0$ and, accordingly, $\partial\tilde{p}_j^R/\partial\alpha_i > 0$. For $q_i = 1 - p_i + \beta p_j$ one can show that $\partial\tilde{p}_j^R/\partial\alpha_i < \partial\tilde{p}_i^R/\partial\alpha_i$ and $\partial q_i(\tilde{p}_i^R, \tilde{p}_j^R)/\partial\alpha_i < 0$ apply. Because of $\partial\tilde{p}_j^R/\partial\alpha_j > 0$ after firm j acquires shares α_j of firm i, the output and profit of firm i would rise even when holding its own price constant. Firm i will only raise its price ($\partial\tilde{p}_j^R/\partial p_j > 0$) if this raises its profit even further, which proves $\partial\pi_{i,c}/\partial\alpha_j > 0$.

Yet, the inequalities $\partial\tilde{p}_i^R/\partial\alpha_i > 0$ and $\partial q_i(\tilde{p}_i^R, \tilde{p}_j^R)/\partial\alpha_i < 0$ suggest an ambiguous effect on the profits of firm i ($\partial\pi_{i,c}/\partial\alpha_i \lesseqgtr 0$). For proving that $\partial\pi_{i,c}/\partial\alpha_i > 0$ if $\alpha_i < \alpha_j + A$, one has to assume a specific functional form for

demand: We use $q_i = 1 - p_i + \beta p_j$.[5] The price and profit of firm i in the product market-equilibrium are then shown in (A.8) and (A.9):

$$\tilde{p}_i^R\left(\tilde{p}_j^R, \alpha_i, \alpha_j\right) = \frac{2 + \left(\frac{1+(\alpha_i-\alpha_j)}{1-\alpha_j}\right)\beta}{4 - \left(\frac{1+(\alpha_i-\alpha_j)}{1-\alpha_j}\right)\beta^2\left(\frac{1-(\alpha_i-\alpha_j)}{1-\alpha_i}\right)}; \tag{A.8}$$

$$\pi_{i,c} = \pi_i\left(\tilde{p}_i^R, \tilde{p}_j^R\right) = q_i(\tilde{p}_i^R, \tilde{p}_j^R)\tilde{p}_i^R = (1 - \tilde{p}_i^R + \beta\tilde{p}_j^R)\tilde{p}_i^R. \tag{A.9}$$

Ideally, one would want to plug $\tilde{p}_i^R$ and $\tilde{p}_j^R$ into (A.9), determine $\partial\pi_{i,c}/\partial\alpha_i$ and solve inequality $\partial\pi_{i,c}/\partial\alpha_i > 0$ for α_i, which would yield a term $\alpha_i < \alpha_j + A$. Yet, the threshold A proves to be a highly nonlinear and lengthy function of α_j and β.

We thus resort to a more subtle proof: The condition $\partial\pi_{i,c}/\partial\alpha_i > 0$ can be stated as in (A.10).

$$\begin{aligned}\frac{\partial\pi_{i,c}}{\partial\alpha_i} =& \left(-\frac{\partial\tilde{p}_i^R}{\partial\alpha_i} + \beta\frac{\partial\tilde{p}_j^R}{\partial\alpha_i}\right)\tilde{p}_i^R + \left(1 - \tilde{p}_i^R + \beta\tilde{p}_j^R\right)\frac{\partial\tilde{p}_i^R}{\partial\alpha_i} > 0 \\ & \frac{1 + \beta\tilde{p}_j^R}{2 - \left(\frac{\partial\tilde{p}_j^R/\partial\alpha_i}{\partial\tilde{p}_i^R/\partial\alpha_i}\right)\beta} > \tilde{p}_i^R\end{aligned} \tag{A.10}$$

When holding α_j constant, a unilateral increase in α_i increases $\pi_{i,c}$ as long as $\tilde{p}_i^R$ is smaller than the left-hand side of (A.10). Because of $\partial\tilde{p}_i^R/\partial\alpha_i > \partial\tilde{p}_j^R/\partial\alpha_i$ an increase in α_i increases $\tilde{p}_i^R$ more strongly than the left-hand side of (A.10), so that $\partial\pi_{i,c}/\partial\alpha_i \leq 0$ will apply once reaching or exceeding the tipping point $\alpha_i \geq \alpha_j + A$. It can be seen that $\partial\pi_{i,c}/\partial\alpha_i > 0$ is a consequence of the strategic complementarity of prices: Inequality (A.10) is more easily satisfied for higher values of both $\partial\tilde{p}_j^R/\partial\alpha_i$ and β. □

Proof Lemma 2 proposes for Bertrand competition with differentiated products $\partial\pi_{i,d}/\partial\alpha_i < 0$ and $\partial\pi_{j,-d}/\partial\alpha_i > 0$. The assumption $p_{j,-d} = p_{j,k}$ implies $\partial p_{j,-d}/\partial\alpha_i = 0$. Reaction function (A.6) causes a higher deviation price $(\partial\tilde{p}_i^R(p_{j,-d}, \alpha_i, \alpha_j)/\partial\alpha_i > 0)$ in the presence of minority shareholdings and thus less business stealing $(\partial q_{j,-d}/\partial\alpha_i > 0)$. This proves $\partial\pi_{j,-d}/\partial\alpha_i = (\partial q_{j,-d}/\partial\alpha_i) \cdot p_{j,-d} > 0$. Finding $\partial\tilde{p}_i^R(p_{j,-d}, \alpha_i, \alpha_j)/\partial\alpha_i > 0$ also implies that with $\alpha_i > 0$ firm i sets a higher than the profit-maximizing price $p_i^R(p_{j,-d}) < \tilde{p}_i^R(p_{j,-d}, \alpha_i, \alpha_j)$, which causes $\pi_i(p_i^R, p_{j,-d}) > \pi_i(\tilde{p}_i^R, p_{j,-d})$. This proves $\partial\pi_{i,d}/\partial\alpha_i < 0$. □

[5] Flath (1991) provides proofs for a Hotelling model. A Salop circle model with three firms and asymmetric, controlling shareholdings was provided by Øystein et al. (2011). Yet, these proofs were done under the assumption of profit function (13).

Appendix to Section 4

The proof of Proposition 1 relies on Lemma 6:

Lemma 6 $(1-\alpha_j)\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = 0.$

Proof For *Cournot competition with homogeneous products*, the equality stated in Lemma 6 can be restated as in (B.1):

$$(1-\alpha_j)\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial q_i}{\partial \alpha_i}\left[(1-\alpha_j)\left(\frac{\partial p}{\partial q_i}q_i + p\right) + \alpha_i \frac{\partial p}{\partial q_i}q_j\right]. \tag{B.1}$$

The bracketed term on the right-hand side of equation (B.1) is the same as the bracketed term in first-order condition (A.2) that takes a value of zero in the optimum. For *Bertrand competition with differentiated products*, the equality stated in Lemma 6 can be restated as in (B.2):

$$(1-\alpha_j)\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial p_i^R}{\partial \alpha_i}\left[(1-\alpha_j)\left(\frac{\partial q_i}{\partial p_i}p_i + q_i\right) + \alpha_i\left(\frac{\partial q_j}{\partial p_i}q_j\right)\right] = 0. \tag{B.2}$$

The bracketed term on the right-hand side of equation (B.2) is the same as the bracketed term in first-order condition (A.5) that takes a value of zero in the optimum. Section 3.2 established that $\partial \pi_{i,d}/\partial \alpha_i = 0$ and $\partial \pi_{j,-d}/\partial \alpha_i = 0$ for *Bertrand competition with homogeneous products*, which proves $(1-\alpha_j)(\partial \pi_{i,d}/\partial \alpha_i) + \alpha_i(\partial \pi_{j,-d}/\partial \alpha_i) = 0$. □

Proof To determine $\partial \tilde{\delta}_i^*/\partial \alpha_i > 0$ (Proposition 1), re-write $\tilde{\delta}_i^*$ as follows:

$$\begin{aligned} \tilde{\delta}_i^* &= \frac{u(\alpha_i, \alpha_j)}{v(\alpha_i, \alpha_j)} \\ \text{with } u(\alpha_i, \alpha_j) &= (1-\alpha_j)(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k}) > 0 \\ \text{and } v(\alpha_i, \alpha_j) &= (1-\alpha_j)(\pi_{i,d} - \pi_{i,c}) + \alpha_i(\pi_{j,-d} - \pi_{j,c}) > 0. \end{aligned} \tag{B.3}$$

If we use $\partial \pi_{i,k}/\partial \alpha_i = \partial \pi_{j,k}/\partial \alpha_i = 0$ and $(1-\alpha_j)(\partial \pi_{i,d}/\partial \alpha_i) + \alpha_i(\partial \pi_{j,-d}/\partial \alpha_i) = 0$ from Lemma 6, $\partial \tilde{\delta}_i^*/\partial \alpha_i$ can be written as in (B.4):

$$\begin{aligned} \frac{\partial \tilde{\delta}_i^*}{\partial \alpha_i} &= \frac{\frac{\partial u(\alpha_i, \alpha_j)}{\partial \alpha_i} \cdot v(\alpha_i, \alpha_j) - \frac{\partial v(\alpha_i, \alpha_j)}{\partial \alpha_i} \cdot u(\alpha_i)}{v(\alpha_i, \alpha_j)^2} \\ \text{with } \frac{\partial u(\alpha_i, \alpha_j)}{\partial \alpha_i} &= \pi_{j,-d} - \pi_{j,k} \\ \text{and } \frac{\partial v(\alpha_i, \alpha_j)}{\partial \alpha_i} &= \pi_{j,-d} - \pi_{j,c} - \left((1-\alpha_j)\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_i}\right). \end{aligned} \tag{B.4}$$

Given $v(\alpha_i, \alpha_j) > 0$ the sign of $\partial\tilde{\delta}^*/\partial\alpha_i$ is the same as that of its numerator as is shown in (B.5):

$$\begin{aligned}&\frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_i}\cdot v(\alpha_i,\alpha_j) - \frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_i}\cdot u(\alpha_i,\alpha_j) = \ldots\\ &(1-\alpha_j)\left[(\pi_{i,k}-\pi_{i,c})\pi_{j,-d} - (\pi_{j,k}-\pi_{j,c})\pi_{i,d} - (\pi_{i,c}-\pi_{j,c})\pi_{i,k}\right]\\ &+\left((1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)u(\alpha_i,\alpha_j).\end{aligned} \tag{B.5}$$

Equation (B.5), and thus $\partial\tilde{\delta}^*/\partial\alpha_i$, is positive if inequality (9) applies. This proves Proposition 1. □

The proof of Proposition 2 relies on Lemma 7:

Lemma 7 $(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = 0.$

Proof For *Cournot competition with homogeneous products*, the equality stated in Lemma 7 can be restated as in (B.6):

$$(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = \frac{\partial q_i}{\partial\alpha_j}\left[(1-\alpha_j)\left(\frac{\partial p}{\partial q_i}q_i + p\right) + \alpha_i\left(\frac{\partial p}{\partial q_i}q_j\right)\right] = 0. \tag{B.6}$$

The bracketed term on the right-hand side of equation (B.6) is the same as the bracketed term in first-order condition (A.2) that takes a value of zero in the optimum. For *Bertrand competition with differentiated products*, the equality that is stated in Lemma 7 can be restated as in (B.7):

$$(1-\alpha_j)\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = \frac{\partial p_i}{\partial\alpha_j}\left[(1-\alpha_j)\left(\frac{\partial q_i}{\partial p_i}p_i + q_i\right) + \alpha_i\left(\frac{\partial q_j}{\partial p_i}q_j\right)\right] = 0. \tag{B.7}$$

The bracketed term on the right-hand side of equation (B.7) is the same as the bracketed term in first-order condition (A.5) that takes a value of zero in the optimum. Section 3.2 established that $\partial\pi_{i,d}/\partial\alpha_j = 0$ and $\partial\pi_{j,-d}/\partial\alpha_j = 0$ for *Bertrand competition with homogeneous products*, which proves $(1-\alpha_j)(\partial\pi_{i,d}/\partial\alpha_j) + \alpha_i(\partial\pi_{j,-d}/\partial\alpha_j) = 0$. □

Proof Lemma 4 proposes $(1-\alpha_j)\partial\pi_{i,c}/\partial\alpha_i + \alpha_i\cdot\partial\pi_{j,c}/\partial\alpha_i < 0$ in Cournot competition with $\partial q_j^R/\partial q_i < 0$. Given $\pi_i = p\cdot q_i$ the first derivative $\partial\pi_{i,c}/\partial\alpha_i$ can be written as in (B.8) if firm i chooses its output $\tilde{q}_i^R$ according to the best response function (A.3). The first derivative $\partial p/\partial\alpha_i$ can be written as in (B.9) if one considers $Q = q_i + q_j$ and $\partial p/\partial Q = \partial p/\partial q_i$ because of $\partial Q/\partial q_i = 1$. Similarly, the derivative $\partial\pi_{j,c}/\partial\alpha_i$ of $\pi_j = p\cdot q_j$ is shown in (B.10):

$$\frac{\partial \pi_{i,c}}{\partial \alpha_i} = \frac{\partial p}{\partial \alpha_i} q_i + \frac{\partial \tilde{q}_i^R}{\partial \alpha_i} p = \frac{\partial \tilde{q}_i^R}{\partial \alpha_i} \left[\frac{\partial p}{\partial q_i} \left(1 + \frac{\partial q_j}{\partial q_i} \right) q_i + p \right]; \tag{B.8}$$

$$\frac{\partial p}{\partial \alpha_i} = \frac{\partial p}{\partial Q} \frac{\partial Q}{\partial \alpha_i} = \frac{\partial p}{\partial q_i} \left[\frac{\partial \tilde{q}_i^R}{\partial \alpha_i} + \frac{\partial q_j}{\partial \tilde{q}_i^R} \frac{\partial \tilde{q}_i^R}{\partial \alpha_i} \right]; \tag{B.9}$$

$$\frac{\partial \pi_{j,c}}{\partial \alpha_i} = \frac{\partial p}{\partial \alpha_i} q_j + \frac{\partial q_j}{\partial \alpha_i} p = \frac{\partial \tilde{q}_i^R}{\partial \alpha_i} \left[\frac{\partial p}{\partial q_i} \left(1 + \frac{\partial q_j}{\partial q_i} \right) q_j + \frac{\partial q_j}{\partial q_i} p \right]. \tag{B.10}$$

Using (B.8) and (B.10), the term $(1-\alpha_j)\partial \pi_{i,c}/\partial \alpha_i + \alpha_i \cdot \partial \pi_{j,c}/\partial \alpha_i < 0$ can be expressed as in (B.11):

$$\begin{aligned} &(1-\alpha_j)\frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,c}}{\partial \alpha_i} = \dots \\ &\frac{\partial \tilde{q}_i^R}{\partial \alpha_i} \left[\left[(1-\alpha_j) \left(\frac{\partial p}{\partial q_i} q_i + p \right) + \alpha_i \frac{\partial p}{\partial q_i} q_j \right] \left(1 + \frac{\partial q_j}{\partial q_i} \right) + \frac{\partial q_j}{\partial q_i} p(\alpha_i + \alpha_j - 1) \right]. \end{aligned} \tag{B.11}$$

The term $(1-\alpha_j)(\partial p/\partial q_i \cdot q_i + p) + \alpha_i \cdot \partial p/\partial q_i \cdot q_j$ takes a value of zero as follows from first-order condition (A.2). Lemma 1 showed that $\partial \tilde{q}_i^R/\partial \alpha_i < 0$. Combining this with $\partial q_j/\partial \tilde{q}_i^R < 0$ and $\alpha_i + \alpha_j - 1 < 0$, one finds $\partial \pi_{i,c}/\partial \alpha_i + \alpha_i \cdot \partial \pi_{j,c}/\partial \alpha_i < 0$, which proves Lemma 4. □

Proof With $\alpha_i = \alpha_j = \alpha$ the firms receive $\pi_{i,c} = \pi_{j,c}$, so that proving that $(1-\alpha) \cdot \partial \pi_{i,c}/\partial \alpha + \alpha \cdot \partial \pi_{j,c}/\partial \alpha > 11$ (Lemma 5) requires that $\partial \pi_{i,c}/\partial \alpha > 0$. This inequality is shown in (B.12) where we made use of (B.13), which requires $Q = q_i + q_j$, $\partial p/\partial Q = \partial p/\partial q_i$, and $\partial q_i/\partial \alpha = \partial q_j/\partial \alpha$ because of symmetry ($q_i = q_j$):

$$\frac{\partial \pi_{i,c}}{\partial \alpha} = \frac{\partial p}{\partial \alpha} q_i + \frac{\partial \tilde{q}_i^R}{\partial \alpha} p = \frac{\partial \tilde{q}_i^R}{\partial \alpha} \left[2 \frac{\partial p}{\partial q_i} q_i + p \right]; \tag{B.12}$$

$$\frac{\partial p}{\partial \alpha} = \frac{\partial p}{\partial Q} \frac{\partial Q}{\partial \alpha} = \frac{\partial p}{\partial q_i} \left[\frac{\partial q_i}{\partial \alpha} + \frac{\partial q_j}{\partial \alpha} \right] = 2 \frac{\partial q_i}{\partial \alpha} \frac{\partial p}{\partial q_i}. \tag{B.13}$$

Under symmetry, first-order condition (A.2) simplifies to the first line in (B.14):

$$\begin{aligned} &\frac{\partial p}{\partial q_i} q_i + (1-\alpha) p = 0 \\ &\frac{1}{1-\alpha} \frac{\partial p}{\partial q_i} q_i + p = 0. \end{aligned} \tag{B.14}$$

Given $\partial p/\partial q_i < 0$ and $1/(1-\alpha) < 2$, this causes (B.15) where the term on the left-hand side was taken from (B.12):

$$2\frac{\partial p}{\partial q_i}q_i + p < \frac{1}{1-\alpha}\frac{\partial p}{\partial q_i}q_i + p = 0. \tag{B.15}$$

Combining inequality (B.15) with $\partial \bar{q}_i^R/\partial\alpha < 0$ from Lemma 1 yields $\partial\pi_{i,c}/\partial\alpha > 0$, which proves Lemma 5. □

Proof Proposition 2 suggests that $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$ if inequality (10) is met. Equation (B.16) provides that $\partial\tilde{\delta}_i^*/\partial\alpha_j$, where the expressions for $u(\alpha_i, \alpha_j)$ and $v(\alpha_i, \alpha_j)$ are the ones given in (B.3):

$$\begin{aligned}
\frac{\partial\tilde{\delta}_i^*}{\partial\alpha_j} &= \frac{\frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_j}\cdot v(\alpha_i,\alpha_j) - \frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_j}\cdot u(\alpha_i,\alpha_j)}{v(\alpha_i,\alpha_j)^2} \\
\text{with } \frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_j} &= \pi_{i,k} - \pi_{i,d} + (1-\alpha_j)\left(\frac{\partial\pi_{i,d}}{\partial\alpha_j} - \frac{\partial\pi_{i,k}}{\partial\alpha_j}\right) + \alpha_i\left(\frac{\partial\pi_{j,-d}}{\partial\alpha_j} - \frac{\partial\pi_{j,k}}{\partial\alpha_j}\right) \\
\text{and } \frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_j} &= \pi_{i,c} - \pi_{i,d} + (1-\alpha_j)\left(\frac{\partial\pi_{i,d}}{\partial\alpha_j} - \frac{\partial\pi_{i,c}}{\partial\alpha_j}\right) + \alpha_i\left(\frac{\partial\pi_{j,-d}}{\partial\alpha_j} - \frac{\partial\pi_{j,c}}{\partial\alpha_j}\right).
\end{aligned} \tag{B.16}$$

Using $\partial\pi_{i,k}/\partial\alpha_j = \partial\pi_{j,k}/\partial\alpha_j = 0$ and $(1-\alpha_j)(\partial\pi_{i,d}/\partial\alpha_j) + \alpha_i(\partial\pi_{j,-d}/\partial\alpha_j) = 0$ from Lemma 7, the partial derivatives can be simplified as is shown in (B.17):

$$\begin{aligned}
\frac{\partial u(\alpha_i,\alpha_j)}{\partial\alpha_j} &= \pi_{i,k} - \pi_{i,d} \\
\frac{\partial v(\alpha_i,\alpha_j)}{\partial\alpha_j} &= \pi_{i,c} - \pi_{i,d} - (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_j} - \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_j}.
\end{aligned} \tag{B.17}$$

Equation (B.16) can thus be simplified as is shown in equation (B.18):

$$\frac{\partial\tilde{\delta}_i^*}{\partial\alpha_j} = \frac{(\pi_{i,k} - \pi_{i,d})\cdot v(\alpha_i,\alpha_j) + \left(\pi_{i,c} - \pi_{i,d} - (1-\alpha_j)\frac{\partial\pi_{i,c}}{\partial\alpha_j} - \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_j}\right)\cdot u(\alpha_i,\alpha_j)}{v(\alpha_i,\alpha_j)^2}. \tag{B.18}$$

Condition $\partial\tilde{\delta}_i^*/\partial\alpha_j > 0$ can be solved to yield (10), which proves Proposition 2. □

Appendix to Section 5

Proof Proposition 3 suggests that $\partial\tilde{\delta}_i^*/\partial\alpha_i > 0$ if inequality (12) is satisfied. Given the detection probability ρ and fine F, firm i's profits can be written as in (C.1) when colluding and as in (C.2) when deviating:

$$\tilde{\pi}_{i,k} = (1-\alpha_j)(\pi_{i,k} - \rho F) + \alpha_i(\pi_{j,k} - \rho F); \tag{C.1}$$

$$\tilde{\pi}_{i,d} = (1-\alpha_j)(\pi_{i,d} - \rho F) + \alpha_i(\pi_{j,-d} - \rho F). \tag{C.2}$$

Plugging (C.1) and (C.2) in (7) and solving for δ_i yields the critical discount factor that is shown in (11). Performing the same calculations done for the proof of Proposition 1 yields inequality (12). It is straightforward to show that the left-hand side of (9) is bigger than the left-hand side of (12), which proves that (12) is easier to satisfy than (9). □

Appendix to Section 6

This section demonstrates that only the functional form but not the economic interpretation of our results changes if one assumes profit function (13) for cross ownership. In this case, the competitive, collusive, and deviations profits can be stated as in (D.1)–(D.3):

$$\hat{\pi}_{i,c} = \frac{\pi_{i,c} + \alpha_i \pi_{j,c}}{1 - \alpha_i \alpha_j}; \tag{D.1}$$

$$\hat{\pi}_{i,k} = \frac{(\pi_{i,k} - \rho F) + \alpha_i(\pi_{j,k} - \rho F)}{1 - \alpha_i \alpha_j}; \tag{D.2}$$

$$\hat{\pi}_{i,d} = \frac{(\pi_{i,d} - \rho F) + \alpha_i(\pi_{j,-d} - \rho F)}{1 - \alpha_i \alpha_j}. \tag{D.3}$$

Lemmas 1-3 were established for profit function (13) by Flath (1991), Flath (1992), and Farrell and Shapiro (1990). We do not repeat these proofs because they resemble the ones that were presented in "Appendix 1".

The analysis of $\partial \hat{\delta}_i^* / \partial \alpha_i$ relies on condition (D.4), which we will show to apply in Cournot competition, in Bertrand competition with differentiated products, and in Bertrand competition with homogeneous products:

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i \frac{\partial \pi_{j,-d}}{\partial \alpha_i} = 0. \tag{D.4}$$

In *Cournot competition with homogeneous products*, the profits of firm i can be written as in (D.5), and its first-order condition as in (D.6). Note that the profit (D.5) is expressed before the eventual payment of ρF, which constitutes a fixed cost and thus does not affect firm i's choice of its strategic variable. Equality (D.4) can be restated as in (D.7):

$$\hat{\pi}_i = \frac{1}{1 - \alpha_i \alpha_j} \left[p(q_i, q_j) q_i + \alpha_i p(q_i, q_j) q_j \right]; \tag{D.5}$$

$$\frac{\partial \hat{\pi}_i}{\partial q_i} = \frac{1}{1-\alpha_i\alpha_j}\left[\frac{\partial p}{\partial q_i}q_i + p(q_i,q_j) + \alpha_i\frac{\partial p}{\partial q_i}q_j\right] = 0; \tag{D.6}$$

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i\frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial q_i^R}{\partial \alpha_i}\left[\left(\frac{\partial p}{\partial q_i}q_i + p\right) + \alpha_i\left(\frac{\partial p}{\partial q_i}q_j\right)\right]. \tag{D.7}$$

The bracketed term on the right-hand side of equation (D.7) is the same as the bracketed term in first-order condition (D.6) that takes a value of zero in the optimum, which proves (D.4).

For *Bertrand competition with differentiated products*, the profit of firm i (before paying ρF) can be written as in (D.8), and its first-order condition as in (D.9). Equality (D.4) can be restated as in (D.10):

$$\hat{\pi}_i = \frac{1}{1-\alpha_i\alpha_j}\left[q_i(p_i,p_j)p_i + \alpha_i q_j(p_j,p_i)p_j\right]; \tag{D.8}$$

$$\frac{\partial \hat{\pi}_i}{\partial p_i} = \frac{1}{1-\alpha_i\alpha_j}\left[\frac{\partial q_i}{\partial p_i}p_i + q_i(p_i,p_j) + \alpha_i\frac{\partial q_j}{\partial p_i}p_j\right] = 0; \tag{D.9}$$

$$\frac{\partial \pi_{i,d}}{\partial \alpha_i} + \alpha_i\frac{\partial \pi_{j,-d}}{\partial \alpha_i} = \frac{\partial p_i^R}{\partial \alpha_i}\left[\left(\frac{\partial q_i}{\partial p_i}p_i + q_i\right) + \alpha_i\left(\frac{\partial q_j}{\partial p_i}q_j\right)\right] = 0. \tag{D.10}$$

The bracketed term on the right-hand side of equation (D.10) is the same as the bracketed term in first-order condition (D.9) that takes a value of zero in the optimum, which proves (D.4).

In *Bertrand competition with homogeneous products* both firms set prices equal to marginal cost regardless of the values of α_i, α_j, which causes $\partial\pi_{i,d}/\partial\alpha_i = 0$ and $\partial\pi_{j,-d}/\partial\alpha_i = 0$. This proves (D.4).

Turning to the finding of main interest, using profit functions (D.1)-(D.3), the critical discount factor $\hat{\delta}_i^*$ can be written as in (D.11):

$$\hat{\delta}_i^* \equiv \frac{(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k})}{(\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{j,-d} - \rho F - \pi_{j,c})}. \tag{D.11}$$

One finds $\partial\hat{\delta}^*/\partial\alpha_i > 0$ if inequality (D.12) is satisfied, which can be interpreted in the same way as inequalities (9) and (12) in the main text:

$$\frac{(\pi_{i,k} - \rho F - \pi_{i,c})(\pi_{i,d} - \pi_{j,-d})}{(\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k})} < \frac{\partial \pi_{i,c}}{\partial \alpha_i} + \alpha_i\frac{\partial \pi_{j,c}}{\partial \alpha_i}. \tag{D.12}$$

To determine $\partial\hat{\delta}_i^*/\partial\alpha_i$, re-write $\hat{\delta}_i^*$ as follows:

$$\begin{aligned} \hat{\delta}_i^* &= \frac{\hat{u}(\alpha_i, \alpha_j)}{\hat{v}(\alpha_i, \alpha_j)} \\ \text{with } \hat{u}(\alpha_i, \alpha_j) &= (\pi_{i,d} - \pi_{i,k}) + \alpha_i(\pi_{j,-d} - \pi_{j,k}) > 0 \\ \text{and } \hat{v}(\alpha_i, \alpha_j) &= (\pi_{i,d} - \rho F - \pi_{i,c}) + \alpha_i(\pi_{j,-d} - \rho F - \pi_{j,c}) > 0. \end{aligned} \tag{D.13}$$

With the use of $\partial\pi_{i,k}/\partial\alpha_i = \partial\pi_{j,k}/\partial\alpha_i = 0$ and (D.4), $\partial\delta_i^*/\partial\alpha_i$ can be written as in (D.14):

$$\begin{aligned} \frac{\partial\hat{\delta}_i^*}{\partial\alpha_i} &= \frac{\frac{\partial\hat{u}(\alpha_i,\alpha_j)}{\partial\alpha_i} \cdot \hat{v}(\alpha_i, \alpha_j) - \frac{\partial\hat{v}(\alpha_i,\alpha_j)}{\partial\alpha_i} \cdot \hat{u}(\alpha_i)}{\hat{v}(\alpha_i, \alpha_j)^2} \\ \text{with } \frac{\partial\hat{u}(\alpha_i, \alpha_j)}{\partial\alpha_i} &= \pi_{j,-d} - \pi_{j,k} \\ \text{and } \frac{\partial\hat{v}(\alpha_i, \alpha_j)}{\partial\alpha_i} &= \pi_{j,-d} - \rho F - \pi_{j,c} - \left(\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right). \end{aligned} \tag{D.14}$$

Given $\hat{v}(\alpha_i, \alpha_j) > 0$, the sign of $\partial\hat{\delta}^*/\partial\alpha_i$ is the same as that of its numerator as is shown in (D.15):

$$\begin{aligned} &\frac{\partial\hat{u}(\alpha_i, \alpha_j)}{\partial\alpha_i} \cdot \hat{v}(\alpha_i, \alpha_j) - \frac{\partial\hat{v}(\alpha_i, \alpha_j)}{\partial\alpha_i} \cdot \hat{u}(\alpha_i, \alpha_j) = \ldots \\ &\left(\pi_{i,k} - \rho F - \pi_{i,c}\right)\left(\pi_{j,-d} - \pi_{i,d}\right) + \left(\frac{\partial\pi_{i,c}}{\partial\alpha_i} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_i}\right)\hat{u}(\alpha_i, \alpha_j). \end{aligned} \tag{D.15}$$

This proves that $\partial\hat{\delta}^*/\partial\alpha_i$ is positive if inequality (D.12) applies.

The analysis of $\partial\hat{\delta}_i^*/\partial\alpha_j$ relies on condition (D.16):

$$\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = 0. \tag{D.16}$$

For *Cournot competition with homogeneous products*, (D.16) can be restated as in (D.17):

$$\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = \frac{\partial q_i}{\partial\alpha_j}\left[\left(\frac{\partial p}{\partial q_i}q_i + p\right) + \alpha_i\left(\frac{\partial p}{\partial q_i}q_j\right)\right] = 0. \tag{D.17}$$

The bracketed term on the right-hand side of equation (D.17) is the same as the bracketed term in first-order condition (D.6) that takes a value of zero in the optimum. For *Bertrand competition with differentiated products*, (D.16) can be restated as in (D.18):

$$\frac{\partial\pi_{i,d}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,-d}}{\partial\alpha_j} = \frac{\partial p_i}{\partial\alpha_j}\left[\left(\frac{\partial q_i}{\partial p_i}p_i + q_i\right) + \alpha_i\left(\frac{\partial q_j}{\partial p_i}q_j\right)\right] = 0. \tag{D.18}$$

The bracketed term on the right-hand side of equation (D.18) is the same as the bracketed term in first-order condition (D.9) that takes a value of zero in the optimum. The equalities $\partial\pi_{i,d}/\partial\alpha_j = 0$ and $\partial\pi_{j,-d}/\partial\alpha_j = 0$ for *Bertrand competition with homogeneous products* also proves (D.16).

With respect to the second main result, inequality $\partial\hat{\delta}_i^*/\partial\alpha_j > 0$ applies if inequality (D.19) is satisfied, whose interpretation is equivalent to that of (10) in the main text:

$$0 < \frac{\partial\pi_{i,c}}{\partial\alpha_j} + \alpha_i \frac{\partial\pi_{j,c}}{\partial\alpha_j}. \tag{D.19}$$

To prove that $\partial\hat{\delta}_i^*/\partial\alpha_j > 0$ applies under condition (D.19), consider that equation (D.20) provides that $\partial\hat{\delta}_i^*/\partial\alpha_j$ when assuming profit function (13) and defining $\hat{u}(\alpha_i, \alpha_j), \hat{v}(\alpha_i, \alpha_j)$ as in (D.13):

$$\begin{aligned}
\frac{\partial\hat{\delta}_i^*}{\partial\alpha_j} &= \frac{\frac{\partial\hat{u}(\alpha_i,\alpha_j)}{\partial\alpha_j} \cdot \hat{v}(\alpha_i,\alpha_j) - \frac{\partial\hat{v}(\alpha_i,\alpha_j)}{\partial\alpha_j} \cdot \hat{u}(\alpha_i,\alpha_j)}{\hat{v}(\alpha_i,\alpha_j)^2} \\
\text{with } \frac{\partial\hat{u}(\alpha_i,\alpha_j)}{\partial\alpha_j} &= \left(\frac{\partial\pi_{i,d}}{\partial\alpha_j} - \frac{\partial\pi_{i,k}}{\partial\alpha_j}\right) + \alpha_i\left(\frac{\partial\pi_{j,-d}}{\partial\alpha_j} - \frac{\partial\pi_{j,k}}{\partial\alpha_j}\right) \\
\text{and } \frac{\partial\hat{v}(\alpha_i,\alpha_j)}{\partial\alpha_j} &= \left(\frac{\partial\pi_{i,d}}{\partial\alpha_j} - \frac{\partial\pi_{i,c}}{\partial\alpha_j}\right) + \alpha_i\left(\frac{\partial\pi_{j,-d}}{\partial\alpha_j} - \frac{\partial\pi_{j,c}}{\partial\alpha_j}\right).
\end{aligned} \tag{D.20}$$

With the use of $\partial\pi_{i,k}/\partial\alpha_j = \partial\pi_{j,k}/\partial\alpha_j = 0$ and (D.16), equation (D.20) can be simplified as is shown in equation (D.21):

$$\frac{\partial\hat{\delta}_i^*}{\partial\alpha_j} = \frac{\left(\frac{\partial\pi_{i,c}}{\partial\alpha_j} + \alpha_i\frac{\partial\pi_{j,c}}{\partial\alpha_j}\right) \cdot \hat{u}(\alpha_i,\alpha_j)}{\hat{v}(\alpha_j)^2}. \tag{D.21}$$

Given $\hat{u}(\alpha_i, \alpha_j) > 0$ and $\hat{v}(\alpha_i, \alpha_j) > 0$, the inequality $\partial\hat{\delta}_i^*/\partial\alpha_j > 0$ applies if inequality (D.19) is satisfied.

As in the case of the common ownership profit function (1), it can be shown for the cross ownership profit function (13) that $\hat{\delta}_i^*$ can be raised more easily by an increase in α_j than by an increase in α_i: Inequality (D.19) is more easily satisfied than is (D.12). The left-hand side of (D.12) is positive and, thus, greater than the left-hand side of (D.19). Assuming symmetry ($\alpha_i = \alpha_j$) before an asymmetric expansion of α_i or α_j takes place causes $\partial\pi_{i,c}/\partial\alpha_i = \partial\pi_{j,c}/\partial\alpha_j$ and $\partial\pi_{j,c}/\partial\alpha_i = \partial\pi_{i,c}/\partial\alpha_j$. Using this in combination with Lemmas 1 and 3, one can show that the right-hand side of (D.19) is weakly greater than that of (D.12). Both observations ensure that inequality (D.19) is more easily satisfied than (D.12).

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